Experimental Investigation of the Dynamic Characteristics of Laminated Composite Beams

Mohammed F. Aly, I. G. M. Goda, and Galal A. Hassan

***Abstract*—— The laminated composite beams are basic structural components used in a variety of engineering structures such as airplane wings, helicopter blades and turbine blades as well as many others in the aerospace, mechanical, and civil industries. An important element in the dynamic analysis of composite beams is the computation of their natural frequencies and mode shapes. This is important because composite beam structures often operate in complex environmental conditions and are frequently exposed to a variety of dynamic excitations. In this paper, a combined finite element and experimental approach is used to characterize the vibration behavior of composite beams. To this end, some beams are made using the hand-lay-up process. Glass fiber is used as reinforcement in the form of bidirectional fabric and general purpose polyester resin as matrix for the composite material of beams. Experimental dynamic tests are carried out using specimens with different fiber orientations. From the results, the influence of fiber orientations on the flexural natural frequencies is investigated. Also, these experiments are used to validate the results obtained from the finite element software ANSYS.**

***Index Term* – Composite beams, Dynamic tests, Finite element method, Natural frequencies**

I. INTRODUCTION

Fiber reinforced composites are finding increasing applications in civil engineering, transportation vehicles, aerospace, marine, aviation, and chemical industries in recent decades. This is due to their excellent features, such as high strength-to-weight and stiffness-to-weight ratios, the ability of being different strengths in different directions and the nature of being tailored to satisfy the strength and stiffness requirements in practical designs. Studies on the behavior of composite beams have recently been important because of their high strength and lightweight properties on modern engineering sought in structures. For any composite structure that may be subjected to dynamic loads, the determination of the natural frequencies is critical in the design process. It is usually the first step in a dynamic analysis since a great deal may be deduced concerning the structural behavior and integrity from knowledge of its natural frequencies. So, the researches pertain to the vibration analysis of composite beams have undergone rapid growth over the past few decades and are still growing.

A number of researchers have been developed numerous solution methods to analysis the dynamic behavior of laminated composite beams. Khdeir and Reddy [[1]] developed analytical solutions of refined beam theories to study the free vibration behavior of cross-ply rectangular beams with arbitrary boundary conditions in conjunction with the state space approach. Krishnaswamy et al. [[2]] developed dynamic equations governing the free vibration of laminated composite beams using Hamilton's principle. The effects of transverse shear and rotary inertia are included in the energy formulation. Matsunaga [[3]] studied the natural frequencies of laminated composite beams by taking into account the complete effects of transverse shear and normal stresses and rotatory inertia. Chen et al. [[4]] presented a new method of state space-based differential quadrature for free vibration of generally laminated beams. Chandrashekhara et al. [[5]] obtained the exact solutions for symmetrically laminated beams based on first order shear deformation theory including rotary inertia.

A large number of investigators address the problem of free vibration analysis of laminated composite beams. Yildirim and Kiral [[6]] studied the out-of-plane free vibration problem of symmetric cross-ply laminated composite beams using the transfer matrix method. The rotary inertia and shear deformation effects are considered in the Timoshenko beam analysis based on the first-order shear deformation theory. Banerjee [[7]] investigated the free vibrations of axially loaded composite Timoshenko beams using the dynamic stiffness matrix method by developing an exact dynamic stiffness matrix of composite beams taking into account the effects of an axial force, shear deformation, and rotatory inertia. Jun et al. [[8]] investigated the free vibration behaviors of axially loaded laminated composite beams having arbitrary lay-up using the dynamic stiffness method taking into account the influences of axial forces, Poisson effect, axial deformation, shear deformation, and rotary inertia. Abramovich and Livshits [[9]] studied the free vibration of non symmetric cross-ply laminated composite beams based on Timoshenko type equations. Eisenberger et al. [[10]] used the dynamic stiffness analysis and the first-order shear deformation theory to study the free vibration of laminated beams. Calım [[11]] make study intended to

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analyze free and forced vibrations of non-uniform composite beams in the Laplace domain. Song and Waas [[12]] studied the free vibration analyses of stepped laminated composite beams using simple higher-order theory (SHOT) which assumes a cubic distribution for the displacement field through the thickness. Yildirim [[13]] used the stiffness method for the solution of the purely in-plane free vibration problem of symmetric cross-ply laminated beams with the rotary inertia, axial and transverse shear deformation effects included by the first-order shear deformation theory. Rao et al. [[14]] developed an analytical method for evaluating the natural frequencies of laminated composite and sandwich beams using higher-order mixed theory and analyzed various beams of thin and thick sections. Kant et al. [[15]] developed an analytical solution to the dynamic analysis of the laminated composite beams using a higher order refined theory. Vinson and Sierakowski [[16]] obtained the exact solution of a simply supported composite beam based on the classical theory, which neglects the effects of the rotary inertia and shearing deformation. Abramovich [[17]] studied free vibration of symmetrically laminated composite beams on Timoshenko type equations.

Many authors have used the finite element technique to analyze the dynamic of laminated beams. Bassiouni et al. [[18]] presented a finite element model to investigate the natural frequencies and mode shapes of the laminated composite beams. Tahani [[19]] developed a new layerwise beam theory for generally laminated composite beam and compared the analytical solutions for static bending and free vibration with the three-dimensional elasticity solution of cross-ply laminates in cylindrical bending and with three-dimensional finite element analysis for angle-ply laminates. Chandrashekhara and Bangera [[20]] investigated the free vibration of angle-ply composite beams by a higher-order shear deformation theory using the shear flexible finite element method. Maiti and Sinha [[21]] developed a finite element method (FEM) to analyze the vibration behavior of laminated composite. Murthy et al. [[22]] derived a refined 2-node beam element based on higher order shear deformation theory for axial-flexural-shear coupled deformation in asymmetrically stacked laminated composite beams. Ramtekkar et al. [[23]] developed a six-node plane-stress mixed finite element model by using Hamilton's principle. Teh and Huang [[24]] presented two finite element models based on a first-order theory for the free vibration analysis of fixed-free beams of general orthotropy. Nabi and Ganesan [[25]] developed a general finite element based on a first-order deformation theory to study the free vibration characteristics of laminated composite beams. Aydogdu [[26]] studied the vibration of cross-ply laminated beams subjected to different sets of boundary conditions. Subramanian [[27]] has investigated the free vibration of laminated composite beams by using two higher order displacement based on shear deformation theories and finite elements.

The main objective of this work is to contribute for a better understanding of the dynamic behavior of components made from fiber reinforced composite materials, specifically for the case of beams. In order to investigate the influence of the fiber orientation on the dynamic behavior of the components, experimental and numerical analysis using the finite element method have been carried out. The results are presented and discussed.

II. PRODUCTION OF THE LAMINATES SPECIMENS

Glass fiber is used as reinforcement in the form of bidirectional fabric and general purpose polyester resin as matrix for the composite material of the laminates specimens.

The steps of manufacturing the composite beams using the hand lay-up process are described below.

A. Preparation of the Mould

The hand lay-up process is open molding technique, only one mould is used. The surface of the mould is thoroughly cleaned to be ready for the use, by removing any dust and dirt from it.

B. Application of the Release Agent

After the mould surface has been cleaned, the release agent is applied. Where, the mould surface is coated with a silicon free wax using a smooth cloth. Then a film of polyvinyl alcohol (PVA) is applied over the wax surface using sponge. PVA is a water soluble material and 15% solution in water is used. When water evaporates, a thin film of PVA is formed on the mould surface. PVA film is dried completely before the application of resin coat. This is very important as the surface of final article will be marred with a partly dried PVA film otherwise release will not be smooth.

C. Preparation of the Matrix Material

The matrix material is prepared using General purpose (GP) Polyester resin. Cobalt Octate (0.35% by volume of resin) is added to act as Accelerator. Methyl ethyl ketone peroxide (MEKP) (1% by volume) is added to act as catalyst. Resin, accelerator and catalyst are thoroughly mixed. The use of accelerator is necessary because without accelerator resin does not cure properly. After adding the accelerator and catalyst to the polyester resin, it has left for some time so that bubbles formed during stirring may die out. The amount of added accelerator and catalyst is not high because a high percentage reduces gel time of polyester resin and may adversely affect impregnation.

D. Preparation of the Reinforcement

E-glass woven roving of 360 g/m^2 (mass per unite area) is used as reinforcement. The fabrics are made of fibers oriented along two perpendicular directions: one is called the warp and the other is called the fill (or weft) direction. The fibers are woven together, which means the fill yarns pass over and under the warp yarns, following a fixed pattern. Fig.1 shows a plain weave where each fill goes over a warp yarn then under a warp yarn and so on. Glass fiber mats (woven – mat), used for making the laminated plate are cut in 12 pieces of required size (1000 mm x 1000 mm).

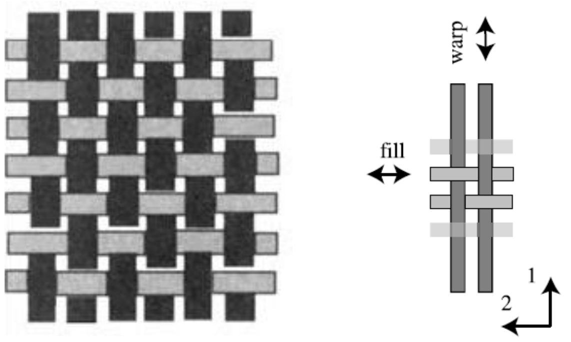


Fig. 1. Schematic representation of woven fabric architecture [[28]]

E. *Preparation of the Laminated Plate*

The first layer of mat is laid and resin is spread uniformly over the mat by means of a brush. The second layer of mat is laid and resin is spread uniformly over the mat by means of a brush. After second layer, to enhance wetting and impregnation, a teethed steel roller is used to roll over the fabric before applying resin. Also resin is tapped and dabbed with spatula before spreading resin over fabric layer. This process is repeated till all the twelve fabric layers are placed. No external pressure is applied while casting or curing because uncured matrix material can squeeze out under high pressure. This results in surface waviness (non-uniform thickness) in the model material. The casting is cured at room temperature for 4 hours and finally removed from the mould to get a fine finished composite plate.

F. *Preparation of the Test Specimens*

After the cure process, test specimens are cut from the sheet of 12 ply laminate of the size 1000 mm x 1000 mm x 5.45 mm by using a diamond impregnated wheel, cooled by running water. All the test specimens are finished by abrading the edges on a fine carborundum paper.
The laminated plate is cut at different off-axis angles (0^0, 15^0, and 30^0) to give beam specimens with different fiber orientation of twelve woven lamina. Since each fabric layer corresponds to 2 different fiber orientations (fibers at 0^0 and 90^0) 2 different layers can be used to simulate each ply as ([0/90], [15/-75], and [30/-60]).

III. MATERIALS AND EXPERIMENTAL TEST SPECIMENS

A. *Materials Characterization*

The mechanical properties of constituents of the test specimens, E-glass woven roving fibers and polyester matrix are listed in Table 1.
The material elastic properties of the laminae of test specimens are determined through the simple rule-of-mixtures. These properties are Young's moduli (E_1 – in direction 1, E_2 – in direction 2, E_3 – in direction 3), Poisson's ratios (v_{12}, v_{13}, and v_{23}), Inplane shear modulus (G_{12}) and transverse shear moduli (G_{13} and G_{23}) as referred in Fig.2. This figure defines the material principal axes for a typical woven fiber reinforced lamina. Axis 1 is along the fiber length and represents the longitudinal direction of the lamina; axes 2 and 3 represent the transverse in-plane and through- the- thickness directions respectively.

TABLE I
MECHANICAL PROPERTIES OF CONSTITUENTS OF TEST SPECIMENS [[29]]

Material	Properties	Value
Glass fiber	Elasticity modulus (GPa)	74
	Shear modulus (GPa)	30
	Density (kg/m^3)	2600
	Poisson ratio	0.25
Polyester resin	Elasticity modulus (GPa)	4.0
	Shear modulus (GPa)	1.4
	Density (kg/m^3)	1200
	Poisson ratio	0.4

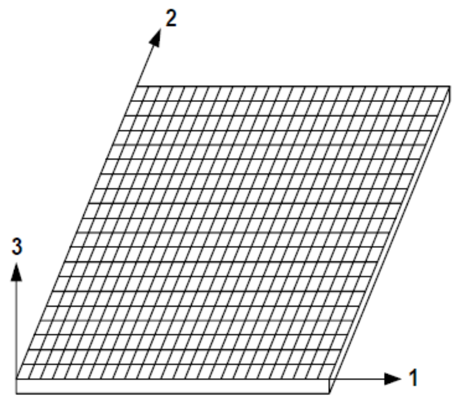


Fig. 2. Lamina reference axes

The volume fractions of the fiber and voids are calculated from the measured weights and densities of fiber, matrix, and composite. The fiber weight fraction of the specimens is measured by burning out the resin of the composite material. In the burning test, three samples have nominal 5.45 mm thicknesses are cut in square sections of 43×43 mm. An electronic balance is used to measure the weight of the three samples of the tested woven fabric composite. The averaged weight of the three samples is measured as 16.48 gm. The average density of the samples is found to be 1610 kg/m3. After the burning process, the resin is removed from the composite and the average weighted of the remaining woven roving fiber becomes 7.55 gm. Then, the fiber weight fraction of the composite material is calculated to be 45.83% and the resin weight fraction is 54.17%.

The void volume fraction v_v is calculated from the measured weights and densities of fiber, matrix, and composite, by equation (1). It is found to be 1.07 %.

$$\nu_{\upsilon} = 1 - \frac{(W_f / \rho_f) + (W_c - W_f) / \rho_m}{W_c / \rho_c} \quad (1)$$

Where W_f, W_m, and W_c are the weights of the fiber, matrix, and composite, respectively.

By using the densities of the fiber ρ_f, matrix ρ_m, and composite ρ_c, respectively, the fiber volume fraction υ_f can be obtained by equation (2):

$$\rho_c = \rho_f \nu_f + \rho_m \nu_m = \rho_f \nu_f + \rho_m (1 - \nu_f - \nu_\upsilon) \quad (2)$$

Where ν_f, ν_m, and ν_υ are the volume fractions of the fiber, matrix, and voids, respectively.

Using the relation of equation (2) the fiber volume fraction (ν_f) is found 30 % according to the densities of fiber and matrix presented in Table 1. Then, the elastic constants of the woven fabric composite material are numerically estimated using the relations which are based on their constituent properties. The young's modulus and the Poisson's ratio of the fill and warp directions are calculated and taken as an average of the longitudinal and transverse values of the corresponding unidirectional layer.

The elastic constants of the unidirectional composite are calculated using the simple rule-of-mixtures by the relations of equation (3) [[16]].

$$\begin{aligned}
E_1 &= E_f \nu_f + E_m (1 - \nu_f), \\
E_2 &= E_m \left[\frac{E_f + E_m + (E_f - E_m)\nu_f}{E_f + E_m - (E_f - E_m)\nu_f} \right], \\
\upsilon_{12} &= \upsilon_f \nu_f + \upsilon_m (1 - \nu_f), \\
\upsilon_{23} &= \upsilon_f \nu_f + \upsilon_m (1 - \nu_f) \left[\frac{1 + \upsilon_m - \upsilon_{12} E_m / E_{11}}{1 - \upsilon_m^2 + \upsilon_m \upsilon_{12} E_m / E_{11}} \right], \\
G_{12} &= G_m \left[\frac{G_f + G_m + (G_f - G_m)\nu_f}{G_f + G_m - (G_f - G_m)\nu_f} \right], \\
G_{23} &= \frac{E_{22}}{2(1 + \upsilon_{23})}.
\end{aligned} \quad (3)$$

Where indices m and f denote matrix and fiber, respectively

After calculating elastic constants of the unidirectional composite, elastic constants of the woven fabric composite material are estimated using the relations of equation (4) [[28]] and the results are listed in Table 2.

$$\begin{aligned}
&\left(\frac{2}{E_1} \frac{E_1(E_1 + (1 - \upsilon_{12}^2)E_2) - \upsilon_{12}^2 E_2^2}{E_1(E_1 + 2E_2) + (1 + 2\upsilon_{12}^2)E_2^2} \right)^{UD} = \left(\frac{1}{E_1} \right)^{WF}, \\
&\left(\frac{4}{E_1} \frac{\upsilon_{12} E_2 (E_1 - \upsilon_{12}^2 E_2)}{E_1(E_1 + 2E_2) + (1 + 2\upsilon_{12}^2)E_2^2} \right)^{UD} = \left(\frac{\upsilon_{12}}{E_1} \right)^{WF}, \\
&\left(\frac{1}{E_1} \frac{E_1(\upsilon_{12} + \upsilon_{23} + \upsilon_{12}\upsilon_{23}) + \upsilon_{12}^2 E_2}{E_1 + (1 + 2\upsilon_{12})E_2} \right)^{UD} = \left(\frac{\upsilon_{13}}{E_1} \right)^{WF}, \\
&\left(\frac{(1 - \upsilon_{23}^2)E_1^2 + (1 + 2\upsilon_{12} + 2\upsilon_{12}\upsilon_{23})E_1E_2 - \upsilon_{12}^2 E_2^2}{E_1E_2(E_1 + (1 + 2\upsilon_{12})E_2)} \right)^{UD} = \left(\frac{1}{E_3} \right)^{WF}, \\
&\left(\frac{1}{G_{12}} \right)^{UD} = \left(\frac{1}{G_{12}} \right)^{WF}, \\
&\left(\frac{1 + \upsilon_{23}}{E_2} + \frac{1}{2G_{12}} \right)^{UD} = \left(\frac{1}{G_{13}} \right)^{WF}.
\end{aligned} \quad (4)$$

Where UD and WF denote unidirectional fiber and woven fiber, respectively

TABLE II
ELASTIC PROPERTIES OF WOVEN FABRIC COMPOSITE LAMINAE

Properties	**Value**
Elastic modulus $E_1 = E_2$ (GPa)	15.70
Elastic modulus E_3 (GPa)	7.85
Shear modulus in plane 1–2 G_{12} (GPa)	2.45
Shear modulus in plane 1–3 G_{13} (GPa)	2.37
Shear modulus in plane 2–3 G_{23} (GPa)	2.37
Poisson ratio in plane 1–2 υ_{12}	0.15
Poisson ratio in plane 1–3 υ_{13}	0.46
Poisson ratio in plane 2–3 υ_{23}	0.46

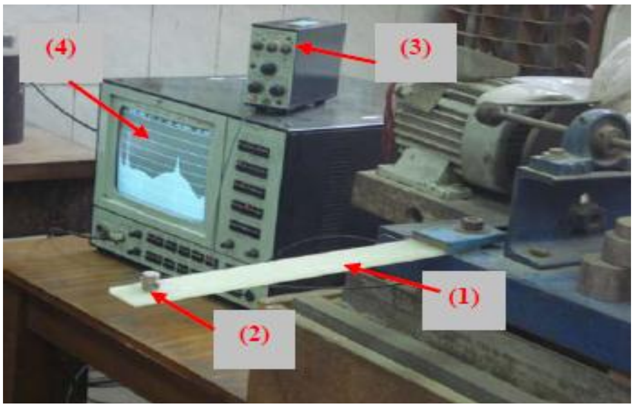
B. *Types of Structures*

All types of structures investigated in this study are in the form of simple beams. The laminate is cut in beams with nominal length of 370 mm, width of 43 mm, thickness of 5.45 mm, and total mass equal to 140 gm. The types of lay up of beams under investigation are varied as 0^0, 15^0, and 30^0 as shown in Fig.3. The total length of the beam specimens is 370 mm and due to the cantilevered fixation of the test specimens the free length becomes 330 mm.

Fig. 3. Laminated composite beam specimens

C. *Experimental Modal Analysis*

Through an impact experimental test, it is determined FRFs (Frequency Response Functions) which relate the response given by the specimen when loaded with a signal, allowing for the determination of the natural frequencies, as shown in Fig.4. This is done by fixing the laminate specimen in a rigid support with one of its side free to vibrate, as a cantilever beam. The impact hammer is used to give the input load (pulse) to the specimen, and the Signal Analyzer is set from 0 Hz to 200 Hz. This output is captured by the accelerometer and is amplified using a conditioning amplifier and then read using the high resolution signal analyzer, giving the FRF.



1- Cantilever laminated beam
2- B&K accelerometer Type 4333
3- B&K conditioning amplifier Type 2626
4- B&K signal analyzer Type 2033

Fig. 4. Experimental set up for modal testing of a cantilever laminated beam

IV. RESULTS AND DISCUSSIONS

Table 3 shows the experimental damped natural frequencies obtained by free vibration test for all laminated beams previously mentioned. Figs. 5-7 present the 1st and 2nd natural frequencies obtained experimentally for the woven laminated beam of fiber angle 0^0, 15^0, and 30^0.

Also, theses beams are modeled using finite element method in order to get the un-damped natural frequencies and mode shapes. The beams are discretized using (type shell99) finite element available in the commercial package ANSYS 10.0. This element has 8 nodes and is constituted by layers that are designated by numbers (LN - Layer Number), increasing from bottom to top of the laminate. The last number quantifies the existent total number of layers in the laminate (NL - Total Number of Layers). The element has six degrees of freedom at each node: translations in the nodal *x*, *y*, and *z* directions and rotations about the nodal *x*, *y*, and *z*-axes.

The constituent laminae are considered to be linear elastic and generally orthotropic therefore the concept of engineering constants is used to describe the laminae elastically. The elastic properties of the woven roving laminae are required as input parameters for the ANSYS. These properties are E_1, E_2, E_3, G_{12}, G_{13}, G_{23}, υ_{12}, υ_{13} and υ_{23} which are given in Table II. The results obtained by ANSYS are presented in Table III, for the first two un-damped natural frequencies.

TABLE III
NATURAL FREQUENCIES (HZ) FROM ANSYS AND EXPERIMENTAL TEST

Lay up	1st mode			2nd mode		
	Ansys	**Exp.**	**% Diff.**	**Ansys**	**Exp.**	**% Diff.**
0	25.1	22.0	12.4	157.0	146.5	6.7
15	22.7	20.0	11.9	141.8	143.5	1.2
30	19.48	17.0	12.7	121.7	121.0	0.6

$$\%Difference = \frac{\left| f_{ANSYS} - f_{Experimental} \right|}{f_{ANSYS}} \times 100$$

From the results of Table III, it has been found that the experimental results show a good agreement with the ANSYS values (maximum difference equal 12.7 % for the 1st mode and 6.7 % for the 2nd mode), proving that the fiber angle has influence on the dynamic behavior of the laminated beams. As the fiber angle increases, the natural frequencies of flexural vibration of beams decrease. From the experimental results, it is observed that increasing the angle of the fibers from 0^0 to 30^0 reduces the natural frequency by about 23% (i.e. from 22 to 17 Hz) for the 1st mode and by about 17.5% (i.e. from 146.5 to 121.0 Hz) for the 2nd mode.

From these results, it is possible to verify the influence of fiber orientation on the free flexural vibration of laminated beams. It is found that the maximum flexural frequency occurs at $\beta = 0^0$ and the minimum occurs at 30^0. This can be explained by the fact that the fibers oriented at 0^0 are more appropriate to flexural loads

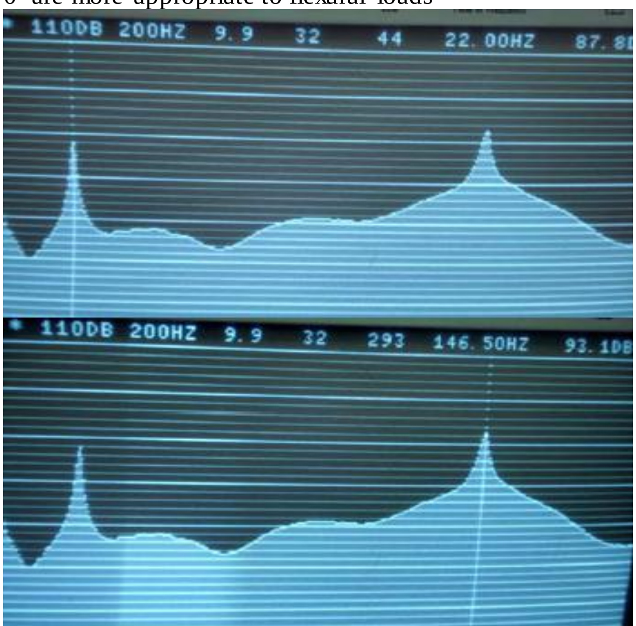


Fig. 5. 1st and 2nd flexural frequencies obtained by experimental dynamic test for 0^0 woven roving laminated beam

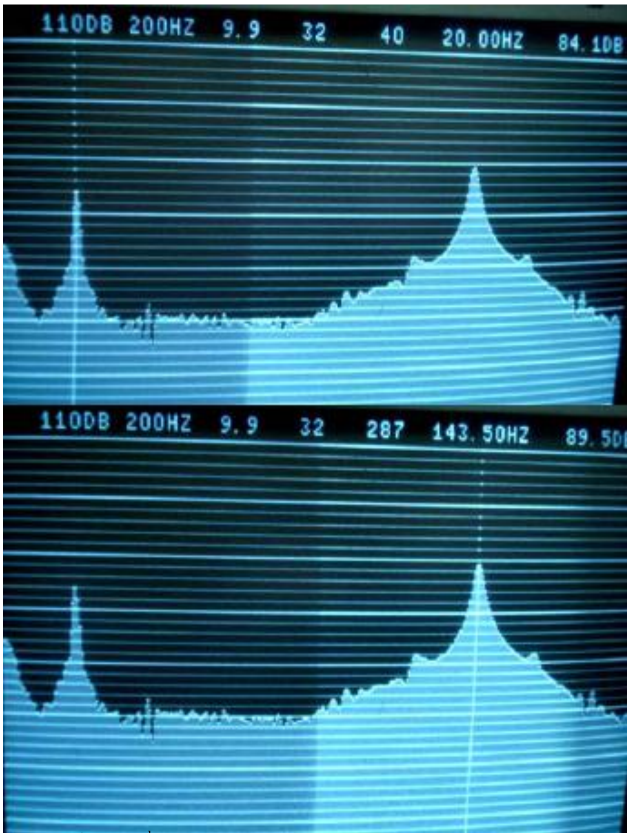


Fig. 6. 1st and 2nd flexural frequencies obtained by experimental dynamic test for 15^0 woven roving laminated beam

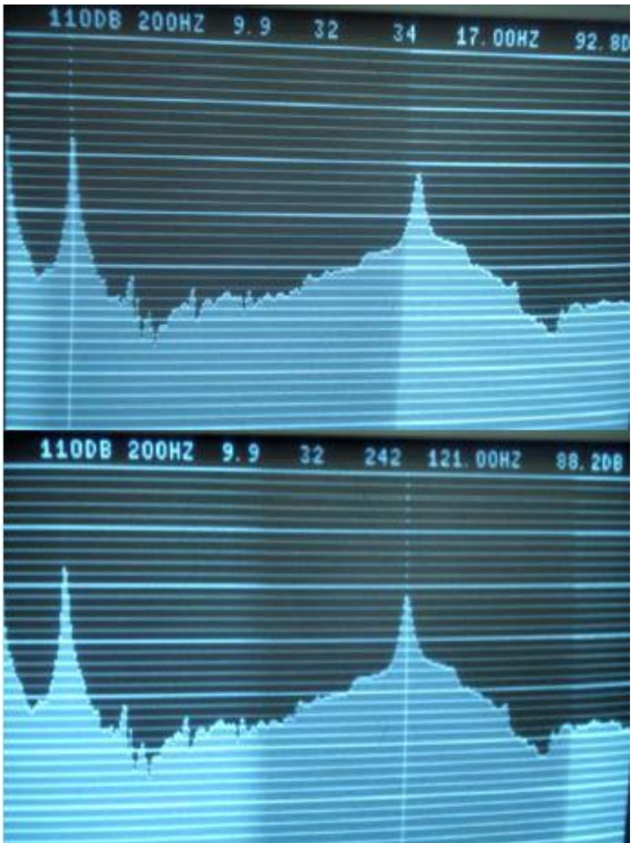


Fig. 7. 1st and 2nd flexural frequencies obtained by experimental dynamic test for 30^0 woven roving laminated beam

Variation of the lowest two flexural frequencies with respect to fiber angle change of woven roving laminated beams are presented in Fig. 8. The experimental frequencies are plotted with the ANSYS results against fiber angle of woven roving laminated beams.

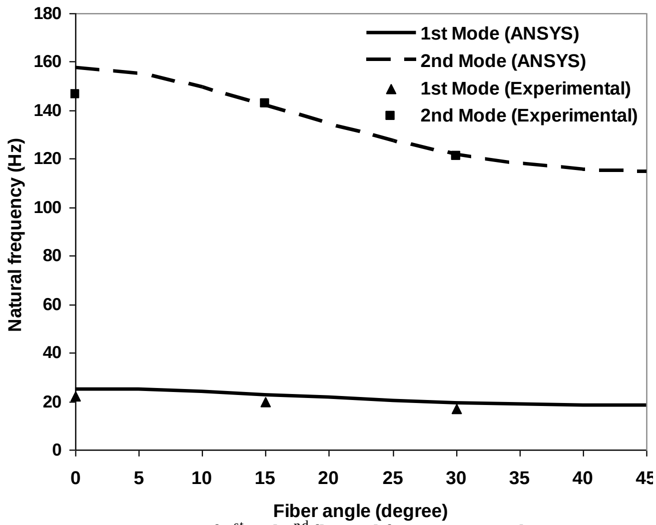
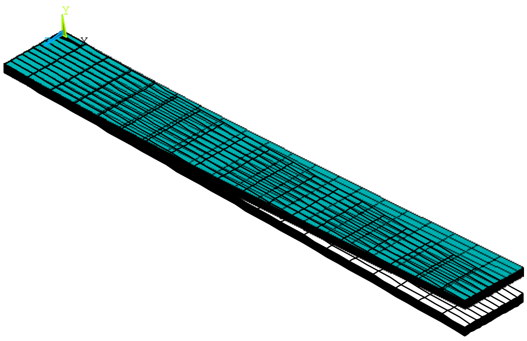


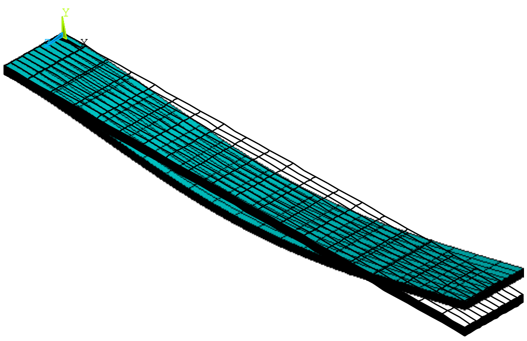
Fig. 8. Variation of 1st and 2nd flexural frequencies with respect to fiber angle change of woven roving laminated beams

With relation to the deviations of the numerical results in relation to the experimental ones, some possible measurement errors can be pointed out such as: measurement noise, positioning of the accelerometers and their mass, non-uniformity in the specimens properties (voids, variations in thickness, non uniform surface finishing). Such factors are not taken into account during the numerical analysis, since the model considers the specimen entirely perfect and with homogeneous properties, what rarely occurs in practice. Another aspect to be considered is that the input properties in the model came from the application of the rule-of-mixtures and they do not take into account effects of the fiber-matrix interface as well as the irregular distribution of resin on the fibers. Also, these models did not include damping effects, which can have a large influence on the structure behavior. Also, the computational package ANSYS does not allow for the consideration of the fibers interweaving present in the fabric used.

The mode shapes associated with the frequencies of 0^0 woven roving beam are illustrated in Fig. 9. They are deduced by ANSYS for the first and second flexural natural frequencies (deformed and un-deformed shapes).



1st Mode - 25.1 Hz



2nd Mode – 157.0 Hz

Fig. 9. Free flexural modes for 0^0 woven roving beam

V. CONCLUSIONS

In this work, the dynamic characteristics of laminated composite beams with different fiber orientations were tested experimentally. The test results were compared to those calculated using finite element software package ANSYS. The main conclusions that can be drawn from this investigation are:

- The changes in fiber angle yield to different dynamic behavior of the component, that is, different natural frequencies for the same geometry, mass and boundary conditions.
- As the fiber angle increases, the flexural natural frequencies decrease, with maximum value occurs at $\beta = 0^0$.
- The results from ANSYS showed in general good agreement with the experimental values.
- The experimental investigation conducted using specially prepared beam specimens with different fiber orientations was reasonable.
- The numerical analysis using finite element package ANSYS to investigate the dynamic characteristics of laminated composite beams, is a successful tool for such applications.
- Finally, this study helps designer in selection of the fiber orientation angle to shift the natural frequencies as desired or to control the vibration level.

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