On Solution of Nonlinear Cubic Non-Homogeneous Schrodinger Equation

Magdy A. El-Tawil[1], Sherif E. Nasr[2], H. El Zoheiry[3]

Abstract In this paper, a perturbing nonlinear cubic non-homogeneous Schrodinger equation, $i\ \partial u(t,z)/\partial z + \alpha\ (\partial^2 u(t,z))/(\partial t^2) + \varepsilon |u(t,z)|^2 u(t,z) + i\gamma u(t,z) = F_1(t,z) + i F_2(t,z),\ (t,z) \in (0,T)\ x\ (0,\infty)$ is studied under limited time interval, complex initial conditions and zero Neumann conditions. The perturbation method and Picard approximation together with the eigen function expansion and variational parameters methods are used to introduce an approximate solution for the perturbative nonlinear case for which a power series solution is proved to exist. Using Mathematica, the solution algorithm is tested through computing the possible orders of approximations. The method of solution is illustrated through case studies and figures. Effect of time interval (T) had been studied through cases studies and figures.

Keywords Nonlinear Schrodinger Equation, Perturbation, Eigen function Expansion, Mathematica, Picard Approximation

I. Introduction

The nonlinear Schrodinger equation (NLS) is the principal equation to be analysed and solved in many fields, see[1-5] for examples. The NLS equation arises in many applications [6-8] such as wave propagation in nonlinear media, surface wave in sufficiently deep waters and signal propagation in optical fibbers. The NLS equation is one of the most important models of mathematical physics arising in a great array of contexts [9, 10] as for conductor electronics, optics in nonlinear media, photonics, plasmas, fundamentals of quantum mechanics, dynamics of accelerators, mean-field theory of Bose-Einstein condensates and bio-molecule dynamics.

In the last ten decades, there are a lot of NLS problems depending on additive or multiplicative noise in the random case [11, 12] or a lot of solution methodologies in the deterministic case. Wang M. et al [13] obtained the exact solutions to NLS equation using what they called the sub-equation method. They got four kinds of exact solutions of the equation for which no sign to the initial or boundary conditions type is made. Xu L. and Zhang J. [14] followed the same previous technique in solving the higher order NLS.

Sweilam N. [15] solved a nonlinear cubic Schrodinger equation which gives rise to solitary solutions using variational iteration method. Zhu S. [16] used the extended hyperbolic auxiliary equation method in getting the exact explicit solutions to the higher order NLS without any conditions. Sun J. et al [17] solved an NLS equation with an initial condition using Lie group method. By using coupled amplitude phase formulation, Parsezian K. and Kalithasan B. [18] constructed the quartic anharmonic oscillator equation from the coupled higher order NLS equation. Two-dimensional grey solitons to the NLS equation were numerically analysed by Sakaguchi H. and Higashiuchi T. [19]. The generalized derivative NLS equation was studied by Two-dimensional grey solitons to the NLS equation were numerically analysed by Sakaguchi H. and Higashiuchi T. [19]. The generalized derivative NLS equation was studied by Huang D. et al [20] introducing a new auxiliary equation expansion method.

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II. The General Linear Case

Consider the non homogeneous linear Schrodinger equation:

$$i\ \frac{\partial u(t,z)}{\partial z} + \alpha\ \frac{\partial^2 u(t,z)}{\partial t^2} = F_1(t,z) + i\ F_2(t,z), (t,z) \in (0,T)\ \mathrm{x}\ (0,\infty) \quad (1)$$

where $u(t,z)$ is a complex valued function which is subjected to:

$$I.C.: u(t,0) = f_1(t) + i\ f_2(t), \quad (2)$$

$f_i(t)$ is a real valued function,

$$B.C.: u(0,z) = u(T,z) = 0. \quad (3)$$

Let $u(t,z) = \psi(t,z) + i\ \phi(t,z)$, ψ, ϕ are real valued functions. The following coupled equations are got as follows:

$$\frac{\partial \phi(t,z)}{\partial z} = \alpha\ \frac{\partial^2 \psi(t,z)}{\partial t^2} + G_1(t,z), \quad (4)$$

$$\frac{\partial \psi(t,z)}{\partial z} = \alpha\ \frac{\partial^2 \phi(t,z)}{\partial t^2} + G_2(t,z), \quad (5)$$

Where $\psi(t,0) = f_1(t)$, $\phi(t,0) = f_2(t)$, $G_1(t,z) = -F_1(t,z)$, $G_2(t,z) = F_2(t,z)$, and all corresponding other I.C. and B.C. are zeros.

Eliminating one of the variables in equations (4) and (5), one can get the following independent equations:

$$\frac{\partial^4 \psi(t,z)}{\partial t^4} + \frac{1}{\alpha^2}\frac{\partial^2 \psi(t,z)}{\partial t^2} = \frac{1}{\alpha^2}\tilde{\psi}_1(t,z), \quad (6)$$

$$\frac{\partial^4 \phi(t,z)}{\partial t^4} + \frac{1}{\alpha^2}\frac{\partial^2 \phi(t,z)}{\partial t^2} = \frac{1}{\alpha^2}\tilde{\psi}_2(t,z), \quad (7)$$

Where

$$\tilde{\psi}_1(t,z) = \frac{\partial\, G_2(t,z)}{\partial z} - \alpha \frac{\partial^2 G_1(t,z)}{\partial t^2}, \quad (8)$$

$$\tilde{\psi}_2(t,z) = \alpha \frac{\partial\, ^2 G_2(t,z)}{\partial t^2} + \frac{\partial\, G_1(t,z)}{\partial z}, \quad (9)$$

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Using the eigen function expansion technique[24], the following solution expressions are obtained:

$$\psi(t,z) = \sum_{n=0}^{\infty} T_n(z)\sin(\tfrac{n\pi}{T})t, \quad (10)$$

$$\phi(t,z) = \sum_{n=0}^{\infty} \tau_n(z)\sin(\tfrac{n\pi}{T})t, \quad (11)$$

Where $T_n(z)$ and $\tau_n(z)$ can be got through the applications of initial conditions and then solving the resultant second order differential equations using the method of variational of parameter[17]. The final expressions can be got as the following :

$$T_n(z) = \left(C_1 + A_1(z)\right)\sin\beta_n z + \left(C_2 + B_1(z)\right)\cos\beta_n z \quad (12)$$

$$\tau_n(z) = \left(C_3 + A_2(z)\right)\sin\beta_n z + \left(C_4 + B_2(z)\right)\cos\beta_n z \quad (13)$$

Where

$$\beta_n = \alpha(\tfrac{n\pi}{T})^2 \quad (14)$$

$$A_1(z) = \frac{1}{\beta_n}\int \tilde{\psi}_{1n}(z;n)\sin(\beta_n z)\,dz, \quad (15)$$

$$B_1(z) = \frac{-1}{\beta_n}\int \tilde{\psi}_{1n}(z;n)\sin(\beta_n z)\,dz, \quad (16)$$

$$A_2(z) = \frac{1}{\beta_n}\int \tilde{\psi}_{2n}(z;n)\cos(\beta_n z)\,dz, \quad (17)$$

$$B_2(z) = \frac{-1}{\beta_n}\int \tilde{\psi}_{2n}(z;n)\cos(\beta_n z)\,dz, \quad (18)$$

In which

$$\tilde{\psi}_{1n}(z;n) = \frac{2}{T}\int \tilde{\psi}_1(t,z)\sin(\tfrac{n\pi}{T}t)dt, \quad (19)$$

$$\tilde{\psi}_{2n}(z;n) = \frac{2}{T}\int \tilde{\psi}_2(t,z)\sin(\tfrac{n\pi}{T}t)dt, \quad (20)$$

The following conditions should also be satisfied:

$$C_2 = \frac{2}{T}\int_0^T f_1(t)\sin\left(\frac{n\pi}{T}t\right)dt - B_1(0), \quad (21)$$

$$C_4 = \frac{2}{T}\int_0^T f_2(t)\sin\left(\frac{n\pi}{T}t\right)dt - B_2(0). \quad (22)$$

Finally the following solution is obtained:

$$u(t,z) = \psi(t,z) + i\,\phi(t,z), \quad (23)$$

Or

$$|u(t,z)|^2 = \psi^2(t,z) + \phi^2(t,z). \quad (24)$$

III. The Non- Linear Case

Consider the non-homogeneous cubic non-linear Schrodinger equation:

$$i\,\frac{\partial u(t,z)}{\partial z} + \alpha\,\frac{\partial^2 u(t,z)}{\partial t^2} + \varepsilon|u(t,z)|^2 u(t,z) + i\,\gamma u(t,z) = F_1(t,z) + i\,F_2(t,z),\ (t,z)\in(0,T)\ x\ (0,\infty) \quad (25)$$

Where $u(t,z)$ is a complex valued function which is subjected to the initial and boundary conditions mentioned before in equations (2), (3) respectively.

Lemma [21-23]

The solution of equation (25) with the constraints (2), (3) is a power series in ε if exist.

Proof

at $\varepsilon = 0$, the following non-homogenous linear equation is got:

$$i\,\frac{\partial u_0(t,z)}{\partial z} + \alpha\,\frac{\partial^2 u_0(t,z)}{\partial t^2} + i\,\gamma u_0(t,z) = F_1(t,z) + i\,F_2(t,z),\ (t,z)\in(0,T)\ x\ (0,\infty) \quad (26)$$

$$u_0(t,z) = \psi_0(t,z) + i\,\phi_0(t,z) \quad (27)$$

Where,

$$\psi_0(t,z) = e^{-\gamma z}\sum_{n=0}^{\infty} T_{0n}(z)\sin(\tfrac{n\pi}{T})t \quad (28)$$

$$\phi_0(t,z) = e^{-\gamma z}\sum_{n=0}^{\infty} \tau_{0n}(z)\sin(\tfrac{n\pi}{T})t \quad (29)$$

Where $T_{0n}(z)$ and $\tau_{0n}(z)$ can be calculated as the linear case equations (12), (13) respectively.

Following Pickard approximation equation (25) can be rewritten as:

$$i\,\frac{\partial u_n(t,z)}{\partial z} + \alpha\,\frac{\partial^2 u_n(t,z)}{\partial t^2} + i\,\gamma u_n(t,z) = F_1(t,z) + i\,F_2(t,z) - \varepsilon|u_{n-1}(t,z)|^2 u_{n-1}(t,z),\ \ n\ge 1 \quad (30)$$

at $n = 1$, the iterative equation takes the form

$$i\,\frac{\partial u_1(t,z)}{\partial z} + \alpha\,\frac{\partial^2 u_1(t,z)}{\partial t^2} + i\,\gamma u_1(t,z) = F_1(t,z) + i\,F_2(t,z) - \varepsilon|u_0(t,z)|^2 u_0(t,z) = \varepsilon k_1(t,z) \quad (31)$$

which can be solved as a linear case with zero initial and boundary conditions. The following general solution can be obtained:

$$\psi_1(t,z) = e^{-\gamma z}\sum_{n=0}^{\infty}(T_{0n}(z) + \varepsilon T_{1n}(z))(z)\sin(\tfrac{n\pi}{T})t, \quad (32)$$

$$\phi_1(t,z) = e^{-\gamma z}\sum_{n=0}^{\infty}(\tau_{0n}(z) + \varepsilon\tau_{1n}(z))(z)\sin(\tfrac{n\pi}{T})t, \quad (33)$$

$$u_1(t,z) = \psi_1(t,z) + i\,\phi_1(t,z), \quad (34)$$

$$u_1(t,z) = u_1^{(0)} + \varepsilon\,u_1^{(1)}, \quad (35)$$

At $= 2$, the following equation is obtained:

$$i\,\frac{\partial u_2(t,z)}{\partial z} + \alpha\,\frac{\partial^2 u_2(t,z)}{\partial t^2} + i\,\gamma u_2(t,z) = F_1(t,z) + i\,F_2(t,z) - \varepsilon|u_1(t,z)|^2 u_1(t,z) = \varepsilon k_2(t,z) \quad (36)$$

Which can be solved as a linear case with zero initial and boundary conditions. The following general solution can be obtained:

$$u_2(t,z) = u_2^{(0)} + \varepsilon\,u_2^{(1)} + \varepsilon^2 u_2^{(2)} + \varepsilon^3 u_2^{(3)} + \varepsilon^4 u_2^{(4)}, \quad (37)$$

Continuing like this, one can get:

$$u_n(t,z) = u_n^{(0)} + \varepsilon\,u_n^{(1)} + \varepsilon^2 u_n^{(2)} + \varepsilon^3 u_n^{(3)} + \cdots + \varepsilon^{(n+m)} u_n^{(n+m)}. \quad (38)$$

As $\rightarrow\infty$, the solution (if exists) can be reached as $u(t,z) = \lim_{n\to\infty} u_n(t,z)$. Accordingly the solution is a power series in ε .

According to the previous lemma, one can assume the solution of equation (25) as the following:

$$u(t,z) = \sum_{n=0}^{\infty}\varepsilon^n u_n(t,z) \quad (39)$$

Let $u(t,z) = \psi(t,z) + i\,\phi(t,z), \psi, \phi$: are real valued functions. The following coupled equations are got:

$$\frac{\partial\phi(t,z)}{\partial z} = \alpha\,\frac{\partial^2\psi(t,z)}{\partial t^2} + \varepsilon(\psi^2 + \phi^2)\psi - \gamma\phi - F_1(t,z), \quad (40)$$

$$\frac{\partial\psi(t,z)}{\partial z} = -\alpha\,\frac{\partial^2\phi(t,z)}{\partial t^2} - \varepsilon(\psi^2 + \phi^2)\phi - \gamma\psi + F_2(t,z), \quad (41)$$

Where $\psi(t,0) = f_1(t)$, $\phi(t,0) = f_2(t)$, and all corresponding other I.C. and B.C. are zeros.

as a perturbation solution, one can assume that The prototype equations to be solved are:

$$\frac{\partial\phi_i(t,z)}{\partial z} = \alpha\,\frac{\partial^2\psi_i(t,z)}{\partial t^2} + G_i^{(1)},\ i\ge 1 \quad (42)$$

$$\frac{\partial\psi_i(t,z)}{\partial z} = \alpha\,\frac{\partial^2\phi_i(t,z)}{\partial t^2} + G_i^{(2)},\ i\ge 1 \quad (43)$$

Where $\psi_i(t,0) = \delta_{i,0}f_1(t)$, $\phi_i(t,0) = \delta_{i,0}f_2(t)$, and all other corresponding initial conditions are zeros. $G_i^{(1)}, G_i^{(2)}$ are functions to be computed from previous steps.

Following the solution algorithm described in the previous section for the linear case, the following final results are obtained.

We can get the following order of approximations,

1. The absolute value of zero order approximation is:

$$\left|u^{(0)}(t,z)\right|^2 = \psi_0^{\,2} + \phi_0^{\,2} \quad (44)$$

where

$$\psi_0(t,z) = e^{-\gamma z}\sum_{n=0}^{\infty} T_{0n}(z)\sin(\tfrac{n\pi}{T})t \quad (45)$$

$$\phi_0(t,z) = e^{-\gamma z}\sum_{n=0}^{\infty} \tau_{0n}(z)\sin(\tfrac{n\pi}{T})t \quad (46)$$

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$$G_1 = -e^{\gamma z} F_1(t,z) \quad (47)$$
$$G_2 = e^{\gamma z} F_2(t,z) \quad (48)$$

2. The absolute value of first order approximation is:

$$\left|u^{(1)}(t,z)\right|^2 = \left|u^{(0)}(t,z)\right|^2 + 2\varepsilon(\psi_0\psi_1 + \phi_0\phi_1) + \varepsilon^2\left({\psi_1}^2 + {\phi_1}^2\right) \quad (49)$$

where

$$\psi_1(t,z) = e^{-\gamma z} \sum_{n=0}^{\infty} T_{1n}(z)\sin\left(\frac{n\pi}{T}\right)t \quad (50)$$
$$\phi_1(t,z) = e^{-\gamma z} \sum_{n=0}^{\infty} \tau_{1n}(z)\sin\left(\frac{n\pi}{T}\right)t \quad (51)$$
$$G_1 = e^{-2\gamma z}\left({\psi_0}^3 + \psi_0{\phi_0}^2\right) \quad (52)$$
$$G_2 = e^{-2\gamma z}\left(-{\phi_0}^3 - \phi_0{\psi_0}^2\right) \quad (53)$$

3. The absolute value of second order approximation is:

$$\left|u^{(2)}(t,z)\right|^2 = \left|u^{(1)}(t,z)\right|^2 + 2\varepsilon^2(\psi_0\psi_2 + \phi_0\phi_2) + 2\varepsilon^3(\psi_1\psi_2 + \phi_1\phi_2) + \varepsilon^4\left({\psi_2}^2 + {\phi_2}^2\right) \quad (54)$$

Where

$$\psi_2(t,z) = e^{-\gamma z} \sum_{n=0}^{\infty} T_{2n}(z)\sin\left(\frac{n\pi}{T}\right)t \quad (55)$$
$$\phi_2(t,z) = e^{-\gamma z} \sum_{n=0}^{\infty} \tau_{2n}(z)\sin\left(\frac{n\pi}{T}\right)t \quad (56)$$
$$G_1 = e^{-2\gamma z}\left(3{\psi_0}^2\psi_1 + 2\psi_0\phi_0\phi_1 + \psi_1{\phi_0}^2\right) \quad (57)$$
$$G_2 = e^{-2\gamma z}\left(-3{\phi_0}^2\phi_1 - 2\phi_0\psi_0\psi_1 - \phi_1{\psi_0}^2\right) \quad (58)$$

IV. Picard Approximation

To validate our previous results, in the absence of the exact solution, let us follow another approximation technique. The Picard approximation is considered in this section.

Solving equation (25) with the same conditions (2) and (3) and following the Picard algorithm, which means that we solve the linear case iteratively[24]. This means that equation (25) can be rewritten as:

$$i\,\frac{\partial u(t,z)}{\partial z} + \alpha\,\frac{\partial^2 u(t,z)}{\partial t^2} + i\,\gamma u(t,z) = -\varepsilon|u(t,z)|^2 u(t,z) + F_1(t,z) + i\,F_2(t,z), \quad (t,z) \in (0,T)\,x\,(0,\infty) \quad (59)$$

Let $u(t,z) = e^{-\gamma z}(\psi(t,z) + i\,\phi(t,z)), \psi, \phi$: are real valued functions. The following coupled equations are got:

$$\frac{\partial\phi(t,z)}{\partial z} = \alpha\,\frac{\partial^2\psi(t,z)}{\partial t^2} + \varepsilon(\psi^2 + \phi^2)\psi - \gamma\phi - F_1(t,z) \quad (60)$$
$$\frac{\partial\psi(t,z)}{\partial z} = -\alpha\,\frac{\partial^2\phi(t,z)}{\partial t^2} - \varepsilon(\psi^2 + \phi^2)\phi - \gamma\phi + F_2(t,z) \quad (61)$$

Where $\psi(t,0) = f_1(t)$, $\phi(t,0) = f_2(t)$, and all other corresponding initial and boundary conditions are zeros.

We can get The following Order of Approximations,

1. The absolute value of Zero order approximation is similar to perturbation method.

2. The absolute value of First order approximation:

$$i\,\frac{\partial u_1(t,z)}{\partial z} + \alpha\,\frac{\partial^2 u_1(t,z)}{\partial t^2} + i\,\gamma u_1(t,z) = -\varepsilon|u_0(t,z)|^2 u_0(t,z) + F_1(t,z) + i\,F_2(t,z), \quad (t,z) \in (0,T)\,x\,(0,\infty) \quad (62)$$

With initial conditions $u_1(t,0) = f_1(t) + i\,f_2(t)$ and boundary conditions $u_1(0,z) = u_1(T,z) = 0$.

$$\left|u^{(1)}(t,z)\right|^2 = {\psi_1}^2 + {\phi_1}^2 \quad (63)$$
$$\psi_1(t,z) = e^{-\gamma z} \sum_{n=0}^{\infty} T_{1n}(z)\sin\left(\frac{n\pi}{T}\right)t \quad (64)$$
$$\phi_1(t,z) = e^{-\gamma z} \sum_{n=0}^{\infty} \tau_{1n}(z)\sin\left(\frac{n\pi}{T}\right)t \quad (65)$$
$$G_1 = -e^{\gamma z}F_1(t,z) + e^{-2\gamma z}\varepsilon({\psi_0}^3 + \psi_0{\phi_0}^2) \quad (66)$$
$$G_2 = e^{\gamma z}F_2(t,z) - e^{-2\gamma z}\varepsilon\left({\phi_0}^3 + \phi_0{\psi_0}^2\right) \quad (67)$$

3. The absolute value of Second order approximation:

$$i\,\frac{\partial u_2(t,z)}{\partial z} + \alpha\,\frac{\partial^2 u_2(t,z)}{\partial t^2} + i\,\gamma u_2(t,z) = -\varepsilon|u_1(t,z)|^2 u_1(t,z) + F_1(t,z) + i\,F_2(t,z), \quad (t,z) \in (0,T)\,x\,(0,\infty) \quad (68)$$

with initial conditions $u_2(t,0) = f_1(t) + i\,f_2(t)$ and boundary conditions $u_2(0,z) = u_2(T,z) = 0$.

$$\left|u^{(2)}(t,z)\right|^2 = {\psi_2}^2 + {\phi_2}^2 \quad (69)$$
$$\psi_2(t,z) = e^{-\gamma z} \sum_{n=0}^{\infty} T_{2n}(z)\sin\left(\frac{n\pi}{T}\right)t \quad (70)$$
$$\phi_2(t,z) = e^{-\gamma z} \sum_{n=0}^{\infty} \tau_{2n}(z)\sin\left(\frac{n\pi}{T}\right)t \quad (71)$$
$$G_1 = -e^{\gamma z}F_1(t,z) + e^{-2\gamma z}\varepsilon({\psi_1}^3 + \psi_1{\phi_1}^2) \quad (72)$$
$$G_2 = e^{\gamma z}F_2(t,z) - e^{-2\gamma z}\varepsilon\left({\phi_1}^3 + \phi_1{\psi_1}^2\right) \quad (73)$$

V. Case Studies

To examine the proposed solution algorithm, we calculated many cases at different conditions of non-homogeneous term and initial conditions too.

1. Perturbation Method

We illustrate here some cases studies (Fig. 1 – Fig. 4)

Taking the case $F_1(t,z) = \rho_1$, $F_2(t,z) = 0$ and $f_1(t) = \rho_1 e^{-t}$, $f_2(t) = 0$, the following selective result for the first and second order approximations are got:

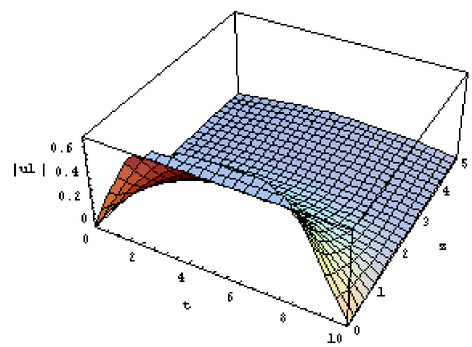

Figure 1. the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 1$, $\gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only one term on the series (M=1)

Taking the case $F_1(t,z) = \rho_1 \sin\left(\frac{m\pi}{T}\right)t$, $F_2(t,z) = 0$ and $f_1(t) = \rho_1 e^{-t}$, $f_2(t) = 0$, the following selective result for the first and second order approximations are got:

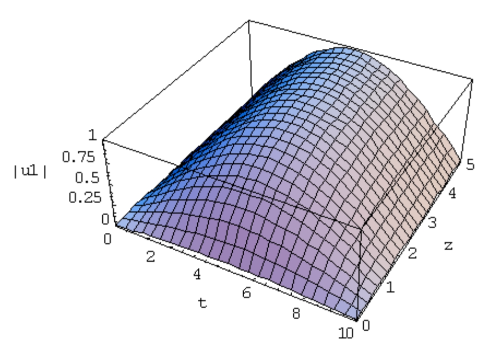

Figure 2. the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 0.2$, $\gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only one term on the series (M=1)

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Taking the case $F_1(t,z) = \rho_1$, $F_2(t,z) = 0$ and $f_1(t) = \rho_1 \sin\left(\frac{m\,\pi}{T}\right) t$, $f_2(t) = 0$, the following selective result for the first and second order approximations are got:

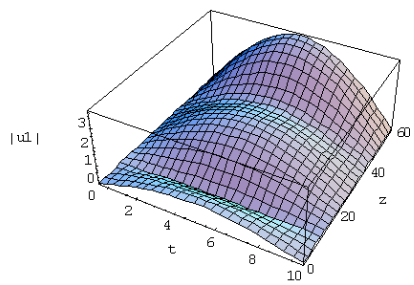

Figure 3. the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 0.2$, $\gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only one term on the series (M=1)

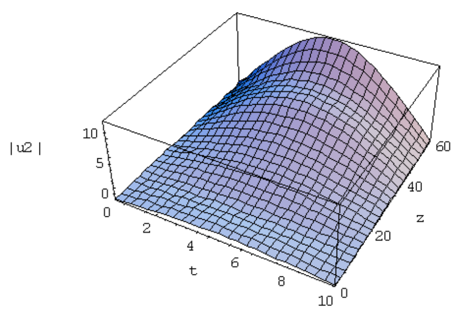

Figure 4. the second order approximation of $\left|u^{(2)}\right|$ at $\varepsilon = 0.2$, $\gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only ten term on the series (M=10)

Note: a lot of other case studies had been studied with combinations between constant, sinusoidal and exponential functions for both non-homogenous and initial conditions.

2. Picard Approximation

We illustrate here some cases of the case studies (Fig. 5 – Fig. 8).Taking the case $F_1(t,z) = \rho_1$, $F_2(t,z) = 0$ and $f_1(t) = \rho_1$, $f_2(t) = 0$, the following selective results for the first and second order approximations are got:

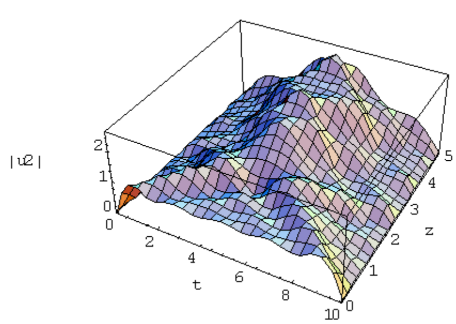

Figure 5. the second order approximation of $\left|u^{(2)}\right|$ at $\varepsilon = 0.2$, $\gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only one term on the series (M=10)

Taking the case $F_1(t,z) = \rho_1 e^{-t}$, $F_2(t,z) = 0$ and $f_1(t) = \rho_1\, e^{-t}$, $f_2(t) = 0$, the following selective results for the first and second order approximations are got:

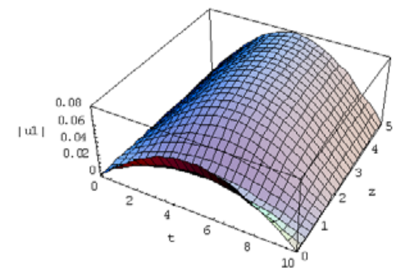

Figure 6. the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 1$, $\gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only one term on the series (M=1)

Taking the case $F_1(t,z) = \rho_1, F_2(t,z) = 0$ and $f_1(t) = \rho_1 \sin\left(\frac{m\,\pi}{T}\right) t, f_2(t) = 0$, the following selective results for the first and second order approximations are got:

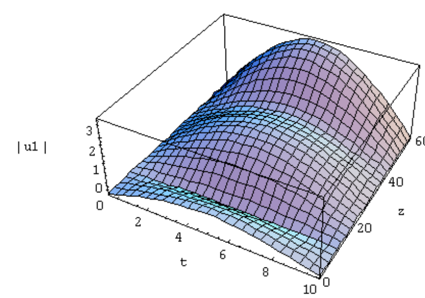

Figure 7. the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 0.2$, $\gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only one term on the series (M=1)

Taking the case $F_1(t,z) = \rho_1 e^{-t}, F_2(t,z) = 0$ and $f_1(t) = \rho_1 \sin\left(\frac{m\,\pi}{T}\right) t, f_2(t) = 0$, the following selective results for the first and second order approximations are got:

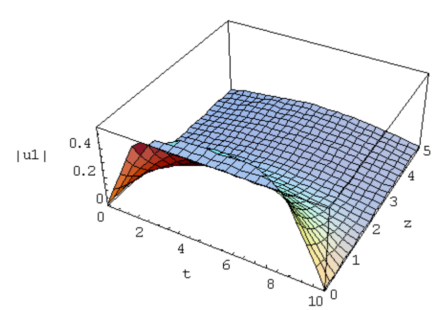

Figure 8. the first order approximation of $\left|u^{(1)}\right|$ at $\varepsilon = 0.2$, $\gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10$ with considering only ten term on the series (M=1)

Note: a lot of other case studies had been studied with combinations between constant, sinusoidal and exponential functions for both non-homogenous and initial conditions.

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VI. Comparison Between Perturbation Method & Picard Approximation

We are here giving both perturbation and Picard results in same graph for some selected cases to compare between two methods, (Fig. 9 – Fig. 11).Taking the case $F_1(t,z) = \rho_1$, $F_2(t,z) = 0$ and $f_1(t) = \rho_1, f_2(t) = 0$.

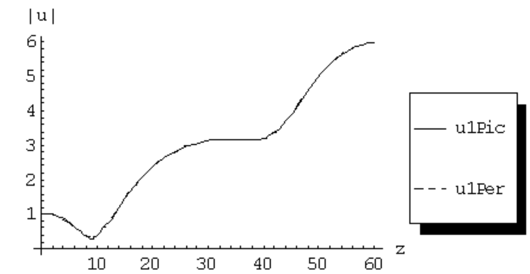

Figure 9. comparison between Picard approximation and Perturbation method for first order at $\varepsilon = 0.2, \gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, T = 10, t = 3$

Taking the case $F_1(t,z) = \rho_1 e^{-t}$, $F_2(t,z) = 0$ and $f_1(t) = \rho_1 e^{-t}, f_2(t) = 0$

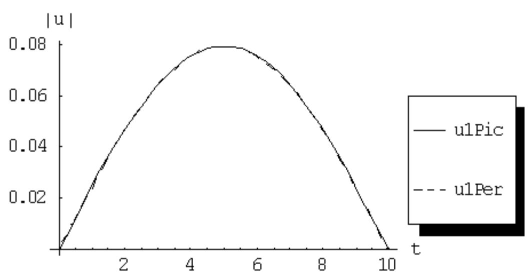

Figure 10. comparison between Picard approximation and Perturbation method for first order at $\varepsilon = 0.2, \gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10,\ z = 5$

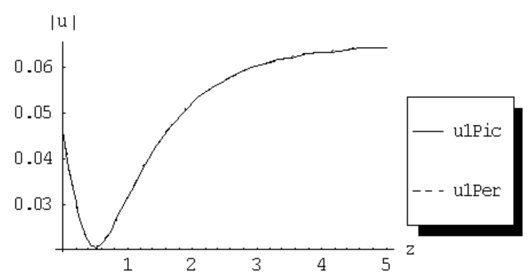

Figure 11. comparison between Picard approximation and Perturbation method for first order at $\varepsilon = 0.2, \gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, T = 10,\ t = 3$

VII. T Study

We are here examining the behavior of Perturbation method and Picard Approximation against different values of T through case studies on the same graph.

1. Perturbation Method

We are here illustrating the effect of the change of the time interval T on the solution and showing that through many case studies, which we summarize some of them through (Fig. 12 – Fig. 13). Taking the case $F_1(t,z) = \rho_1, F_2(t,z) = 0$, $f_1(t) = \rho_1, f_2(t) = 0$, the following selective results for the first and second order approximations are got:

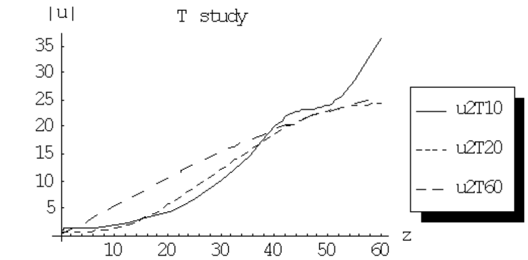

Figure 12. the second order approximation of $|u^{(2)}|$ at $\varepsilon = 0.2$, $\gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, M = 10, t = 4$ for different values of T =10, 20 and 60 respectively

Taking the case $F_1(t,z) = \rho_1 e^{-t}, F_2(t,z) = 0$, $f_1(t) = \rho_1 \sin\left(\frac{m\pi}{T} t\right), f_2(t) = 0$, the following selective result for the first and approximation are got:

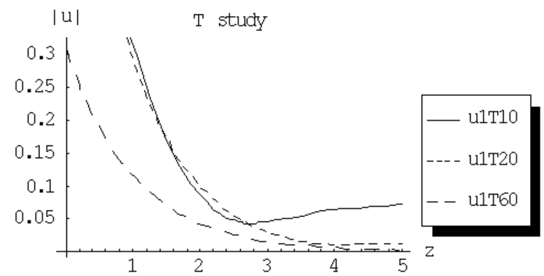

Figure 13. the first order approximation of $|u^{(1)}|$ at $\varepsilon = 0.2$, $\gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, M = 1, t = 6$ for different values of T =10, 20 and 60 respectively

It is clear from case studies that as T increase from $T = 10, T = 20, T = 60$ the magnitude of $u(t,z)$ decrease accordingly.

2. Picard Approximation

We are here do study the effect of change of the time interval T on the solution through many case studies. Some of them are illustrated (Fig. 14 – Fig. 15). Taking the case $F_1(t,z) = \rho_1, F_2(t,z) = 0$, $f_1(t) = \rho_1, f_2(t) = 0$ and following the algorithm, the following selective result for the first and second order approximations are got:

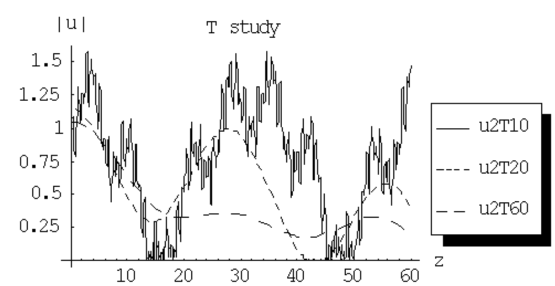

Figure 14. the second order approximation of $|u^{(2)}|$ at $\varepsilon = 0.002$, $\gamma = 0$ and $\alpha, \rho_1, \rho_2 = 1, M = 10, t = 4$ for different values of T =10, 20 and 60 respectively

Taking the case $F_1(t,z) = \rho_1 \sin\left(\frac{m\pi}{T} t\right), F_2(t,z) = 0$, $f_1(t) = \rho_1 e^{-t}, f_2(t) = 0$ and following the algorithm, the following selective result for the first approximation are got:

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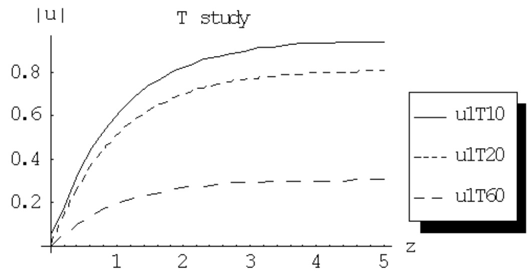

Figure 15. the first order approximation of $|u^{(1)}|$ at $\varepsilon = 0.2$, $\gamma = 1$ and $\alpha, \rho_1, \rho_2 = 1, M = 1, t = 6$ for different values of T =10, 20 and 60 respectively

It is clear from case studies that as T increase from $T = 10, T = 20, T = 60$ the magnitude of $u(t,z)$ decrease accordingly.

VIII. Conclusions

The stability of the solution of the nonlinear cubic non-homogeneous Schrodinger equation is highly affected in the absence of gamma (γ). The perturbation method as well as the Picard approximation introduce approximate solutions for such problems where second or third order of approximations can be obtained from which some parametric studies can be achieved to illustrate the solution behaviour under the change of the problem physical parameters. The use of Mathematica, or any other symbolic code, makes the use of the solution algorithm possible and can develop a solution procedure which can help in getting some knowledge about the solution.

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