Chapter 4

Alternating Current Circuits
Chapter 4:

4-1 AC Sources
4.2 Resistors in an AC Circuit
4.3 Inductors in an AC Circuit
4.4 Capacitors in an AC Circuit
4.5 The RLC Series Circuit
4.6 Power in an AC Circuit
4.7 Resonance in a Series RLC Circuit
4.8 The Transformer and Power Transmission
4.9 Rectifiers and Filters
Objecties: The students should be able to:

- Describe the sinusoidal variation in ac current and voltage, and calculate their effective values.

- Write and apply equations for calculating the inductive and capacitive reactance for inductors and capacitors in an ac circuit.

- Describe, with diagrams and equations, the phase relationships for circuits containing resistance, capacitance, and inductance.
Write and apply equations for calculating the impedance, the phase angle, the effective current, the average power, and the resonant frequency for a series ac circuit.

Describe the basic operation of a step up and a step-down transformer.

Write and apply the transformer equation and determine the efficiency of a transformer.
4-1 AC Circuits

- An AC circuit consists of a combination of circuit elements and a power source.
- The power source provides an alternative voltage, $\Delta v$.
- Notation Note
  - Lower case symbols will indicate instantaneous values.
  - Capital letters will indicate fixed values.
AC Voltage

- The output of an AC power source is sinusoidal and varies with time according to the following equation:
  \[ \Delta v = \Delta V_{\text{max}} \sin \omega t \]
  - \( \Delta v \) is the instantaneous voltage
  - \( \Delta V_{\text{max}} \) is the maximum output voltage of the source
    - Also called the voltage amplitude
  - \( \omega \) is the angular frequency of the AC voltage
AC Voltage, cont.

- The angular frequency is
  \[ \omega = 2\pi f = \frac{2\pi}{T} \]
  - \( f \) is the frequency of the source
  - \( T \) is the period of the source
- The voltage is positive during one half of the cycle and negative during the other half
AC Voltage, final

- The current in any circuit driven by an AC source is an alternating current that varies sinusoidally with time.
- Commercial electric power plants in the US use a frequency of 60 Hz.
  - This corresponds with an angular frequency of 377 rad/s.
4-2 Resistors in an AC Circuit

- Consider a circuit consisting of an AC source and a resistor
- The AC source is symbolized by
- $\Delta v_R = \Delta V_{max} = V_{max} \sin \omega t$
- $\Delta v_R$ is the instantaneous voltage across the resistor
The instantaneous current in the resistor is

\[ i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{\text{max}}}{R} \sin \omega t = I_{\text{max}} \sin \omega t \]

The instantaneous voltage across the resistor is also given as

\[ \Delta v_R = I_{\text{max}} R \sin \omega t \]
Resistors in an AC Circuit, 3

- The graph shows the current through and the voltage across the resistor.
- The current and the voltage reach their maximum values at the same time.
- The current and the voltage are said to be in phase.
The current and vary identically with time, because $i_R$ and $\Delta v_R$ both vary as $\sin \omega t$ and reach their maximum values at the same time, as shown in Figure, they are said to be in phase.

- The phasor diagram for the resistive circuit showing that the current is in phase with the voltage.
Resistors in an AC Circuit, 4

- For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor
- The direction of the current has no effect on the behavior of the resistor
- Resistors behave essentially the same way in both DC and AC circuits
Phasor Diagram

- To simplify the analysis of AC circuits, a graphical constructor called a *phasor diagram* can be used.
- A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents.
Phasors, cont.

- The vector rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable.
- The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.
rms Current and Voltage

- The average current in one cycle is zero
- The rms current is the average of importance in an AC circuit
  - rms stands for root mean square
    - \( I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max} \)
- Alternating voltages can also be discussed in terms of rms values
  - \( \Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707 \Delta V_{max} \)
Power

- The rate at which electrical energy is dissipated in the circuit is given by
  \[ P = i^2 R \]
  - \( i \) is the *instantaneous current*

- The heating effect produced by an AC current with a maximum value of \( I_{\text{max}} \) is not the same as that of a DC current of the same value

- The maximum current occurs for a small amount of time
Power, cont.

- The average power delivered to a resistor that carries an alternating current is

\[ P_{av} = I_{rms}^2 R \]
Notes About rms Values

- rms values are used when discussing alternating currents and voltages because

- AC ammeters and voltmeters are designed to read rms values

- Many of the equations that will be used have the same form as their DC counterparts
Example 1:

- The voltage output of an AC source is given by the expression
  \[ \Delta v = (200 \text{ V}) \sin \omega t. \]

Find the rms current in the circuit when this source is connected to a 100 Ohm resistor.

Solution:

Comparing this expression for voltage output with the general form
\[ \Delta v = \Delta V_{\text{max}} \sin \omega t, \]
we see that
\[ \Delta V_{\text{max}} = 200 \text{ V}. \]
Thus, the rms voltage is

\[ \Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = \frac{200 \text{ V}}{\sqrt{2}} = 141 \text{ V} \]

\[ I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = \frac{141 \text{ V}}{100 \Omega} = 1.41 \text{ A} \]
4-3 Inductors in an AC Circuit

Kirchhoff’s loop rule can be applied and gives:

\[ \Delta v + \Delta v_L = 0, \text{ or} \]
\[ \Delta v - L \frac{di}{dt} = 0 \]
\[ \Delta v = L \frac{di}{dt} = \Delta V_{\text{max}} \sin \omega t \]
Current in an Inductor

- The equation obtained from Kirchhoff's loop rule can be solved for the current

\[ i_L = \frac{\Delta V_{\text{max}}}{L} \int \sin \omega t \, dt = -\frac{\Delta V_{\text{max}}}{\omega L} \cos \omega t \]

\[ i_L = \frac{\Delta V_{\text{max}}}{\omega L} \sin \left( \omega t - \frac{\pi}{2} \right) \]

\[ I_{\text{max}} = \frac{\Delta V_{\text{max}}}{\omega L} \]

- This shows that the instantaneous current \( i_L \) in the inductor and the instantaneous voltage \( \Delta v_L \) across the inductor are out of phase by \((\pi/2)\) rad = 90°
Phase Relationship of Inductors in an AC Circuit

- The current is a maximum when the voltage across the inductor is zero
  - The current is momentarily not changing
- For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by $90^\circ$ ($\pi/2$)
Phasor Diagram for an Inductor

- The phasors are at 90° with respect to each other.
- This represents the phase difference between the current and voltage.
- Specifically, the current lags behind the voltage by 90°.
Inductive Reactance

- The factor $\omega L$ has the same units as resistance and is related to current and voltage in the same way as resistance.
- Because $\omega L$ depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies.
- The factor is the **inductive reactance** and is given by:
  - $X_L = \omega L$
Inductive Reactance, cont.

- Current can be expressed in terms of the inductive reactance
  \[ I_{\text{max}} = \frac{\Delta V_{\text{max}}}{X_L} \quad \text{or} \quad I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} \]

- As the frequency increases, the inductive reactance increases
  - This is consistent with Faraday’s Law:
    - The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the reactance and a decrease in the current
Voltage Across the Inductor

- The instantaneous voltage across the inductor is

\[ \Delta v_L = -L \frac{di}{dt} \]

\[ = -\Delta V_{\text{max}} \sin \omega t \]

\[ = -I_{\text{max}} X_L \sin \omega t \]
Example 33.2  **A Purely Inductive AC Circuit**

In a purely inductive AC circuit, $L = 25.0 \, mH$ and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

\[
X_L = \omega L = 2\pi f L = 2\pi (60.0 \, Hz) (25.0 \times 10^{-3} \, H) = 9.42 \, \Omega
\]

\[
I_{\text{rms}} = \frac{\Delta V_{L,\text{rms}}}{X_L} = \frac{150 \, V}{9.42 \, \Omega} = 15.9 \, A
\]
What If?  What if the frequency increases to 6.00 kHz? What happens to the rms current in the circuit?

Answer  If the frequency increases, the inductive reactance increases because the current is changing at a higher rate. The increase in inductive reactance results in a lower current.

Let us calculate the new inductive reactance:

\[ X_L = 2\pi(6.00 \times 10^3 \text{ Hz})(25.0 \times 10^{-3} \text{ H}) = 942 \, \Omega \]

The new current is

\[ I_{\text{rms}} = \frac{150 \, \text{V}}{942 \, \Omega} = 0.159 \, \text{A} \]
4-4 Capacitors in an AC Circuit

- The circuit contains a capacitor and an AC source

- Kirchhoff’s loop rule gives:
  \[ \Delta v + \Delta v_C = 0 \text{ and so} \]
  \[ \Delta v = \Delta v_C = \Delta V_{\text{max}} \sin \omega t \]

- \( \Delta v_C \) is the instantaneous voltage across the capacitor
Capacitors in an AC Circuit, cont.

- The charge is \( q = C\Delta V_{\text{max}} \sin \omega t \)
- The instantaneous current is given by

\[
i_C = \frac{dq}{dt} = \omega C\Delta V_{\text{max}} \cos \omega t
\]

or \( i_C = \omega C\Delta V_{\text{max}} \sin \left( \omega t + \frac{\pi}{2} \right) \)

- The current is \( \pi/2 \) rad = 90° out of phase with the voltage
More About Capacitors in an AC Circuit

- The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.

- The current leads the voltage by 90°.
Phasor Diagram for Capacitor

- The phasor diagram shows that for a sinusoidally applied voltage, the current always leads the voltage across a capacitor by 90°.
Capacitive Reactance

- The maximum current in the circuit occurs at \( \cos \omega t = 1 \) which gives

\[
I_{\text{max}} = \omega C \Delta V_{\text{max}} = \frac{\Delta V_{\text{max}}}{(1/\omega C)}
\]

- The **impeding effect** of a capacitor on the current in an AC circuit is called the **capacitive reactance** and is given by

\[
X_C \equiv \frac{1}{\omega C} \quad \text{which gives} \quad I_{\text{max}} = \frac{\Delta V_{\text{max}}}{X_C}
\]
Voltage Across a Capacitor

- The instantaneous voltage across the capacitor can be written as $\Delta v_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t$
- As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases.
- As the frequency approaches zero, $X_C$ approaches infinity and the current approaches zero.
  - This would act like a DC voltage and the capacitor would act as an open circuit.
Example 3 A Purely Capacitive AC Circuit

An 8.00-μF capacitor is connected to the terminals of a 60.0-Hz AC source whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit.

\[ \omega = 2\pi f = 377 \text{ s}^{-1} \]

\[
X_C = \frac{1}{\omega C} = \frac{1}{(377 \text{ s}^{-1})(8.00 \times 10^{-6} \text{ F})} = 332 \text{ Ω}
\]

\[
I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{150 \text{ V}}{332 \text{ Ω}} = 0.452 \text{ A}
\]
Thank you!