Nonlinear Quintic Schrodinger Equations with Complex Initial Conditions, Limited Time Response

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Abstract: **In this paper, a perturbing nonlinear quintic Schrodinger equation is studied under limited time interval, complex initial conditions and zero Neumann conditions. The perturbation and Picard approximation methods together with the eigenfunction expansion and variational of parameters methods are used to introduce an approximate solution for the perturbative nonlinear case for which a power series solution is proved to exist. Using Mathematica, the solution algorithm is tested through computing the possible orders of approximations. The method of solution is illustrated through case studies and figures.**

Keywords: **Nonlinear quitntic Schrodinger equation, Perturbation, Eigenfunction expansion, Mathematica, Picard Approximatio**n.

I. INTRODUCTION

The nonlinear Schrodinger equation (NLS) is the principal equation to be analyzed and solved in many fields, see [1-5] for examples. The NLS equation arises in many application areas [6-8] such as wave propagation in nonlinear media, surface wave in sufficiently deep waters and signal propagation in optical fibers. The NLS are one of the most important models of mathematical physics arising in a great array of contexts [9, 10] as for conductor electronics, optics in nonlinear media, photonics, plasmas, fundamentation of quantum mechanics, dynamics of accelerators, mean-field theory of Bose-Einstein condensates or in biomolecule dynamics. It was also the second nonlinear partial differential equation (PDE) whose initial value problem was discovered to be solvable via the inverse scattering transform (IST) method. In the last ten decades, there are lot of NLS problems depending on additive or multiplicative noise in the random case [11, 12] or lot of solution methodologies in the deterministic case. Wang M. and et al [13] obtained the exact solutions to NLS using what they called the sub-equation method. They got four kinds of exact solutions of the equation for which no sign to the initial or boundary conditions type is made. Xu L. and Zhang J. [14] followed the same Previous technique in solving the higher order NLS. Sweilam N. [15] solved a nonlinear cubic Schrodinger equation which gives rise to solitary solutions using variational iteration method. Zhu S. [16] used the extended hyperbolic auxiliary equation method in getting the exact explicit solutions to the higher order NLS without any conditions. Sun J. and et al [17] solved an NLS with an initial condition using Lie group method. By using coupled amplitude phase formulation, Parsezian K. and Kalithasan B. [18] constructed the quartic anharmonic oscillator equation from the coupled higher order NLS. Two-dimensional grey solitons to the NLS were numerically analyzed by Sakaguchi H. and Higashiuchi T. [19]. The generalized derivative NLS was studied by Huang D. and et al [20] introducing a new auxiliary equation expansion method.

In this paper, a straight forward solution algorithm is introduced using the transformation from a complex solution to coupled equations in two real solutions, eliminating one of the solutions to get separate independent and higher order equations, and finally introducing a perturbative approximate solution to the system.

II. THE GENERAL LINEAR CASE

Consider the linear Schrodinger equation:

$$i\frac{\partial u(t,z)}{\partial z}+\alpha\,\frac{\partial^2 u(t,z)}{\partial t^2}=0,(t,z)\in(0,T)\;x\;(0,\infty) \quad (1)$$

where $u(t,z)$ is a complex valued function which is subjected to:

$$I.C.: u(t,0)=f_1(t)+\,i\,f_2(t) \quad (2)$$

$$B.Cs.: u(0,z)=\,u(T,z)=0. \quad (3)$$

Let $u(t,z)=\,\psi(t,z)+\,i\,\phi(t,z),\ \psi,\phi$ are real valued functions and following the same analysis illustrated in [21], we can find that:

$$\psi(t,z)=\,\sum_{n=0}^{\infty}T_n(z)\sin\,(\frac{n\,\pi}{T})t, \quad (4)$$

$$\phi(t,z)=\sum_{n=0}^{\infty}\tau_n(z)\sin\,(\frac{n\,\pi}{T})t, \quad (5)$$

where $T_n(z)$ and $\tau_n(z)$ can be got through the applications of initial conditions and then solving the resultant second order differential equations using the method of Variational of Parameters [22]. The final expressions can be got as the following:

$$T_n(z)=\big(C_1+\,A_1(z)\big)\sin\beta_n z+\big(C_2+\,B_1(z)\big)\cos\beta_n z\ , \quad (6)$$

$$\tau_n(z) = \left(C_3 + A_2(z)\right)\sin\beta_n z + \left(C_4 + B_2(z)\right)\cos\beta_n z \qquad (7)$$

where β_n, $A_1(z)$, $B_1(z)$, $A_2(z)$, $B_2(z)$, C_2 and C_4 can be expressed as illustrated in [22]. Finally the following solution is obtained:

$$|u(t,z)|^2 = \psi^2(t,z) + \phi^2(t,z). \qquad (8)$$

III. THE NON-LINEAR CASE

Consider the quintic non-linear Schrodinger equation:

$$i\frac{\partial u(t,z)}{\partial z} + \alpha\frac{\partial^2 u(t,z)}{\partial t^2} + \varepsilon|u(t,z)|^4 u(t,z) + i\,\gamma u(t,z) = 0, \quad (t,z) \in (0,T)\; x\; (0,\infty) \qquad (9)$$

Where $u(t,z)$ is a complex valued function which is subjected to the initial and boundary conditions mentioned before in equations (2) and (3) respectively.

Lemma [23, 24]

The solution of equation (9) with the constraints (2), (3) is a power series in ε if exist.

Proof: at $\varepsilon = 0$, the following linear homogenous equation is got:

$$i\,\frac{\partial u_0(t,z)}{\partial z} + \alpha\,\frac{\partial^2 u_0(t,z)}{\partial t^2} + i\,\gamma u_0(t,z) = 0, \quad (t,z) \in (0,T)\; x\; (0,\infty) \qquad (10)$$

$$u_0(t,z) = \psi_0(t,z) + i\,\phi_0(t,z) \qquad (11)$$

where,

$$\psi_0(t,z) = e^{-\gamma z}\sum_{n=0}^{\infty} T_{0n}(z)\sin\left(\frac{n\,\pi}{T}\right)t, \qquad (12)$$

$$\phi_0(t,z) = e^{-\gamma z}\sum_{n=0}^{\infty} \tau_{0n}(z)\sin\left(\frac{n\,\pi}{T}\right)t \qquad (13)$$

where $T_{0n}(z)$ and $\tau_{0n}(z)$ can be calculated as the linear case equations (6), (7) respectively.

Following Pickard approximation equation (1) can be rewritten as:

$$i\frac{\partial u_n(t,z)}{\partial z} + \alpha\frac{\partial^2 u_n(t,z)}{\partial t^2} + i\,\gamma u_n(t,z) = -\varepsilon|u_{n-1}(t,z)|^4 u_{n-1}(t,z), \qquad n \geq 1 \qquad (14)$$

at $n = 1$, the iterative equation takes the form

$$i\frac{\partial u_1(t,z)}{\partial z} + \alpha\frac{\partial^2 u_1(t,z)}{\partial t^2} + i\,\gamma u_1(t,z) = -\varepsilon|u_0(t,z)|^4 u_0(t,z) = \varepsilon k_1(t,z) \qquad (15)$$

which can be solved as a linear case with zero initial and boundary conditions. The following general solution can be obtained:

$$\psi_1(t,z) = e^{-\gamma z}\sum_{n=0}^{\infty} T_{1n}(z)(z)\sin\left(\frac{n\,\pi}{T}\right)t, \qquad (16)$$

$$\phi_1(t,z) = e^{-\gamma z}\sum_{n=0}^{\infty} \tau_{1n}(z)(z)\sin\left(\frac{n\,\pi}{T}\right)t, \qquad (17)$$

$$u_1(t,z) = u_1{}^{(0)} + \varepsilon\, u_1{}^{(1)} \qquad (18)$$

at $n = 2$, the following equation is obtained:

$$i\frac{\partial u_2(t,z)}{\partial z} + \alpha\frac{\partial^2 u_2(t,z)}{\partial t^2} + i\,\gamma u_2(t,z) = -\varepsilon|u_1(t,z)|^4 u_1(t,z) = \varepsilon k_2(t,z) \qquad (19)$$

which can be solved as a linear case with zero initial and boundary conditions. The following general solution can be obtained:

$$u_2(t,z) = u_2{}^{(0)} + \varepsilon\, u_2{}^{(1)} + \varepsilon^2 u_2{}^{(2)}, \qquad (20)$$

Continuing like this, one can get:

$$u_n(t,z) = u_n{}^{(0)} + \varepsilon\, u_n{}^{(1)} + \varepsilon^2 u_n{}^{(2)} + \varepsilon^3\, u_n{}^{(3)} + .. + \varepsilon^n\, u_n{}^{(n)} \qquad (21)$$

As $n \to \infty$, the solution (if exists) can be reached as $u(t,z) = \lim_{n\to\infty} u_n(t,\ z)$. Accordingly the solution is a power series in ε.

According to the previous lemma, one can assume the solution of equation (1) as the following:

$$u_n(t,z) = \sum_{i=0}^{n} \varepsilon\, u_n{}^{(i)} \quad ,i = 0,1,2,\ldots,n \qquad (22)$$

Let $u(t,z) = \psi(t,z) + i\,\phi(t,\ z), \psi, \phi$: are real valued functions. The following coupled equations are got:

$$\frac{\partial\phi(t,z)}{\partial z} = \alpha\,\frac{\partial^2\psi(t,z)}{\partial t^2} + \varepsilon(\psi^2 + \phi^2)^2\psi - \gamma\phi, \qquad (23)$$

$$\frac{\partial\psi(t,z)}{\partial z} = -\,\alpha\,\frac{\partial^2\phi(t,z)}{\partial t^2} - \varepsilon(\psi^2 + \phi^2)^2 - \gamma\psi, \qquad (24)$$

where $\psi(t,0) = f_1(t)$, $\phi(t,0) = f_2(t)$, and all corresponding other I.C. and B.C. are zeros. As a perturbation solution for second order approximation, one can assume that:

$$\psi(t,z\ ) = \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2 \qquad (25)$$

$$\phi(t\,,z\,) = \phi_0 + \varepsilon\phi_1 + \varepsilon^2\phi_2 \qquad (26)$$

Where $\psi_0(t,0) = f_1(t)$, $\phi_0(t,0) = f_2(t)$, and all corresponding other I.C. and B.Cs. are zeros.

Substituting equations (25) and (26) into equations (23) and (24) and then equating the equal powers of ε, one can get the following set of coupled equations:

$$\frac{\partial\phi_0(t,z)}{\partial z} = \alpha\,\frac{\partial^2\psi_0(t,z)}{\partial t^2} - \gamma\phi_0, \qquad (27)$$

$$\frac{\partial\psi_0(t,z)}{\partial z} = -\,\alpha\,\frac{\partial^2\phi_0(t,z)}{\partial t^2} - \gamma\psi_0 \qquad (28)$$

$$\frac{\partial\phi_1(t,z)}{\partial z} = \alpha\,\frac{\partial^2\psi_1(t,z)}{\partial t^2} - \gamma\phi_1 + ({\psi_0}^2 + {\phi_0}^2)^2\,\psi_0, \qquad (29)$$

$$\frac{\partial\psi_1(t,z)}{\partial z} = -\alpha\,\frac{\partial^2\phi_1(t,z)}{\partial t^2} - \gamma\psi_1 - ({\phi_0}^2 + {\psi_0}^2)^2\,\phi_0, \qquad (30)$$

$$\frac{\partial\phi_2(t,z)}{\partial z} = \alpha\,\frac{\partial^2\psi_2(t,z)}{\partial t^2} - \gamma\phi_2 + \left({\phi_0}^2 + {\psi_0}^2\right)^2\psi_1 + (4\,{\psi_0}^3\psi_1 + 4\,{\phi_0}^3\phi_1 + 4\psi_0\psi_1{\phi_0}^2 + \;4{\psi_0}^2\phi_0\phi_1), \qquad (31)$$

$$\frac{\partial\psi_2(t,z)}{\partial z} = -\,\alpha\,\frac{\partial^2\phi_2(t,z)}{\partial t^2} - \gamma\psi_2 - \left({\phi_0}^2 + {\psi_0}^2\right)^2\phi_1 - (4\,{\psi_0}^3\psi_1 + 4\,{\phi_0}^3\phi_1 + 4\psi_0\psi_1{\phi_0}^2 + \;4{\psi_0}^2\phi_0\phi_1)\phi_0, \qquad (32)$$

and so on. The prototype equations to be solved are:

$$\frac{\partial\phi_i(t,z)}{\partial z} = \alpha\,\frac{\partial^2\psi_i(t,z)}{\partial t^2} + {G_i}^{(1)}, \qquad i \geq 1 \qquad (33)$$

$$\frac{\partial\psi_i(t,z)}{\partial z} = \alpha\,\frac{\partial^2\phi_i(t,z)}{\partial t^2} + {G_i}^{(2)}, \qquad i \geq 1 \qquad (34)$$

where $\psi_i(t,0) = \delta_{i,0}f_1(t)$, $\phi_i(t,0) = \delta_{i,0}f_2(t)$, and all other all corresponding conditions are zeros. ${G_i}^{(1)}, {G_i}^{(2)}$ are functions to be computed from previous steps.

Following the solution algorithm described in the previous section for the linear case, the following final results are obtained.

1. The absolute value of the zero order approximation is:

$$|u_0(t\,,z)|^2 = {\psi_0}^2 + \;{\phi_0}^2 \qquad (35)$$

2. The absolute value of the first order approximation is:

$$|u_1(t\,,z)|^2 = |u_0(t\,,z)|^2 + 2\varepsilon(\psi_0\psi_1 + \phi_0\phi_1) + \varepsilon^2\left({\psi_1}^2 + \;{\phi_1}^2\right) \qquad (37)$$

3. The absolute value of the second order approximation is:

$$\left|u^{(2)}(t\,,z)\right|^2 = \left|u^{(1)}(t\,,z)\right|^2 + 2\varepsilon^2(\psi_0\psi_2 + \phi_0\phi_2) + \varepsilon^3(\psi_1\psi_2 + \phi_1\phi_2) + \varepsilon^4\left({\psi_2}^2 + {\phi_2}^2\right) \qquad (39)$$

IV. CASE STUDIES

To examine the proposed solution algorithm, some case studies are illustrated.

IV.1. Case study 1:

Taking the case $f_1(t) = \rho_1$, $f_2(t) = \rho_1$ and following the algorithm, the following selective results for the first and second order approximations, (Fig.1 – Fig. 7), are got:

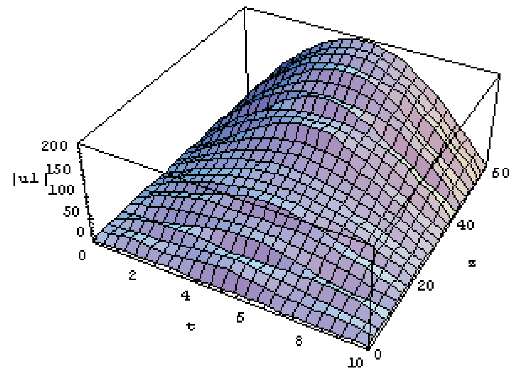


Fig.1 the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 1$, $\gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only ten terms on the series (M=10).

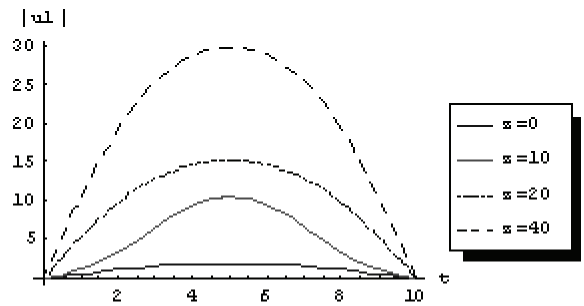


Fig.2 the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 0.2, \gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, M = 10$ for different values of z.

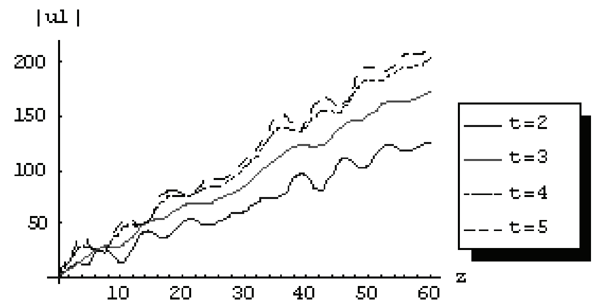


Fig.3 the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 1, \gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, M = 10$ for different values of t.

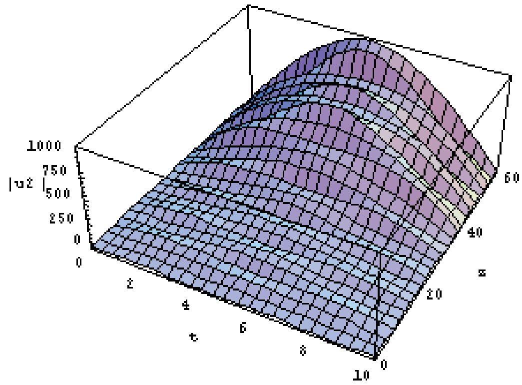


Fig.4 the second order approximation of $\left|u^{(2)}\right|$ at $\varepsilon = 0.2$, $\gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only ten term on the series (M=10).

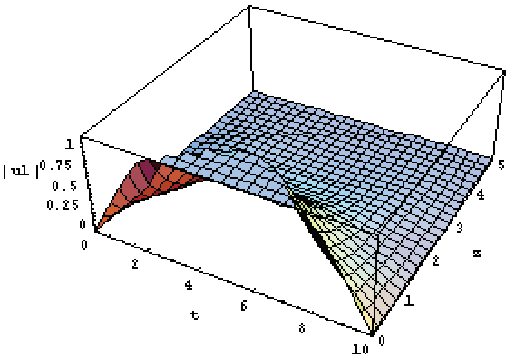


Fig. 5 the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 1$, $\gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only ten terms on the series (M=10).

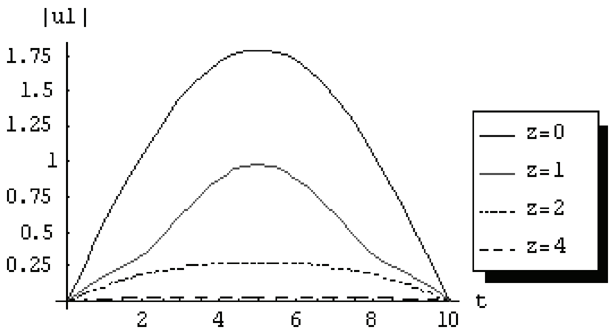


Fig. 6 the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 0.2, \gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, M = 10$ for different values of z.

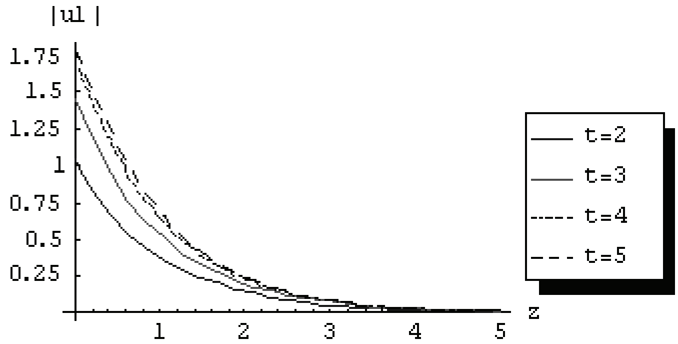


Fig.7 the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 0.2, \gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, M = 10$ for different values of t.

We notice that, in case of absence of gamma term ($\gamma = 0$) the absolute value of u increases with the increase of z which can be considered a case of instability. While, we can notice the tremendous effect of the presence of gamma term ($\gamma = 1$) on the stability of the solution, even for high values of epsilon (ε).

IV.2. Case study 2:

Taking the case $f_1(t) = \rho_1$, $f_2(t) = \rho_2 \sin\left(\frac{m\,\pi}{T}\right) t$ and following the algorithm, the following selective results for the first and second order approximations, (Fig. 8 – Fig. 10), are got:

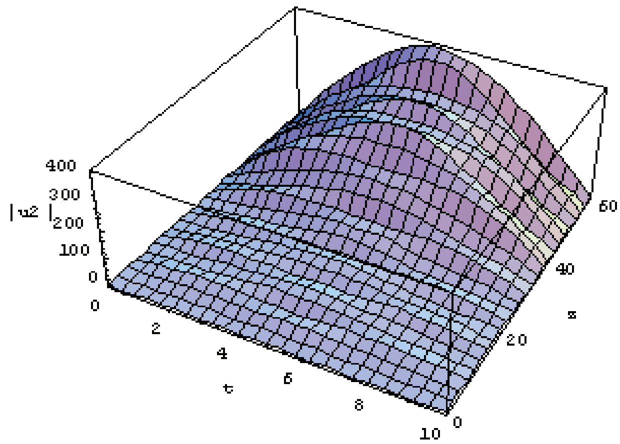


Fig. 8 the second order approximation of $\left|u^{(2)}\right|$ at $\varepsilon = 0.2$, $\gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only ten terms on the series (M=10).

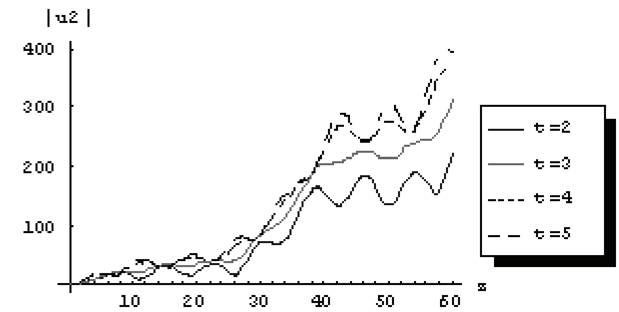


Fig. 9 the second order approximation of $|u^{(2)}|$ at $\varepsilon = 0.2, \gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, M = 10$ for different values of t.

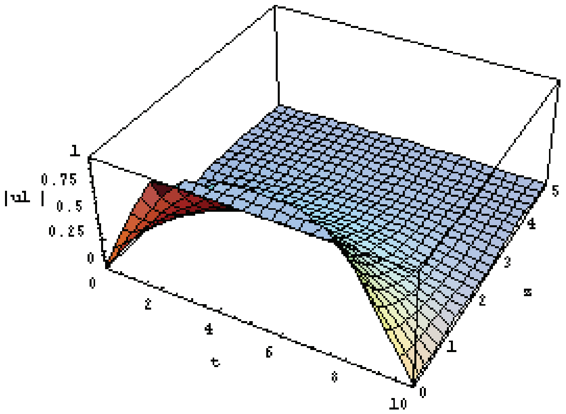


Fig. 10 the first order approximation of $|u^{(1)}|$ at $\varepsilon = 0.2$, $\gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only ten terms on the series (M=10).

We still notice the instability of the solution with absence of gamma ($\gamma = 0$).

V. PICARD APPROXIMATION

We are here introducing another method to solve equation (9) subject to the same initial and boundary conditions, equations (2) and (3) respectively.

Consider the quintic non-linear Schrodinger equation (9) and following Picard Approximation [25, 26]; we can rewrite equation (9) as:

$$i\frac{\partial u_n(t,z)}{\partial z} + \alpha\frac{\partial^2 u_n(t,z)}{\partial t^2} + i\,\gamma u_n(t,z) = -\varepsilon|u_{n-1}(t,z)|^4 u_{n-1}(t,z), n \geq 1 \qquad (40)$$

Let $u(t,z) = \psi(t,z) + i\,\phi(t,z), \psi, \phi$: are real valued functions. The following coupled equations are got:

$$\frac{\partial\phi(t,z)}{\partial z} = \alpha\frac{\partial^2\psi(t,z)}{\partial t^2} + \varepsilon(\psi^2 + \phi^2)^2\psi - \gamma\phi, \qquad (41)$$

$$\frac{\partial\psi(t,z)}{\partial z} = -\alpha\frac{\partial^2\phi(t,z)}{\partial t^2} - \varepsilon(\psi^2 + \phi^2)^2\phi - \gamma\phi, (42)$$

Where $\psi(t,0) = f_1(t)$, $\phi(t,0) = f_2(t)$, and all corresponding other I.C. and B.Cs. are zeros.

$$\frac{\partial\phi_i(t,z)}{\partial z} = \alpha\frac{\partial^2\psi_i(t,z)}{\partial t^2} + G_i^{(1)}, \qquad i \geq 1 \qquad (43)$$

$$\frac{\partial\psi_i(t,z)}{\partial z} = \alpha\frac{\partial^2\phi_i(t,z)}{\partial t^2} + G_i^{(2)}, \qquad i \geq 1 \qquad (44)$$

where $\psi_i(t,0) = \delta_{i,0}f_1(t)$, $\phi_i(t,0) = \delta_{i,0}f_2(t)$, and all other corresponding conditions are zeros. $G_i^{(1)}, G_i^{(2)}$ are functions to be computed from previous steps. Computing some iterations, the following order of approximations are obtained.

$$u_i(t.z) = \psi_i(t,z) + \phi_i(t,z), \quad i = 0,1,2,.... \qquad (45)$$

where $\psi_i(t,z)$ and $\phi_i(t,z)$ can be calculated using linear case algorithm.

VI. CASE STUDIES, PICARD

To examine the proposed solution algorithm, some case studies are illustrated.

VI.1. Case study 1:

Taking the case $f_1(t) = \rho_1$, $f_2(t) = \rho_2$ and following the algorithm of Picard Approximation, the following selective results for the first and second order approximations, (Fig.11 – Fig.14) are got:

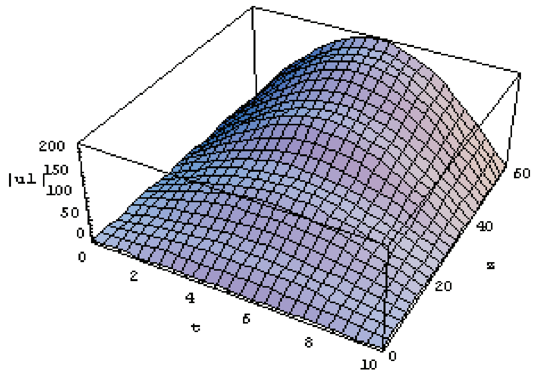
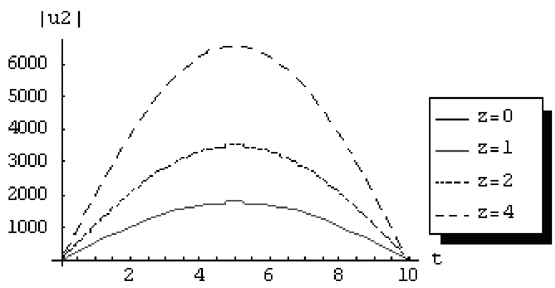


Fig. 11 the first order approximation of $|u^{(1)}|$ at $\varepsilon = 1$,



$\gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only one term on the series (M=1).

Fig. 12 the second order approximation of $|u^{(2)}|$ at $\varepsilon = 0.2, \gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, M = 1$ for different values of z.

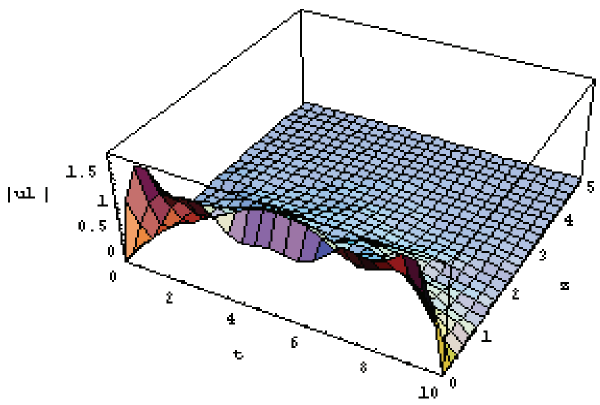


Fig. 13 the first order approximation of $|u^{(1)}|$ at $\varepsilon = 1$, $\gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only ten terms on the series (M=10).

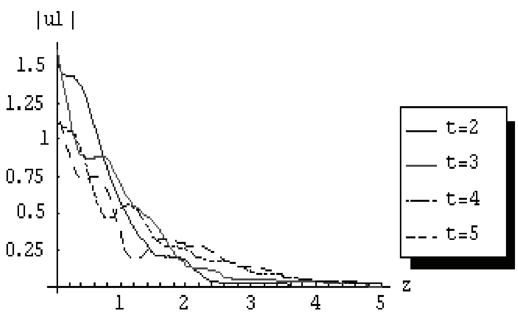


Fig. 14 the first order approximation of $|u^{(1)}|$ at $\varepsilon = 0.2, \gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, M = 10$ for different values of t.

We notice that, in case of absence of gamma term ($\gamma = 0$) the absolute value of u increases with the increase of z which can be considered a case of instability. While, we can notice the tremendous effect of the presence of gamma term ($\gamma = 1$) on the stability of the solution, even for high values of epsilon (ε).

IV.2. Case study 2:

Taking the case $f_1(t) = \rho_1$, $f_2(t) = \rho_1 \sin\left(\frac{m\pi}{T}\right) t$ and following the algorithm of Picard Approximation, the following selective results for the first and second order approximations, (Fig. 15 – Fig. 17) are got:

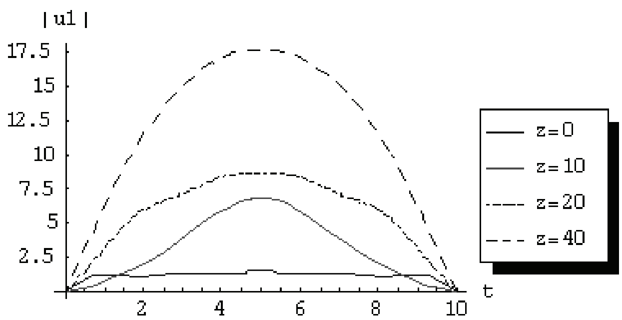


Fig. 15 the first order approximation of $|u^{(1)}|$ at $\varepsilon = 0.2, \gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, M = 10$ for different values of z.

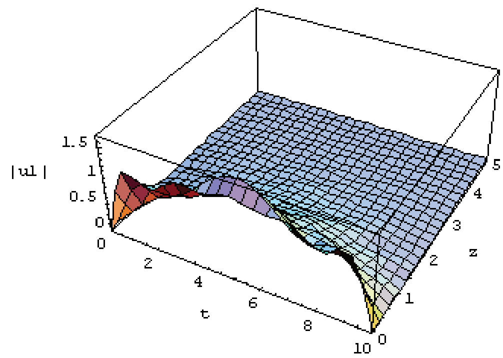


Fig. 16 the first order approximation of $|u^{(1)}|$ at $\varepsilon = 1$, $\gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only ten terms on the series (M=10).

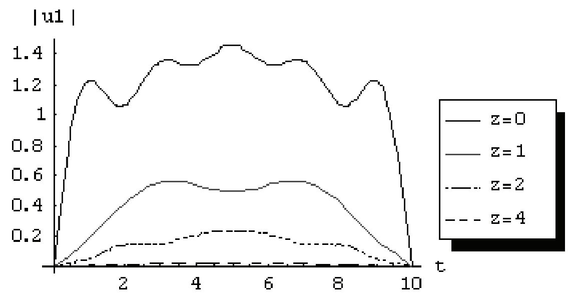


Fig. 17 the first order approximation of $|u^{(1)}|$ at $\varepsilon = 0.2, \gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, M = 10$ for different values of z.

VII. COMPARISON BETWEEN PERTURBATION METHOD AND PICARD APPROXIMATION

Let us compare between two methods.

VII.1. Case study 1:

Taking the case $f_1(t) = \rho_1, f_2(t) = \rho_1$, (Fig. 18 – Fig. 20).

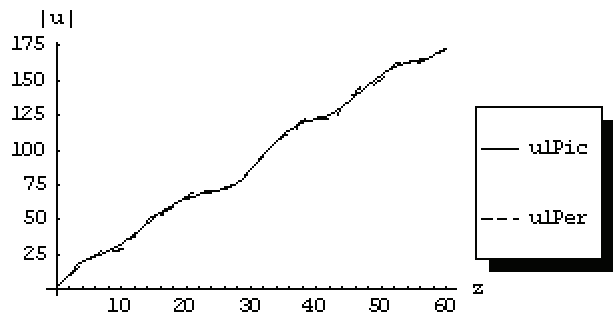


Fig. 18 comparison between Picard approximation and Perturbation method for first order at $\varepsilon = 1, \gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, t = 3$.

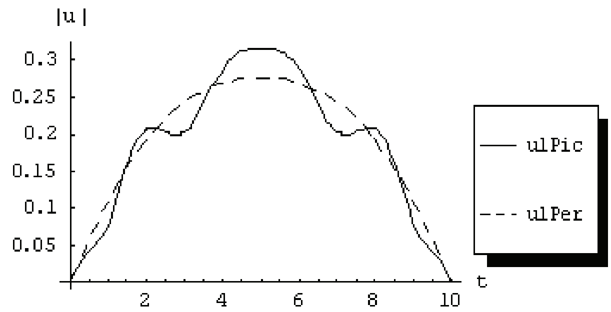


Fig. 19 comparison between Picard approximation and Perturbation method for first order at $\varepsilon = 1, \gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, z = 2$.

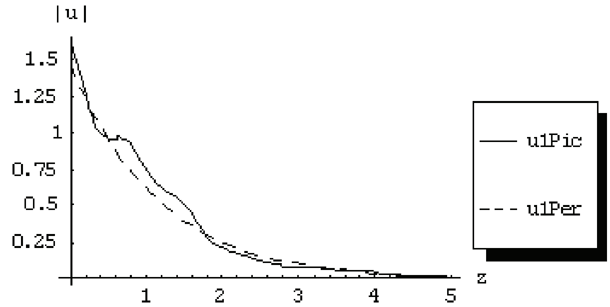


Fig. 20 comparison between Picard approximation and Perturbation method for first order at $\varepsilon = 1, \gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, t = 3$.

VII.2. Case study 2:

Taking the case $f_1(t) = \rho_1, f_2(t) = \rho_2 \sin\left(\frac{m\pi}{T}\right) t$, (Fig. 21 – Fig. 22).

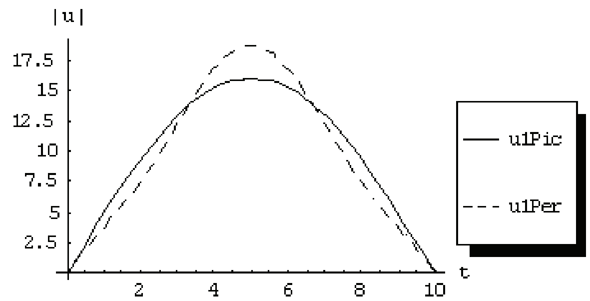


Fig. 21 comparison between Picard approximation and Perturbation method for first order at $\varepsilon = 1, \gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, z = 5$.

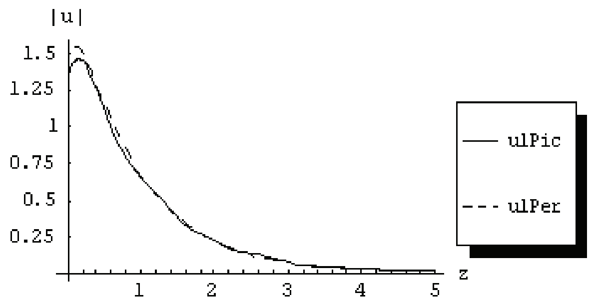


Fig. 22 comparison between Picard approximation and Perturbation method for first order at $\varepsilon = 0.2, \gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10,\ t = 6$.

VIII. CONCLUSION

The stability of the solution of the quintic nonlinear homogeneous Schrodinger equation is highly affected in the absence of gamma. The perturbation as well as the Picard Approximation methods introduce approximate solutions for such problems where first and second order approximations can be obtained from which some parametric studies can be achieved to illustrate the solution behavior under the change of the problem physical parameters. The use of Mathematica, or any other symbolic code, makes the use of the solution algorithm possible and can develop a solution procedure which can help in getting some knowledge about the solution.

REFERENCES

[1] Cazenave T. and Lions P., Orbital stability of standing waves for some nonlinear Schrodinger equations, Commun. Math. Phys. 85(1982) 549-561.

[2] Faris W.G. and Tsay W.J., Time delay in random scattering, SIAM J. on applied mathematics 54(2) (1994) 443-455.

[3] Bruneau C., Menza L., and Lehner T., Numerical resolution of some nonlinear Schrodinger –like equations in plasmas, Numer. Math. PDEs 15(6) (1999) 672-696.

[4] Abdullaev F. and Granier J., Solitons in media with random dispersive perturbations, Physica (D)134(1999) 303-315.

[5] Corney J.F. and Drummond P., Quantum noise in Optical fibers.II.Raman Jitter in soliton communications, J. Opt.Soc.Am. B 18(2)(2001) 153-161.

[6] Biswas A. and Milovic D., bright and dark solitons of generalized Schrodiner equation, Communications in nonlinear Science and Numerical Simulation, doi:10.1016/j.cnsns.2009.06.017

[7] Staliunas K., Vortices and dark solitons in the two-dimensional nonlinear Schrodinger equation, Chaos, Solitons and Fractals 4(1994) 1783-1796.

[8] Carretero R., Talley J.D., Chong C. and Malomed B.A., Multistable solitons in the cubic-quintic discrete nonlinear Schrodinger equation, Physics letter A 216(2006) 77-89.

[9] Seenuvasakumaran P., Mahalingam A. and Porsezian K., dark solitons in N-coupled higher order nonlinear Schrodinger equations, Communications in Nonlinear science and Numerical simulation 13(2008) 1318-1328.

[10] Xing Lü, Bo Tian, Tao Xu, Ke-Jie Cai and Wen-Jun Liu, Analytical study of nonlinear Schrodinger equation with an arbitrary linear-time potential in quasi one dimensional Bose-Einstein condensates , Annals of Physics 323(2008) 2554-2565.

[11] Debussche A. and Menza L., Numerical simulation of focusing stochastic nonlinear Schrodinger equations, Physica D 162(2002) 131-154.

[12] Debussche A. and Menza L., Numerical resolution of stochastic focusing NLS equations, Applied mathematics letters 15(2002) 661-669.

[13] Wang M. and et al, various exact solutions of nonlinear Schrodinger equation with two nonlinear terms, Chaos, Solitons and Fractals 31(2007) 594-601.

[14] Xu L. and Zhang J., exact solutions to two higher order nonlinear Schrodinger equations, Chaos, Solitons and Fractals 31(2007) 937-942.
[15] Sweilam N., variational iteration method for solving cubic nonlinear Schrodinger equation, J. of computational and applied mathematics 207(2007) 155-163.
[16] Zhu S., exact solutions for the high order dispersive cubic-quintic nonlinear Schrodinger equation by the extended hyperbolic auxiliary equation method, Chaos, Solitons and Fractals (2006) 960-779.
[17] Sun J., Qi Ma Z., Hua W. and Zhao Qin M., New conservation schemes for the nonlinear Schrodinger equation, applied mathematics and computation 177(2006) 446-451.
[18] Porsezian K. and Kalithasan B., Cnoidal and solitary wave solutions of the coupled higher order nonlinear Schrodinger equations in nonlinear optics, Chaos, Solitons and Fractals 31(2007) 188-196
[19] Sakaguchi H. and Higashiuchi T., two- dimensional dark soliton in the nonlinear Schrodinger equation, Physics letters A 39(2006) 647-651.
[20] Huang D., Li D.S. and Zhang H., explicit and exact traveling wave solution for the generalized derivative Schrodinger equation, Chaos, Solitons and Fractals 31(3) (2007) 586-593.
[21] Magdy A. El-Tawil, H. El Zoheiry and Sherif E. Nasr "Nonlinear Cubic Homogenous Schrodinger Equations with Complex Intial Conditions, Limited Time Response", The Open Applied mathematics Journal, 4(2010) 1-12
[22] Farlow S., P.D.E. for scientists and engineers, John Wiely & sons, N.Y., 1982.
[23] El-Tawil A., El-Hazmy A., Perturbative nonlinear Schrodinger equations under variable group velocity dissipation, Far East Journal math. Sciences, 27(2) (2007) 419 – 430.
[24] El-Tawil A., El Hazmy A., On perturbative cubic nonlinear Schrodinger equations under complex non-homogenities and complex intial conditions, Journal Differential Equations and Nonlinear Mechanics, doi: 10.1155/2009/395894
[25] Pipes L. and Harvill L., applied mathematics for engineers and physicists, McGraw –Hill, Tokyo, 1970.
[26] Zwillinger D., Handbook of differential Equations, third edition, Acadimic Press, 1997.