An Improvement Prioritization Model Integrating Analytic Hierarchy Process and Genetic Algorithm

Mohammed F. Aly, Hagag M. Abd El-hameed

Abstract

Multi-criteria decision making (MCDM) is one of the most well known branches of decision making and it is referring to making decision in the presence of multiple criteria. Analytic Hierarchy Process (AHP) is a common tool used to solve the MCDM problem. Since AHP has been introduced, it has been applied in numerous situations with impressive results. However, AHP has been also criticized mainly in priority derivation procedure. One of the main problems in current AHP as priority derivation procedure is; Inconsistency of the judgment, and accuracy performance of the prioritization method. To solve the criticism and the problems; this paper proposes new and more reliable model (Analytic Hierarchy Process-Genetic Algorithm prioritization model, AHPGA) to drive and optimize the weights of AHP. The propose framework combines the power of genetic algorithm (GA) with Analytic Hierarchy Process (AHP). The new model minimizes Euclidian distance of Least Squire Method as objective function. Effectiveness of new proposed model is measured by comparing model results other prioritization methods in the literature. The proposed model gives better results compare to the other prioritization methods based on accuracy of derived priorities.

Keywords: Multi- criteria Decision-Making (MCDM), Analytic Hierarchy Process (AHP), Genetic Algorithm (GA), Evolutionary Computing.

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1. Introduction

AHP is a Multi-Attribute Decision-Making (MADM) model proposed by Saaty (1980). Given its advantages of integral structure, simple theory, and ease-of-operation, this method is often used in decision making when addressing events of uncertainty and under various evaluation criteria. For the decision-makers, the hierarchical structure contributes to provide a better understanding; however, it is often necessary to evaluate alternatives based on other criterion, in order to determine priorities. AHP contains an inherit analytical framework, wherein complex and non structural situations are divided into hierarchical elements. Then, the relative significance of every element is scored subjectively by numerical value, and the level of priority is obtained from these values as the factor weights.

The AHP framework provides a comprehensive and rational methodology, which encompasses the following steps: (1) structuring a decision problem in a hierarchy, (2) obtaining the judgment matrix based on pair-wise comparisons between alternatives and between criteria, (3) testing consistency until satisfactory, and (4) synthesizing comparisons across various levels to obtain the final weights of alternatives. Users of AHP make judgments on pair-wise comparisons according to Saaty's discrete 9- value scale method. The matrix is called a pair-wise weighting matrix (PWM).

Saaty (1980) pointed out that, AHP is an efficient auxiliary tool for addressing several issues, including generating a set of alternatives, choosing a best policy alternative, and determining requirements. Feng, Chen, and Jiang[1] evaluated and selected a combination of suppliers through AHP. Chiang[2] suggested that, AHP is a dynamic solution that can successfully address change and evaluation of suppliers. Lee, Chen, and Chang [3] applied AHP to evaluate the performance of IT departments in the manufacturing industry for a standard and persuasive evaluation. Sha and Che [4], [5], Che, Wang, and Sha [6] used AHP to build the solving model for supply chain network design. önüt and Soner [7] computed the relative weights by AHP, and applied in a fuzzy environment. Many researchers used AHP as MCDM process for supplier selection problem Xinyang Deng et al.[8], Dr.p.parthiban et al.[9] and Junyi Chai et al.[10].

In addition, the use of AHP as weighting and driving priority tool is also widespread. Bojan Srdjevic [11] proposes a multi-criteria approach for combining prioritization methods within the AHP. The leading assumption is that for each particular decision problem and related hierarchy, AHP must not necessarily employ only one prioritization method. If more available methods are used to identify the best estimates of local priorities for each comparison matrix in the hierarchy, then the estimate of final alternatives' priorities should also be the best possible. Ying-Ming et al. [12] proposes a linear programming method for generating the most favorable weights (LP-GFW) from pair-wise comparison matrices, which incorporates the variable weight concept of DEA into the priority scheme of the AHP to generate the most favorable weights. Ramanathan [13] use data envelopment analysis (DEA) to generate local weights of alternatives from pair-wise comparison judgment matrices used in the AHP. Based on the above discussions, AHP is an efficient method of solving the relative significance under several evaluation criteria.

Despite its wide acceptance, AHP has been criticized on the ground that decision makers (DMs) often cannot provide strictly consistent comparisons. This problem is of a particular concern when the numbers of criteria and alternatives are large. In Saaty's work, consistency is verified by the Consistency Ratio (CR) that indicates the probability that the matrix ratings are randomly generated. The rule of thumb is that a CR over 0.1 indicates the pair-wise comparison matrix (PWM) should be revised. To deal with the problem of inconsistent comparisons of judgment and performance of the prioritization method, Genetic algorithm (GA) is used and new prioritization model (AHPGA) is developed.

GA procedure is a stochastic optimization technique was mainly applied to research topics in the area of artificial intelligence. However during the past decade, GA has become one of the most well-known search heuristics and is widely used in many combinatorial optimization problems including prioritization AHP vector. GA is inspired by Darwin’s evolution theory based on the survival of the fittest species as introduced by Holland (1975) and further described by Goldberg (1989). According to the mechanism of natural selection and the exchange of genetic information, the species with the optimal fitness will govern the world. GA is often used as a search algorithm, which is based on the biological principles of selection, reproduction and mutation. It searches an optimal solution to the problems by manipulating a population of strings (chromosomes) that represent different potential solutions, each corresponding to a sample point from the search space. For each generation, all the populations are evaluated based on their fitness. An individual with a larger fitness has a higher chance of evolving into the next generation. By searching many peaks simultaneously, GA reduces the possibility of trapping into a local minimum. GA works with a coding of parameters instead of parameters themselves. The coding of parameters helps the genetic operator to evolve the current state into the next state with minimum computations.

GA has been applied to many problems in various domains such as improve weighting methods. Fong et al. [14] integrates GA mechanism and case-based reasoning (CBR) system to assist in assigning the suitable weights to each level of BSC. Based on the BSC design, the study proposed a three-level feature weights design to enhance CBR’s inference performance. GA is employed to facilitate weighting all of levels in BSC and to determine the most appropriate three-level feature weights. Pavlos and Nikolaos [15] proposed an innovating strategy planning for enterprise resources allocation based on a performance measurement view using BSC and genetic algorithm.

Numerous researchers are used GAs to recover the real number weightings of the various criteria in AHP and provides a function for automatically improving the consistency ratio of pair-wise comparisons. Common examples are presented by; Alberto et al. [16], Zakaria et al. [17], Wang et al. [18], Cziner and Hurme [19] and Ludmil [20]. Xuesong et al. [21] combine Genetic Algorithm and AHP to make the decision of machining selection. First, the weights of evaluation factors are formulated and obtained using AHP, then, weights, objectives and constraints are combined and solved using Genetic Algorithm and a set of machining scheme is presented to the user as an outcome. Ludmil et al. [22] are deriving priorities from pair-wise comparison judgments in the framework of AHP. The elicitation of priorities is represented as a multi-criteria optimization problem and the multi-objective evolutionary algorithm PESA-II is applied for its solving.

The main objective of this paper was to develop a new and more reliable model (AHPGA prioritization model) to determine and optimize the weights of AHP. The proposed model combines between AHP and GA. The new model minimizes Euclidian distance of Least

Squire Method as objective function. Effectiveness of new proposed model is measured by comparing model results with AHP original model which exploits the Average Normalized Column (ANC) weighting method to estimate priority vector and with other prioritization methods in the literature.

2. Materials and Methods

This paper used AHP prioritization tool which contains several methods. In this section we describe and discuses the tools used to develop the new model. Procedure and steps of the new model also discussed in this section.

2.1. Analytic Hierarchy Process Prioritization Methods

Analytic hierarchy process has many prioritization methods. The proposed AHPGA prioritization model uses two methods, Average Normalized Columns (ANC) and Direct Least Square. The following part discusses these methods.

2.1.1. Average Normalized Columns (ANC)

The decision weights for each preferential matrix can be obtained after the consistency check by the aforementioned methods related to eigenvector and consistency ratios. Consequently, the final decision weights of the alternatives can be aggregated by a series of multiplications of the rearrangements of the preferential matrixes, average of normalized columns (ANC) one of these methods. If we have consistent matrix ANC is to divide the elements of each column by the sum of the column and then add the element in each resulting row to form average normalized matrix A^*, and divide this sum by the number of elements in the row (n). This is a process of averaging over the normalized columns. In mathematical form, the vector of priorities can be calculated as follows:

$$W_{\mathrm{i}} = \frac{1}{n} \sum_{j=1}^{n} \frac{a_{\mathrm{ij}}}{\sum_{i}^{n} a_{\mathrm{ij}}} \;, \quad i, j = 1,2, \dots n \qquad (1)$$

We can conceder this method an approximation of EV, Consistency ratio (CR) can be calculated from the following equations:

$$\mathrm{CR} = [(\boldsymbol{\lambda}_{\mathrm{max}} - n) / (n - 1)] / \mathrm{RI} \qquad (2)$$

$$\boldsymbol{\lambda}_{max} = \frac{1}{n} \sum_{i=1}^{n} \frac{(a_{\mathrm{ij}}\, w)_i}{w_i} \qquad (3)$$

Where: CR consistency ratio, n matrix size, $\boldsymbol{\lambda}_{\mathbf{max}}$ maximum Eigen-value, *w* is the item weight referring to priority vector and RI random index.

Table1: Random index values [23].

Size of matrix n	1	2	3	4	5	6	7	8	9	10
Random Index RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Some distance minimizing methods such as the least squares method (LSM), logarithmic least square method (LLS) are of the possible tools for computing the priorities of the alternatives. All these are optimization methods.

2.1.2. Direct Least Square

In the DLS method the objective is to find a consistent ratio-scale matrix which minimizes the Euclidean distance from consistent ratio-scale matrix. That is,

$$Min \sum_{i=1}^{n} \sum_{j=1}^{n} \left(a_{\mathrm{ij}} - \frac{w_{\mathrm{i}}}{w_{\mathrm{j}}} \right)^2 \tag{4}$$

$$s.t. \sum_{i=1}^{n} w_{\mathrm{i}} = 1 \tag{5}$$

The nonlinear optimization problem in this equation has no special tractable form and therefore is difficult to solve numerically.

2.2. Proposed AHPGA Prioritization Model

In solving optimization problem for deriving priorities by using AHPGA, it involves three operators which are selection, crossover, and mutation. In this model these operators are selected as follow, rank selection for selection, one point crossover for crossover and uniform mutation for mutation. In order to ensure that only the best population always survives, elitism has also been applied as an additional selection strategy.

2.2.1. Chromosome Representation

This model is using a set of binary numbers (binary encoding) for each population or chromosome in initial population. Then this binary numbers translated to real numbers or (Permutation encoding) for fitness calculation. Real numbers are used because it is more natural and useful representation of priorities in AHP.

2.2.2. Initialization

The initial population of candidate solution is generated randomly across the search space. Search space is the space for all possible feasible solutions. Every solution can be marked by its value of the fitness of the problem. Random numbers are used for initial population to give AHPGA starting point. Each single population is generating randomly based on number of criteria or alternative in AHP hierarchy setting.

2.2.3. Fitness Function

In general, fitness function F(x) is first derived from the objective function and used in successive genetic operations. Total deviation (TD) equation used as an objective function to be optimized. Once an offspring population is created or the population is initialized, the fitness values of candidate solution are evaluated.

$$F(x) = TD, \qquad TD = \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\left(a_{\mathrm{ij}} - \frac{w_{\mathrm{i}}}{w_{\mathrm{j}}}\right)^{2}\right)^{1/2} \tag{6}$$

After calculate each initial population fitness function, that chromosome will set as parent. That parent also will produce offspring and store the offspring chromosome. Besides that, that parent also will go to selection step.

2.2.4. Parameter Setting

By using GA, parameter setting is the important part in getting the better result. The performance of GA is greatly dependent on its turning of parameter. This model proposes a new parameter setting. Table 2 shows the proposed parameter setting.

Table 2: Proposed GA parameter setting.

Parameter Name	**Value**
Population size	1000
Number of generation	Not Specified
Crossover rate	50 %
Mutation rate	10 %

Flow chart that shows procedure of AHPGA prioritization model in deriving priorities is presented in figure 1.

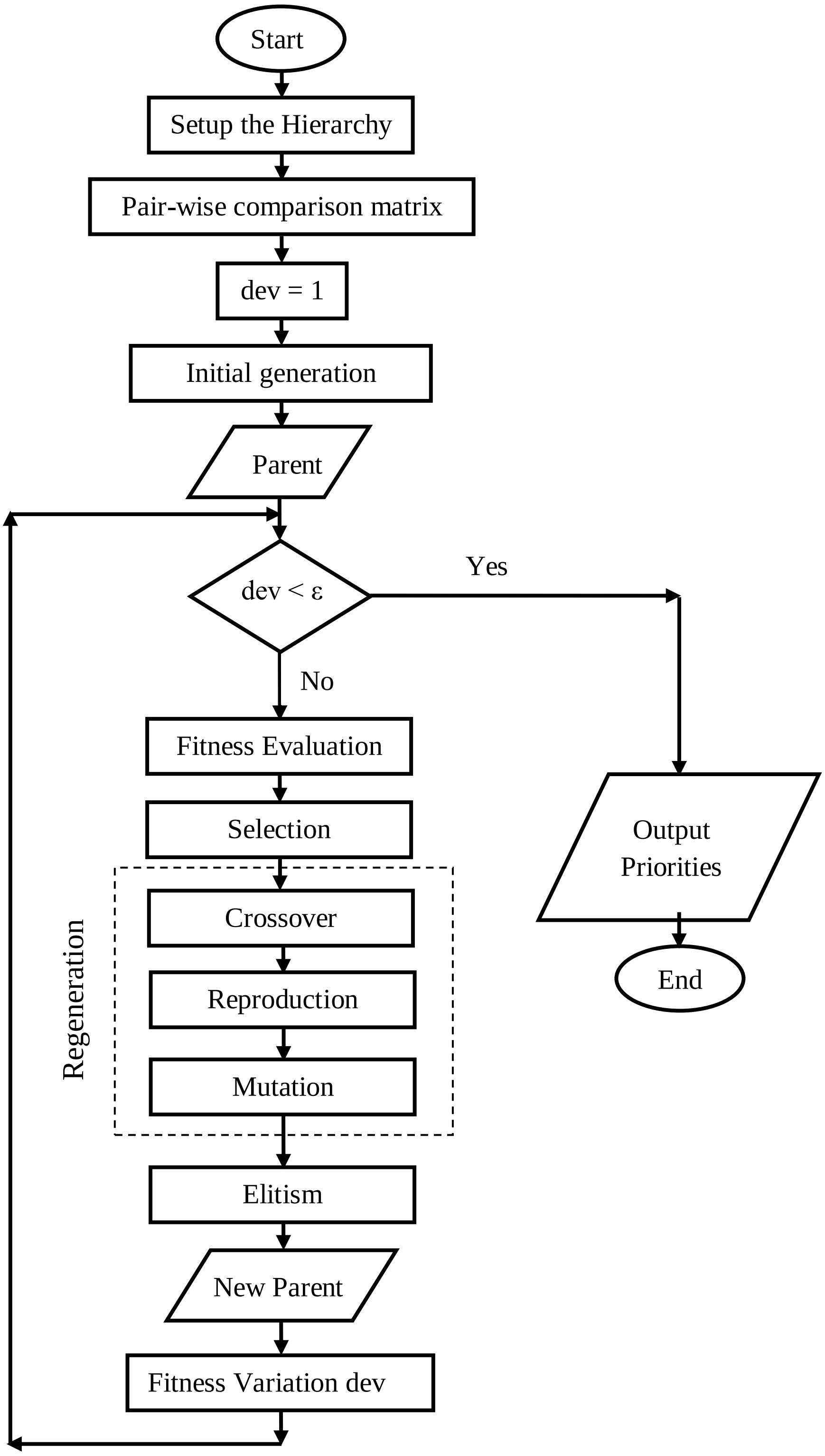


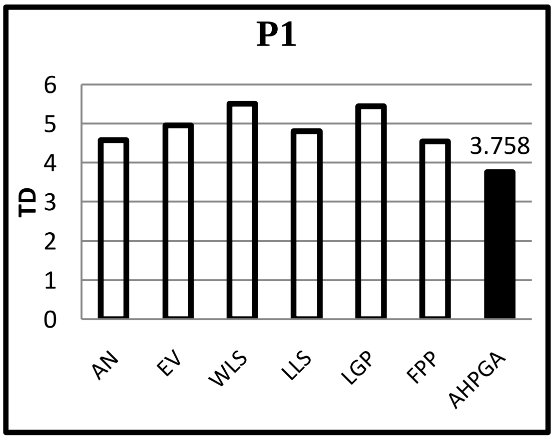
Figure 1: AHPGA procedure for deriving priorities in AHP.

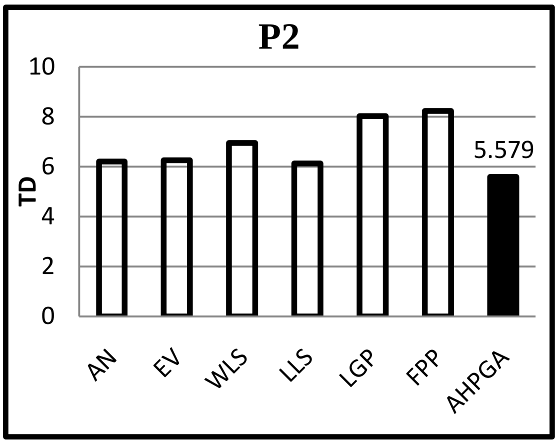
3. Verification of Proposed Model

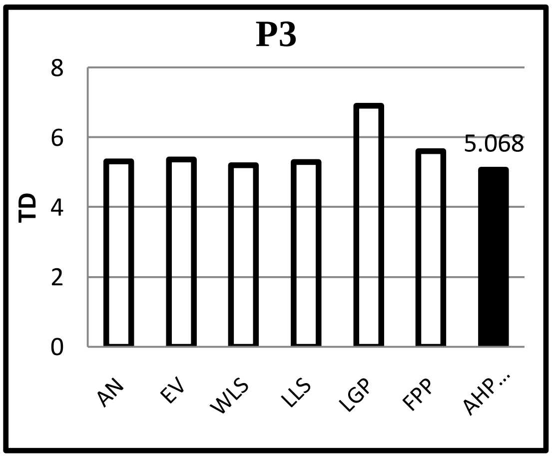
Two examples are used to illustrate and verify the potential applications of the proposed AHPGA model. The illustration presents the advantages of the proposed model and verifies model consistency with the previous work.

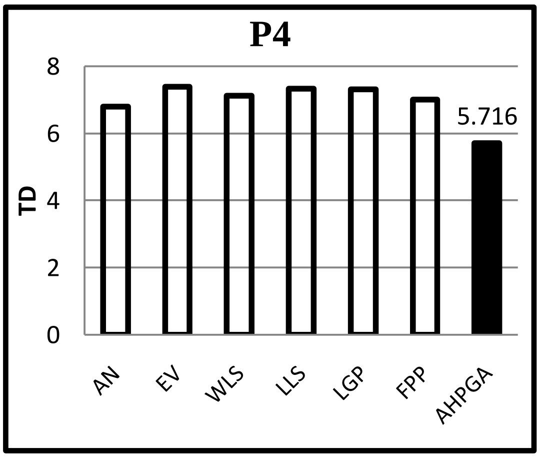
Example (1)

In this example, a case has been conducted based on data that are taken from Bojan's study [11]. Bojan's study proposes a multi-criteria approach for combining prioritization methods within the AHP. Prioritization methods used in this example are Additive normalization (AN), Eigenvector (EV), Weighted Least-Squares (WLS), Logarithmic Least-Square (LLS), Logarithmic Goal Programming (LGP), Fuzzy Preference Programming (FPP) and Analytical Hierarchy Presses Genetic Algorithm (AHPGA).









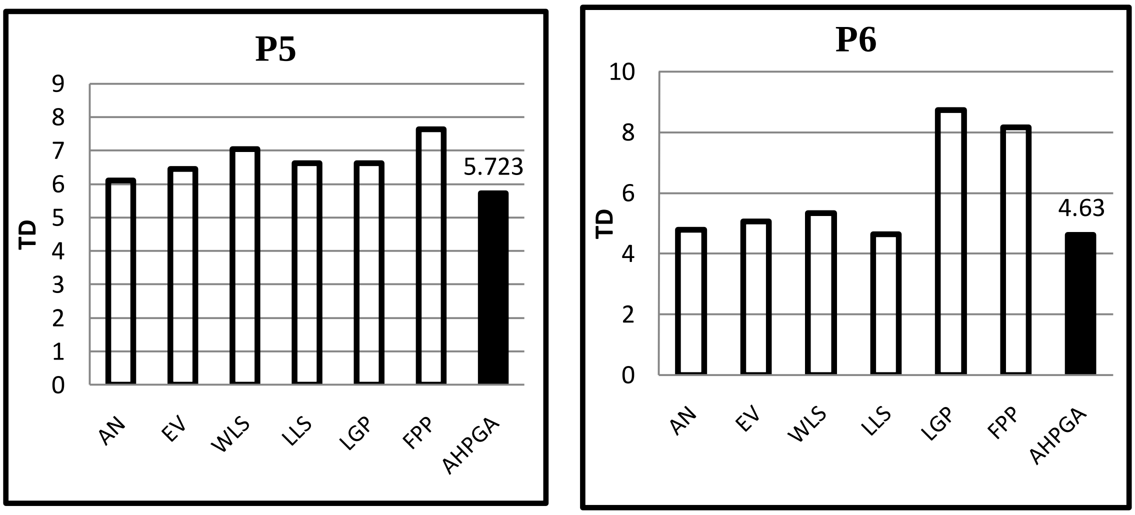


Figure 2: Total deviations for deferent prioritization methods

The results of this case, reservoir storage allocation problem (figure 2) shows that, AHPGA prioritization model produces the smaller or close to zero the value of TD (as comparing criteria) for every single matrix (i.e. P1, P2… and P6). These results also show that the integrated AHPGA model can be used to estimates priorities for both situation of inconsistency and consistency of the judgment.

Example (2)

In this case, the study has been conducted based on data that are taken from example used in study of Delgado-Galván et al. [24] which uses AHP Eigen-value method (EVM) for assessing externalities in water leakage management.

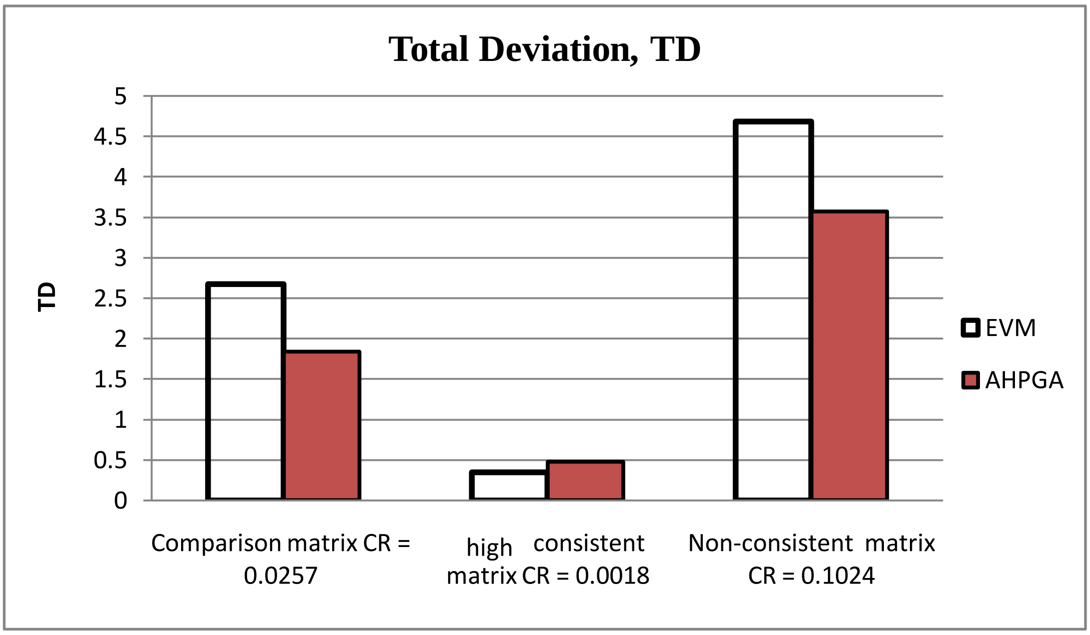


Figure 3: Water leakage management problem results

Results of this example (figure 3) show that, in general AHPGA model is a good prioritization model for AHP priority vector generation. AHPGA model gives better solutions (minimum TD value and the same ranking) in case of moderate consistent and inconsistent pair-wais compression matrices. The AHPGA results in case of height consistent matrices not the best solution. Results of examples one and two shows that, AHPAG model has better results and the smallest value of TD over the other prioritization methods in case of moderate consistent and inconsistent matrices.

4. Conclusion

AHP has been criticized on the ground that decision makers (DMs) often cannot provide strictly consistent comparisons. This problem is of a particular concern when the numbers of criteria and alternatives are large. So, the purpose of this paper is to develop new and more reliable prioritization model to deal with this problem, and to improve accuracy and performance of AHP method. The propose framework combines the power of genetic algorithm (GA) with Analytic Hierarchy Process (AHP) to develops the new (AHPGA) model. AHPGA prioritization model accuracy and applicability are validated by comparing its results with other prioritization methods reported in the literature. AHPGA model results are in close agreement with other prioritization methods results.

The following conclusions may be drawn from this paper: Analytic Hierarchy Process (AHP) using average normalizing column (ANC) as weighting method is an acceptable prioritization method and gets acceptable results in this work. The proposed AHPGA prioritization model is a successful and applicability prioritization tool. In general AHPGA model gives acceptable (logic ranking of criteria) and more accurate results (minimum total deviation value) in all cases used to verify the model in this work. In case of high consistent matrix ($CR< 0.003$), the AHPGA results lead to the same ranking of criteria compared with another prioritization methods and optimization techniques used to optimize AHP. Although that, the total error (TD value) may be greater than the TD values of the other methods. So, AHPGA model may need some modification in GA parameter setting.

The proposed AHPGA model gives good results in case of consistent matrices within the range tested in this paper. As a recommendation for the future work, more deep research is required to improve accuracy of proposed AHPGA prioritization and Modification in GA parameter setting may be needed in case of high consistent ($CR < 0.003$) and high inconsistent matrices.

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