EFFECTS OF FIBER ORIENTATION AND LAMINATE STACKING SEQUENCE ON OUT-OF-PLANE AND IN-PLANE BENDING NATURAL FREQUENCIES OF LAMINATED COMPOSITE BEAMS

Galal.A.Hassan
Mechanical Design and Production Department
Faculty of Engineering, Cairo University, Giza 12613
Egypt
Galalhassaan@yahoo.com

Mohammed.A.Fahmy
Industrial and Manufacturing Engineering Department
Faculty of Engineering, Fayoum University, Fayoum
Egypt
Mfahmy_aly@yahoo.com

Ibrahim Goda Mohammed
Mechanical Design and Production Department
Faculty of Engineering, Cairo University, Giza 12613
Egypt
Eng_Ibrahimgoda@yahoo.com

ABSTRACT

Advanced fiber reinforced plastic (FRP) composites are increasingly being used in weight sensitive structural applications due to their high specific stiffness, high specific strength characteristics, and resistance in fatigue. Beam analysis plays an important role in mechanical and civil structural design such as railway, car suspension system, and structural foundation. Finding free vibration characteristics of laminated composite beams (LCBs) is one of the bases for designing and modeling of industrial products. Anisotropy of these composites allows the designer to tailor the material in order to achieve the desired performance requirements. Thus, it is of fundamental importance to develop tools that allow the designer to obtain optimized designs considering the structural requirements, functional characteristics and restrictions imposed by the production process. Within these requirements, this work considers the dynamic behavior of components of unidirectional symmetric LCBs. In this study, the flexural vibrations of LCBs are analyzed analytically using Bernoulli-Navier hypothesis and Timoshenko's first-order shear beam theory. The commercial finite element program ANSYS 10.0 is used to perform a dynamic modelling to the laminated beams. Mindlin eight-node isoparametric layered shell elements (SHELL 99) are employed in the modeling for describing the bending vibrations of these laminated beams. The influence of fiber directions and stacking arrangements of laminates on out-of-plane and in-plane vibrations were investigated. The results obtained by the commercial software ANSYS 10.0 were compared to those from Euler-Bernoulli beam theory and Timoshenko's first-order shear beam theory and are presented for the purpose of comparison.

KEYWORDS

Fiber reinforced plastic, Laminated composite beams, Free vibration, Natural frequencies, Shear deformation, Finite element analysis

1. INTRODUCTION

Fiber-reinforced composite laminates are commonly used in the construction of aerospace, civil, marine, automotive and other high performance structures due to their high specific stiffness and strength, excellent fatigue resistance, longer durability as compared to metallic structures, and ability to be tailored for specific applications. Composite materials can be tailored to meet the particular requirements of stiffness and strength by altering lay-up and fiber orientations. The ability to tailor a composite material to its job is one of the most significant advantages of a composite material over an ordinary material. So the research and development of composite materials in the design of mechanical, aerospace, and civil structures has grown tremendously in the past few decades as studied by (Jun et al., 2008) and (Subramanian, P., 2006).

It is essential to know the vibration characteristics of these structures, which may be subjected to dynamic loads in complex environmental conditions. If the frequency of the loads variation matches one of the resonance frequencies of the structure, large translation/torsion deflections and internal stresses can occur, which may lead to failure of structure components. In order to achieve the right combination of material properties and service performance, the dynamic behavior is one of the main points to be considered. To avoid the typical problems caused by vibrations, it is important to determine: a) the natural frequencies of the structure and b) the modal shapes to reinforce the most flexible regions or to locate the right positions where weight should be reduced or damping should be increased. With respect to these dynamic aspects, the composite materials represent an excellent possibility to design components with requirements of dynamic behavior as mentioned by (Tita et al., 2003). A variety of structural components made of composite materials such as turbine blades, vehicle axles, aircraft wing, and helicopter blade can be approximated as laminated composite beams, which requires a deeper understanding of the vibration characteristics of the composite beams as mentioned by (Kapuria and Alam, 2006).

Due to the composite beams widely used in a variety of structures as well as their substantial benefits and great promise for future application, the dynamic behaviors of the laminated composite beams have received widespread attention and have been investigated extensively by many researchers. A number of researchers have been developed numerous solution methods to analysis the dynamic behaviors of the laminated composite beams (Khdeir and Reddy, 1994), (Krishnaswamy et al., 1992), and (Matsunaga, H., 2001).

The classical laminated beam theory, developed by Euler-Bernoulli, is used only for thin beams because this theory has neglected both transverse shear and normal strains and it is inaccurate for a moderately deep laminated beam with relatively soft transverse shear modulus and for highly anisotropic composites. The inaccuracy is due to neglecting the transverse shear and normal strains in the laminate. In order to take into account the effects of low ratio of transverse shear modulus to the in-plane modulus, the first order shear deformation theory of Timoshenko has been developed (Matsunaga, H., 2001). However, since in the theory the transverse shear strain is assumed to be constant in the depth direction, a shear correction factor has to be incorporated to adjust the transverse shear stiffness for studying the dynamic problems of beams (Matsunaga, H., 2001). The accuracy of solutions of the first order shear deformation theory will be strongly dependent on predicting better estimates for the shear correction factor (Matsunaga, H., 2001).

Theoretical analyses of flexural vibration of layered beams have been studied by several researchers. Miller and Adams (1975) have been studied the vibration characteristics of orthotropic clamped-free beams without including the effect of the shear deformation. Bhimaraddi and Chandrashekhara (1991) considered the modeling of laminated beams by a systematic reduction of the constitutive relations of the three-dimensional anisotropic body and concluded that these relations should be adopted while modeling especially angle-ply laminated composite beams. Matsunaga, H. (2001) analyzed natural frequencies and buckling stresses of general cross-ply laminated composite beams by taking into account the complete effects of transverse shear and normal stresses and rotary inertia. Abramovich, H. (1992) gave exact solutions, based on the Timoshenko type equations, for symmetrically laminated composite beams with 10 different boundary conditions. The rotary inertia and shear deformation effects were investigated for simply supported beams with h/b=1 for the axial and out-of plane bending vibration cases (Abramovich, H., 1992). Zapfe and Lesieutre (1997) presented an iterative smeared model for the vibration analysis of laminated beams. Jun, L. et al. (2008) investigated the free vibration and buckling behaviors of axially loaded laminated composite beams having arbitrary lay-up using the dynamic stiffness method. Qiao Pizhong and Zou Guiping (2002) presented an analytical study for dynamic behavior of pultruded fiber-reinforced plastic (FRP) composite cantilever I-beams based on a Vlasov-type linear hypothesis.

A significant amount of research has been conducted on the vibration analysis of laminated beams with focusing on classical lamination theory, Timoshenko first order beam theory, and higher order beam theory. Abramovich and Livshits (1994) studied the free vibration of non symmetric Cross-ply laminated Composite Beams based on Timoshenko type equations. The effect of coupled longitudinal and transversal displacements, shear deformation and rotary inertia are included in the analysis (Abramovich and Livshits, 1994). Eisenberger, M. et al. (1995) used the dynamic stiffness analysis and the first-order shear deformation theory to study the free vibration of laminated beams. Vinson and Sierakowski (2004) obtained the exact solution of a simply supported composite beam based on the classical theory, which neglects the effects of the rotary inertia and shearing deformation. Khdeir and Reddy (1994) have been studied free vibrations of cross-ply laminated beams with arbitrary boundary conditions. Krishnaswamy, S. et al. (1992) gave analytical solutions for the free vibration problem of laminated composite beams. Also, Dynamic equations governing the free vibration of laminated composite beams are developed using Hamilton's principle; the effects of transverse shear and rotary inertia are included in the energy formulation (Krishnaswamy, S. et al., 1992). Song and Waas (1997) have been studied both buckling and free vibration analyses of laminated composite beams. They Song and Waas (1997) also investigated the shear deformation effects. Yildirim, V. (2000) used the stiffness method for the solution of the purely in-plane free vibration problem of symmetric cross-ply laminated beams with the rotary inertia, axial and transverse shear deformation effects included by the first-order shear deformation theory. Banerjee, J. (1998) has investigated the free vibration of axially laminated composite Timoshenko beams using dynamic stiffness matrix method. Yõldõrõm and Kõral (2000) studied the out-of-plane free vibration problem of symmetric cross-ply laminated beams using the transfer matrix method. Also, the effects of the rotary inertia and shear deformation are investigated under various boundary conditions. Kant, T. et al. (1998) developed an analytical solution to the dynamic analysis of the laminated composite beams using a higher order refined theory. This model also fails to satisfy the traction- free surface conditions at the top and bottom surfaces of the beam but has included the effect of transverse normal strain. Rao et al. (2001) proposed a higher-order mixed theory for determining the natural frequencies of a diversity of laminated Simply-Supported beams. Also they Rao et al. (2001) developed an analytical method for evaluating the natural frequencies of laminated composite and sandwich beams using higher-order mixed theory and analyzed various beams of thin and thick sections.

Many authors have given finite element solutions to analysis the dynamic of laminated beams. Bassiouni et al. (1999) presented a finite element model to investigate the natural frequencies and mode shapes of the laminated composite beams. The model required all lamina had the same lateral displacement at a typical cross-section, but allowed each lamina to rotate a different amount from the other. The transverse shear deformation was included. Tahani, M. (2007) developed a new layerwise beam theory for generally laminated composite beam and compared the analytical solutions for static bending and free vibration with the three-dimensional elasticity solution of cross-ply laminates in cylindrical bending and with three-dimensional finite element analysis for angle-ply laminates. Chandrashekhara and Bangera (1992) investigated the free vibration of angle-ply composite beams by a higher-order shear deformation theory using the shear flexible FEM. Maiti and Sinha (1994) developed a finite element method (FEM) to analyze the vibration behavior of laminated composite beams and investigated the effects of various parameters. Murthy et al. (2005) derived a refined 2-node beam element based on higher order shear deformation theory for axial-flexural-shear coupled deformation in asymmetrically stacked laminated composite beams. Ramtekkar et al. (2002) developed a six-node plane-stress mixed finite element model by using Hamilton’s principle. Natural frequencies of cross-ply laminated beams were obtained and various mode shapes were presented. Teh and Huang (1979) presented two finite element models based on a first-order theory for the free vibration analysis of fixed-free beams of general orthotropy. Nabi and Ganesan (1994) examined bi-axial bending, axial and torsional vibrations using the finite element method and the first-order shear deformation theory. Harmonic response of tapered composite beams was examined using the finite element analysis based on the higher order shear deformation theory by (Rao and Ganesan, 1997). Suresh and Malhotra (1998) studied the vibration and damping behaviour of layered composite box beams using the finite element method and the finite element formulation is based on first order shear deformation theory which takes shear deformation of the beam into consideration. Aydogdu, M. (2005) studied the vibration of cross-ply laminated beams subjected to different sets of boundary conditions. The analysis is based on a three-degree-of-freedomshear deformable beam theory. Jun et al. (2008) presented a dynamic finite element method for free vibration analysis of generally laminated composite beams on the basis of first order shear deformation theory. Subramanian, P. (2006) has investigated free vibration analysis of LCBs by using two higher order displacement based on shear deformation theories and finite elements. Both theories assume a quintic and quartic variation of in-plane and transverse displacements in the thickness coordinates of the beam respectively. Results indicate application of these theories and finite element model results in natural frequencies with higher accuracy.

Most of the previous works are focused on the study of dynamic behavior of laminated composite beams, especially the out of plane bending vibration (flexural vibration). In the present study, the effects of fiber angle and laminate stacking sequence on the out-of-plane and in-plane bending frequencies for the laminated beams are investigated separately. In this study the laminated beams is modeled and analyzed by the FEM. The finite element software package ANSYS is used to perform the numerical analyses using an eight-node layered shell element. The rotary inertia and shear deformation effects are taken into account. Also, in the present work, an analytical study for dynamic behavior of laminated beams is presented; where the out-of-plane and in-plane bending vibrations of LCBs are analyzed analytically using Bernoulli-Nervier hypothesis and Timoshenko beam analysis.

2. MATERIALS AND MECHANICAL PROPERTIES

A generally laminated composite beam, as shown in Figure 1, is considered. The laminated beam is made of many plies of orthotropic materials, and the principal material axes of a ply may be oriented at an arbitrary angle with respect to the x-axis. In the right-handed Cartesian coordinate system, the x-axis is coincident with the beam axis and its origin is on the mid-plane of the beam. The length, breadth and thickness of the beam are represented by L, b and h, respectively.

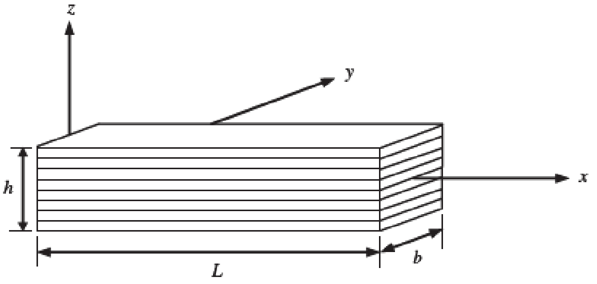


Figure 1 Geometry of a laminated composite beam

2.1. Materials characterization

Glass fiber (E-Glass) is used as reinforcement in the form of unidirectional fibers with epoxy resin as matrix for the laminated composite beams. The mechanical properties for fiber and matrix are presented in Table 1 (Danial et al., 2003). For all finite element and analytical models, their associate material elastic properties were calculated analytically using the simple rule-of-mixtures as given in (Vinson and Sierakowski, 2004). More accurate values can be further obtained with some mechanical testing.

The constituent laminae were considered to be linear elastic and generally orthotropic therefore the concept of engineering constants was used to describe the laminae elastically. A certain set of elastic properties is required as input parameters for the finite element code and for the analytical model. The set of properties required as an input parameter at a material level were E_1, E_2, E_3, G_{12}, G_{13}, G_{23}, ν_{12}, ν_{13} and ν_{23} as shown in Table 1; Where 1, 2, and 3 are principal material directions.

Table 1 Material elastic properties

Material	Properties	Value
Glass fiber	Fiber longitudinal modulus in ℓ direction Ef_ℓ (GPa)	74
	Fiber transverse modulus in t direction Ef_t (GPa)	74
	Fiber shear modulus $Gf_{\ell t}$ (Gpa)	30
	Density ρ_f (kg/m^3)	2600
	Fiber Poisson ratio $\nu f_{\ell t}$	0.25
Epoxy resin	Elastic modulus E (Gpa)	4.5
	Shear modulus G (Gpa)	1.6
	Density ρ_m (kg/m3)	1200
	Poisson ratio ν	0.4
Laminae (orthotropic)	Lamina longitudinal modulus E_1 (GPa)	46.2
	Lamina transverse modulus E_2 (GPa)	14.70
	Lamina transverse modulus E_3 (GPa)	14.70
	Density of composite ρ_c (kg/m3)	2040
	Lamina shear modulus in plane 1–2 G_{12} (GPa)	5.35
	Lamina shear modulus in plane 1–3 G_{13} (GPa)	5.35
	Lamina shear modulus in plane 2–3 G_{23} (GPa)	5.22
	Major Poisson ratio in plane 1–2 υ_{12}	0.31
	Major Poisson ratio in plane 1–3 υ_{13}	0.31
	Major Poisson ratio in plane 2–3 υ_{23}	0.41
	Fiber volume fraction ν_f	60%

3. DYNAMIC MODELING BY THE FINITE ELEMENT METHOD, ANSYS

3.1. Element type

The beams were discretized using (type shell99) finite element Figure 2, available in the commercial package ANSYS10.0. This element has 8 nodes and is constituted by layers that are designated by numbers (LN - Layer Number), increasing from the bottom to the top of the laminate; the last number quantifies the existent total number of layers in the laminate (NL - Total Number of Layers).The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. The choice of shell99 element type is based on layered applications of a structural shell model, and the type of results that need to be calculated.

x_{IJ} = Element x-axis if ESYS is not supplied.

x = Element x-axis if ESYS is supplied.

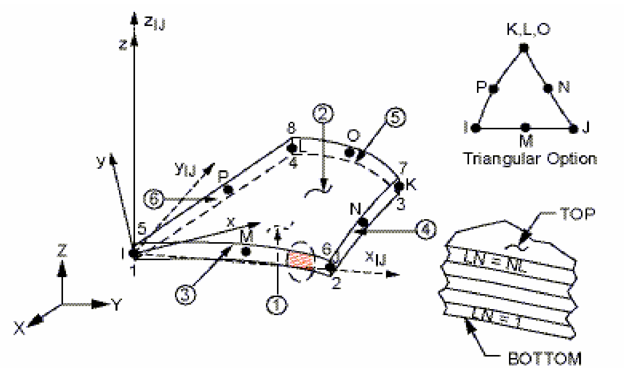


Figure 2 Shell99 geometry

3.2. Analysis type

Modal analysis will be carried out with ANSYS 10.0 finite element software. A modal analysis typically is used to determine the vibration characteristics (natural frequencies and mode shapes) of a structure or machine component in the design stage. It can also serve as a starting point for another, more detailed, dynamic analysis, such as a transient dynamic analysis, a harmonic response analysis, or a spectrum analysis.

4. DYNAMIC MODELING BY ANALYTICAL METHODS

In the present paper, the out-of-plane and in-plane bending free vibration of symmetric laminated beams are studied by the Bernoulli-Euler and Timoshenko's first-order shear beam theories.

4.1. Bernoulli-Navier hypothesis

The oldest and the well-known beam theory is the Euler–Bernoulli beam theory (or classical beam theory—CBT) which assumed that straight lines perpendicular to the mid-plane before bending remain straight and perpendicular to the mid-plane after bending. As a result of this assumption, transverse shear strain is neglected. Although this theory is useful for slender beams, it does not give accurate solutions for thick beams.

The beams to be analyzed are orthotropic and its cross section has two axes of symmetry y and z. The mass is also symmetrical with respect to these axes, and, accordingly, the center of mass coincides with the origin of the y-z coordinate system. A beam with two cross-sectional planes of symmetry may undergo flexural vibration in either of the two planes of symmetry and torsional vibration (La'szlo' and George, 2003). Flexural vibrations are focused in this study.

The natural frequencies of orthotropic beams are:

Vibration in x-z plane is given by,

$$(\omega_{yi}^{B})^2 = \frac{\overline{EI}_{yy}}{2\pi * \rho} \frac{\mu_{Bi}^4}{L^4} \qquad (1)$$

Vibration in x-y plane is given by

$$(\omega_{zi}^{B})^2 = \frac{\overline{EI}_{zz}}{2\pi * \rho} \frac{\mu_{Bi}^4}{L^4} \qquad (2)$$

Where:

$\overline{EI}_{yy}$ is the bending stiffness about y axis; in N.m^2,

$\overline{EI}_{zz}$ is the bending stiffness about z axis; in N.m^2,

L is the length of the beam, ρ is the mass per unit length, and μ^4_{Bi} for fixed-free beam are given in Table 2 .The subscript i = 1,2,.. indicates the first, second, and so forth, modes.
For symmetric orthotropic laminated beam;

The bending stiffness about y axis $\overline{EI}_{yy}$ can be obtained by this relation

$$\overline{EI}_{yy} = \frac{b}{d_{11}} \text{ in (N.m}^2\text{)} \qquad (3)$$

The bending stiffness about z axis $\overline{EI}_{zz}$ can be obtained by this relation

$$\overline{EI}_{zz} = \frac{1}{a_{11}} \frac{b^3}{12} \text{ in (N.m}^2) \tag{4}$$

Where:

d_{11}: element 1–1 of the laminate bending compliance matrix (1/N .m)

a_{11}: element 1–1 of the laminate extensional compliance matrix (m/N)

Table 2 The constants μ_{Bi} and μ_{Gi} for fixed free end support

μ_B	μ_G
$\mu_{B1} = 1.875$ $\mu_{B2} = 4.694$ $\mu_{Bi} \approx$ (i-0.5)	$\mu_{Gi} \approx$ (i-0.5)π

4.2. Timoshenko's first-order shear beam theory

The theory, based on the assumption that cross sections remain plane but not perpendicular to the axis is frequently called first-order shear theory. A beam, in which shear deformation is taken into account, is called a Timoshenko beam.

Timoshenko beam theory was developed to account for shear deformation with the assumption that the displacement field through the beam thickness does not restrict plane sections to remain perpendicular to the deformed centroidal line. However, the theory still imposes planar normals to the centroidal line to remain planar after deformation. The constant shear strain distribution throughout the beam thickness violates the shear traction free condition on the top and bottom surfaces of the beam. Shear correction factors are then employed to correct the discrepancy between results derived from exact solutions and solutions obtained via shear deformation theory.

The natural frequencies of the orthotropic beams with shear deformation are:

Vibration in x-z plane is given by, ω_{yi}

$$(\omega_{yi})^2 \approx \left[\frac{1}{(\omega_{yi}^B)^2} + \frac{1}{(\omega_{yi}^S)^2} \right]^{-1} \tag{5}$$

$$(\omega_{yi}^S)^2 = \frac{\overline{S}_{ZZ}}{2\pi * \rho} \frac{\mu_{Si}^2}{L^2} \tag{6}$$

Vibration in x-y plane is given by, ω_{Zi}

$$(\omega_{Zi})^2 \approx \left[\frac{1}{(\omega_{Zi}^B)^2} + \frac{1}{(\omega_{Zi}^S)^2} \right]^{-1} \tag{7}$$

$$(\omega_{Zi}^S)^2 = \frac{\overline{S}_{yy}}{2\pi * \rho} \frac{\mu_{Si}^2}{L^2} \tag{8}$$

Where ω^B is the natural frequency of a beam undergoing bending deformation only, ω^S is the natural frequency of a beam undergoing shear deformation only, S_{zz} is the shear stiffness in x-z plane; in N, S_{yy} is the shear stiffness in x-y plane; in N, and $\mu_{Si} = \mu_{Gi}$ for fixed-free beam is given in Table 2.

The shear stiffness in Z direction (Transverse shear stiffness) is given by, S_{zz}

$$S_{ZZ} = \frac{5}{6} b \int_{-h/2}^{h/2} \overline{Q}_{55} dz \tag{9}$$

$$\overline{Q}_{55} = G_{13} \cos^2 \vartheta + G_{23} \sin^2 \vartheta \tag{10}$$

Where $\overline{Q}_{55}$ is the transformed shear stiffness and ϑ is the angle between the fiber direction and longitudinal axis of the beam.

The shear stiffness in y direction (Lateral shear stiffness) is given by, S_{yy}

$$S_{yy} = \frac{b}{1.2 a_{66}} \tag{11}$$

Where a_{66} is element 6–6 of the laminate extensional compliance matrix (m/N)

5. NUMERICAL RESULTS AND DISCUSSION

5.1. Influence of fiber angle on out of plane bending frequencies

The influences of fiber orientation are investigated by modeling laminated beams of different lay-up construction of fixed – free boundary condition. The evaluation of the dynamic behavior was performed on beams with different lay-up with the same length, width, and thickness. The analysis was performed to 8-layered symmetrically laminated beam with length

400 mm, width 40 mm and thickness 3.2 mm and the lamination scheme of beams is ranging from $\theta = 0^0$ to 90^0, in increments of 5^0.

The following results are obtained after modeling the composite beams by FE ANSYS, Euler–Bernoulli theory, and Timoshenko beam theory. The variations of the lowest six out-of-plane bending frequencies of the laminated beams with respect to fiber angle are presented in Figures 3 and 4. From the results, it is noticed that out-of plane bending frequencies decreases, in general, as the fiber angle increases; in the case of the effect of fiber orientation, it is found that the maximum out-of-plane bending vibration frequencies occur at $\theta = 0^0$.

5.2. Influence of fiber angle on in plane bending frequencies

The influences of fiber orientation on natural frequencies of plane x-y vibration modes are investigated for the laminated beams previously mentioned, and the results are plotted in Figure 5.

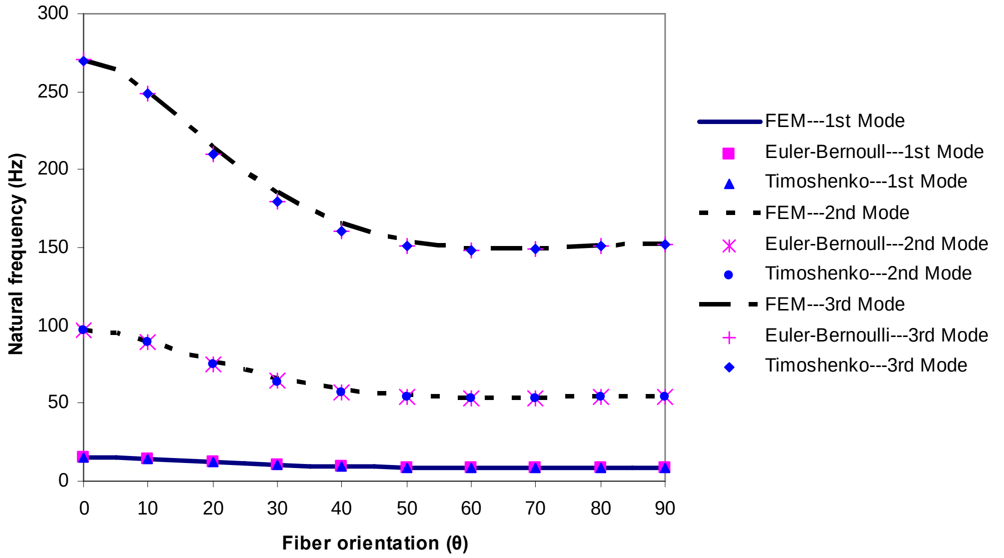


Figure 4 Variation of the out-of-plane bending frequencies of 1st, 2nd, and 3rd mode with respect to fiber angle for fixed-free boundary condition

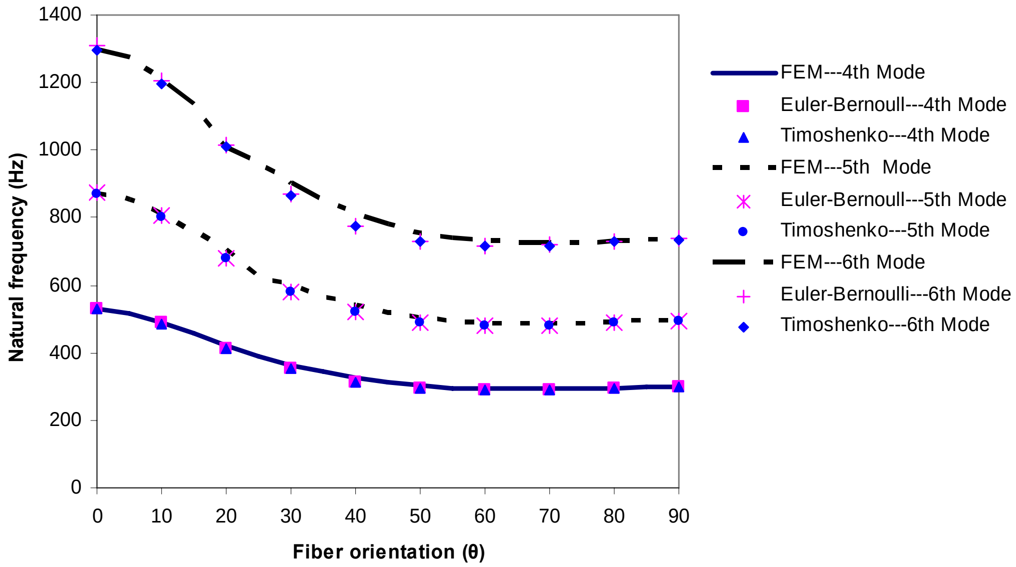


Figure 5 Variation of the out-of-plane bending frequencies of 4^{th}, 5^{th}, and 6^{th} mode with respect to fiber angle for fixed-free boundary condition.

The variation of the lowest three in-plane bending frequencies of the laminated beams with respect to fiber angle is presented in Figure 5. From the results, it was found that the in-plane bending frequencies decrease gradually with increasing fiber angle up to 70^0 then it increases by small values or nearly constant until it reaches to 90^0.

5.3. Influence of laminate stacking sequences on out-of-plane and in-plane natural frequencies

The influences of laminate stacking sequences on natural frequencies of out of plane bending and in plane bending modes are investigated by FE ANSYS, Euler–Bernoulli theory, and Timoshenko beam theory; by modeling laminated beams of different stacking sequences configuration of fixed – free boundary condition.

The dynamic modeling of the laminated beams is performed to 3 set of symmetrical laminates with a total of 8 layers and dimension of 400 mm length, 40 mm width and total thickness 3.2 mm. Assume each layer has the same thickness. The lamination schemes of the laminated beams to be modeled are as follow: (0/90)2S, (45/-45)2S, and (45/-45/0/90)S. The numerical results obtained are tabulated in Tables 3 and 4.

The mode shapes associated with the frequencies of (0/90)2S laminated beam are illustrated in Figures 6 and 7 They are deduced by FEM ANSYS for the first six out of plane bending frequencies and for the first three in plane bending frequencies.

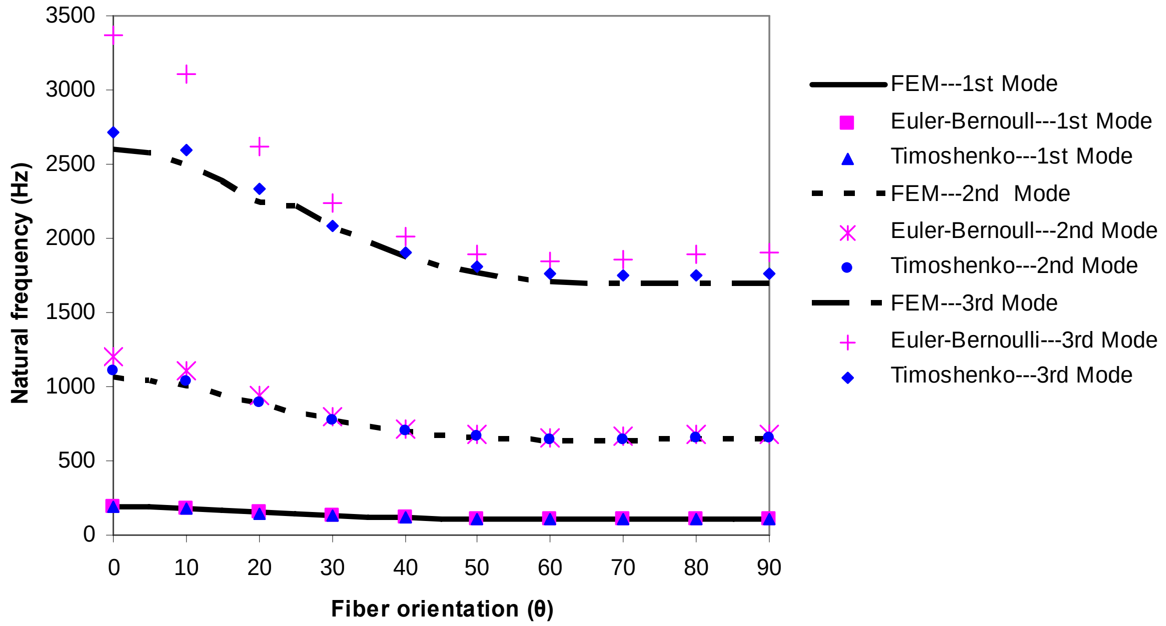


Figure 6 Variation of the lowest three in-plane bending frequencies with respect to fiber angle for fixed-free boundary condition

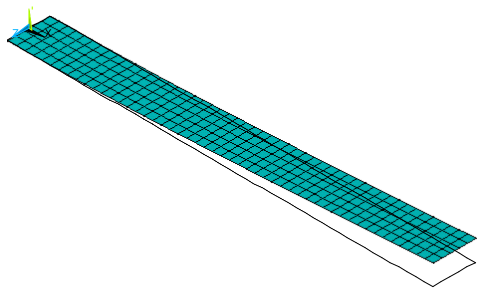
Table 3 The first six out of plane bending natural frequencies (Hz) for different stacking sequence laminates

Lamination schemes	Theory	Modes					
		1	2	3	4	5	6
$(0/90)_{2S}$	FEM ANSYS	13.71	85.85	240.14	470.00	775.67	1156.00
	Euler–Bernoulli	13.70	85.80	240.30	470.70	778.1	1162.40
	Timoshenko	13.70	85.70	240.00	469.50	774.7	1155.00
$(45/-45)_{2S}$	FEM ANSYS	9.38	58.78	165.20	325.90	542.94	817.42
	Euler–Bernoulli	9.15	57.32	160.54	314.50	520.00	776.70
	Timoshenko	9.15	57.30	160.40	314.15	518.85	774.36

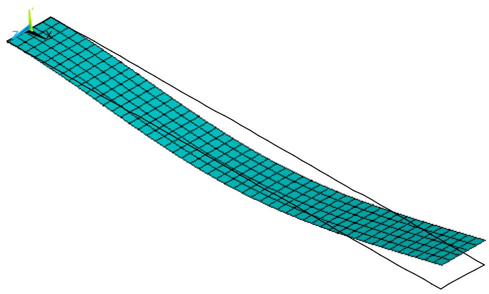
(45/-45/0/90)s	FEM ANSYS	10.27	64.30	180.45	355.20	590.20	886.24
	Euler–Bernoulli	10.06	63.07	176.70	346.00	572.00	854.60
	Timoshenko	10.06	63.05	176.53	345.60	570.70	851.56

Table 4 The first three in plane bending natural frequencies (Hz) for different stacking sequence laminates

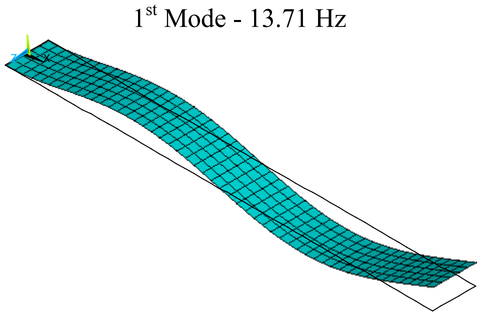
Lamination schemes	Theory	Modes		
		1	2	3
$(0/90)_{2S}$	FEM ANSYS	154.4	893.0	2256.0
	Euler–Bernoulli	156.5	980.8	2747.0
	Timoshenko	154.3	924.6	2361.3
$(45/-45)_{2S}$	FEM ANSYS	114.9	701.4	1890.0
	Euler–Bernoulli	115.0	720.5	2018.0
	Timoshenko	114.6	711.0	1945.0
(45/-45/0/90)s	FEM ANSYS	139.5	836.2	2202.0
	Euler–Bernoulli	140.2	879.0	2462.0
	Timoshenko	139.3	854.7	2283.5



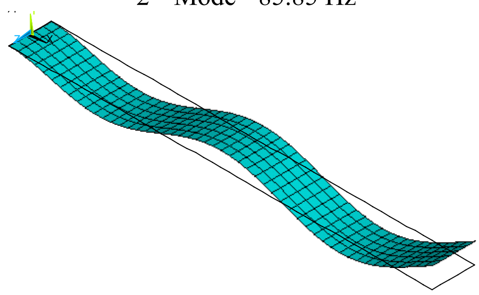
1st Mode - 13.71 Hz



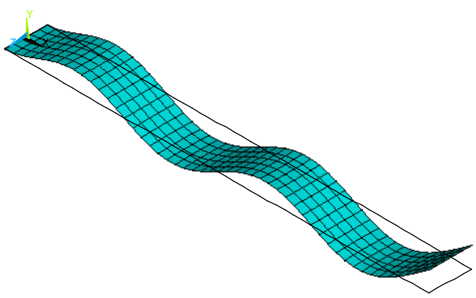
2nd Mode - 85.85 Hz



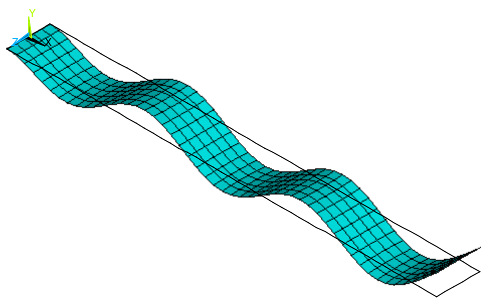
3rd Mode - 240.14 Hz



4th Mode - 470 Hz

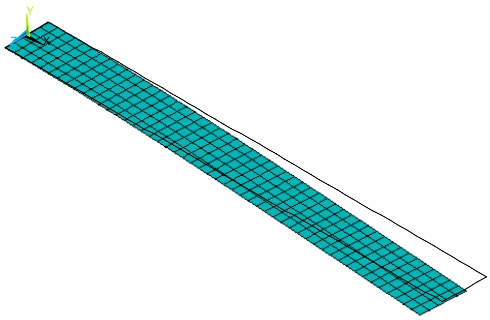


5th Mode - 775.67 Hz

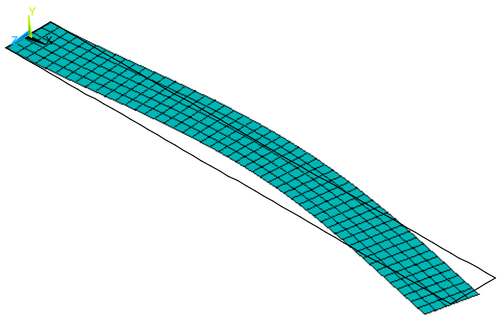


6th Mode - 1156 Hz

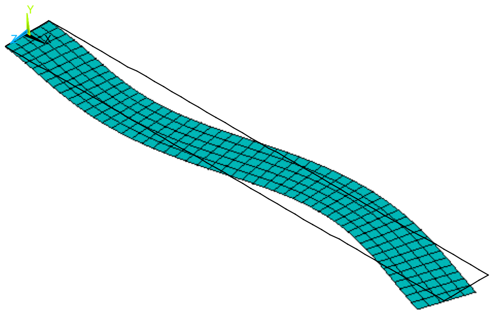
Figure 7 The first six out of plane bending vibration modes of fixed-free beam



1st Mode - 154.4 Hz



2nd Mode - 893 Hz



3rd Mode - 2256 Hz

Figure 8 The first three in plane bending vibration modes of fixed-free beam

The results presented in Figure 8 are obtained by FE package after modeling the three laminate configurations. It represents the variations of the out-of-plane bending frequencies with respect to mode number of the 3 laminate schemes for fixed free boundary conditions. From the results, it is already possible to verify the influence of the stacking sequence of the laminates on out-plane frequencies; the laminate with fibers at +/-45° has in general smaller natural frequencies than the laminate with +/-45/0/90° layers and than the laminate with fibers at 0° and 90°.

The laminate with fibers at 0° and 90° has a larger natural frequency for out of plane bending (Flexural Modes) than the laminate of +/-45/0/90° and than the (+/-45°) laminate. Because 50% of the fibers are oriented at 0^0 direction for $0/90^0$ laminate, and thus appropriate for bending (Flexural Modes). This can be explained by the fact that the fibers oriented at 0° are more appropriate to flexural loads.

The variation of the in-plane bending frequencies of the 3 laminate schemes with respect to mode number for fixed free boundary conditions is presented in Figure 9. From the results it is already possible to verify the influence of the stacking sequence of the laminates on in-plane bending frequencies (Lateral modes). The laminate with fibers at $0/90^0$ has in general larger natural frequencies than the laminate with $+/-45/0/90^0$ layers and than the laminate with fibers at $+/-45^0$. This is because 50% of the fibers are oriented at 0^0 for $0/90^0$ laminate.

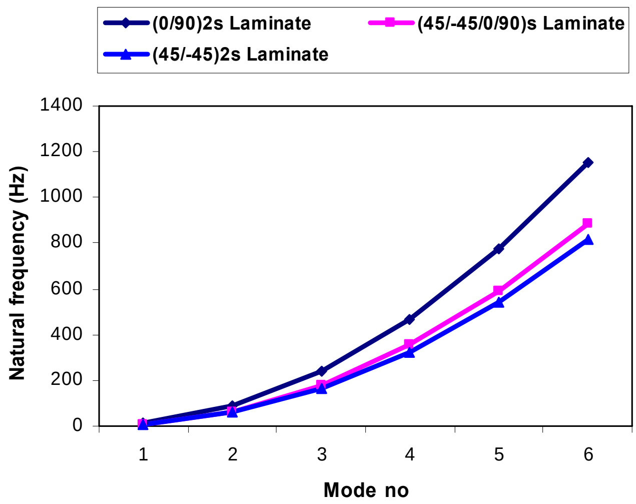


Figure 9 Influence of laminate stacking sequence on natural frequencies of out of plane vibration modes for fixed free boundary condition

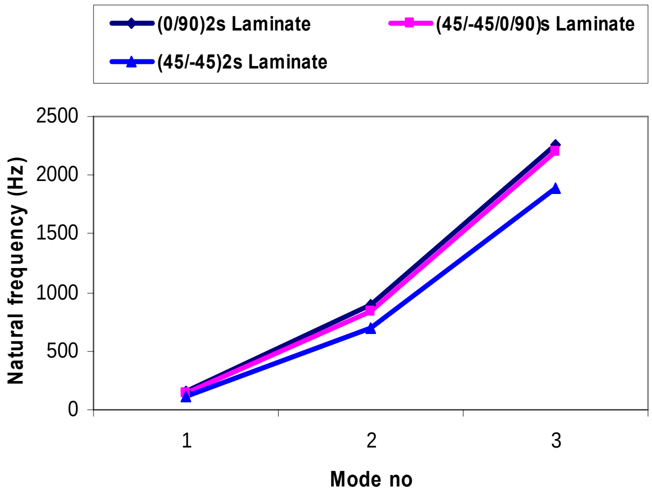


Figure 10 Influence of laminate stacking sequence on natural frequencies of in-plane vibration modes for fixed free boundary condition.

6. CONCLUSION

From the results, it is clear that changes in fiber angle as well as laminate stacking sequences yield to different dynamic behavior of the component, that is, different natural frequencies for the same geometry, mass and boundary conditions. This gives the designer one additional degree of freedom to design the laminate - the possibility to change fiber orientations in order to get more (or less) structure stiffness. This possibility makes once more these materials very attractive since it makes possible to obtain the desired natural frequencies without increasing mass or changing geometry. In practical applications, it means that if a natural frequency excites the structure, the designer can change the material properties by changing the laminate stacking sequence, instead of re-design the complete structure.

Also from the results, it has found that the out-of plane and in-plane bending frequencies decrease, in general, as the fiber angle increases and the maximum out-of-plane and in-plane bending vibration frequencies occur at $\theta = 0^0$.

The theoretical results from Finite Element Analysis, ANSYS showed in general a good agreement with the numerical results values obtained by Bernoulli hypothesis and Timoshenko beam theory. This agreement is clearly shown for the out-of-plane lower modes than the higher modes.

The finite element software package ANSYS is an efficient vibration prediction tool, because of its ability to model the laminated composite beam and reveal fundamental modal frequencies and modal shapes. (By using of shell element SHELL99), it is useful for simpleness (saving of computing time). Finally this study is useful for the designer in order to select the fiber orientation angle to shift the natural frequencies as desired or to control the vibration level.

REFERENCES

Abramovich A, Livshits A., (1994), "Free vibration of non-symmetric cross-ply laminated composite beams", Journal of Sound and Vibration, Vol. 38, pp. 597-612.

Abramovich H., (1992), "Shear deformations and rotary inertia effects of vibrating composite beams", Composite Structures, Vol. 20, pp. 165-73.

Aydogdu M., (2005), "Vibration analysis of cross-ply laminated beams with general boundary conditions by Ritz method", International Journal of Mechanical Sciences, Vol.47, pp. 1740–1755.

Banerjee, J.R., (1998), "Free vibration of axially loaded composite timoshenko beams using the dynamic stiffness matrix method", Computers and Structures, Vol. 69, pp. 197-208.

Bassiouni AS, Gad-Elrab RM, Elmahdy TH., (1999), "Dynamic analysis for laminated composite beams", Composite Structures, Vol. 44 pp. 81–87.

Bhimaraddi A., Chandrashekhara K., (1991), "Some observations on the modeling of laminated composite beams with general lay-ups", Composite Structures, Vol. 19, pp. 371-380.

Chandrashekhara K., Bangera K.M., (1992), "Free vibration of composite beams using a refined shear flexible beam element", Computer and Structures, Vol. 43, pp. 719-727.

Danial Gay, Hoa SV, Tsai SW., (2003), Composite materials design and applications, Boca Raton, London New York, Washington.

Eisenberger M, Abramovich H, Shulepov O., (1995), "Dynamic stiffness analysis of laminated beams using a first-order shear deformation theory", Composite Structure, Vol.31, pp. 265-271.

Jun L, Hongxing H, Rongying S., (2008), "Dynamic finite element method for generally laminated composite beams", International Journal of Mechanical Sciences, Vol. 50, pp. 466–480.

Jun L, Hongxing H, Rongying S., (2008), "Dynamic stiffness analysis for free vibrations of axially loaded laminated composite beams", Composite Structures, Vol. 84, pp. 87–98.

Kant T, Marurb SR, Rao GS., (1998), "Analytical solution to the dynamic analysis of laminated beams using higher order refined theory", Composite Structures, Vol.40, pp. 1–9.

Kapuria S, Alam N., (2006), "Efficient layerwise finite element model for dynamic analysis of laminated piezoelectric beams", Computer Methods in Applied Mechanics and engineering, Vol. 195, pp. 2742–2760.

Khdeir A.A., Reddy J.N., (1994), "Free vibration of cross-ply laminated beams with arbitrary boundary conditions", International Journal of Engineering Science, Vol. 32, pp.1971-1980.

Krishnaswamy S., Chandrashekhara K., Wu W.Z.B., (1992), "Analytical solutions to vibration of generally layered composite beams"., Journal of Sound and Vibration, Vol. 159, pp. 85-99.

La'szlo' P., George S., (2003), Mechanics of composite structures, Cambridge University, New York, United States of America.

Maiti D.K., Sinha P.K., (1994), "Bending and free vibration analysis of shear deformable laminated composite beams by finite element method", Composite Structures, Vol.29, pp. 421-431.

Matsunaga H., (2001), "Vibration and buckling of multilayered composite beams according to higher order deformation theories", Journal of Sound and Vibration, Vol. 246, pp.47-62.

Miller AK, Adams DF., (1975), "An analytic means of determining the flexural and torsional resonant frequencies of generally orthotropic beams", Journal of Sound and Vibration, Vol. 41, pp. 433-449.

Murthy M.V.V.S., Mahapatra D.R., Badarinarayana K., Gopalakrishnan S., (2005) "A refined higher order finite element for asymmetric composite beams", Composite Structures, Vol.67, pp. 27-35.

Nabi MS, Ganesan N., (1994), "Generalized element for the free vibration analysis of composite beams", Computer and Structures, Vol.51, pp. 607-610.

Qiao Pizhong, Zou Guiping., (2002), "Free vibration analysis of fiber-reinforced plastic composite cantilever I-beams", Mechanics of Advanced Materials and Structure, Vol. 9, pp. 359–373.

Ramtekkar G.S., Desai Y.M., Shah A.H., (2002), "Natural vibrations of laminated composite beams by using mixed finite element modeling", Journal of Sound and Vibration, Vol.257, pp. 635-651.

Rao MK, Desai YM, Chitnis MR., (2001), "Free vibrations of laminated beams using mixed theory",Composite Structures, Vol. 52, pp. 149–60.

Rao SM, Ganesan N., (1997), "Dynamic response of non-uniform composite beams", Journal of Sound and Vibration, Vol. 200, pp. 563-77.

Song SJ, Waas A., (1997), "Effects of shear deformation on buckling and free vibration of laminated composite beams", Composite Structures, Vol.37, pp.33-43.

Subramanian, P., (2006), "Dynamic analysis of laminated composite beams using higher order theories and finite elements", Composite Structures, Vol. 73, pp. 342-353.

Suresh KJ, Venkatesan C, Ramamurti V., (1990), "Structural dynamic analysis of composite beams", Journal of Sound and Vibration, Vol.143, pp. 503-519.

Suresh R, Malhotra SK.,(1998), "Vibration and damping analysis of thin-walled box beams", Journal of Sound and Vibration, Vol. 215, pp. 201-210.

Tahani M., (2007) "Analysis of laminated composite beams using layerwise displacement theories", Composite Structures, Vol.79, pp.535–47.

Teh KK, Huang CC., (1979), "The vibration of generally orthotropic beams-A finite element approach", Journal of Sound and Vibration, Vol.62, pp.195-206.

Tita V, Carvalho J, Lirani J., (2003), "Theoretical and experimental dynamic analysis of fiber reinforced composite beams", Journal of the Brazilian Society of Mechanical Sciences and Engineering, Vol. 25, pp. 306-310.

Vinson JR, Sierakowski RL., (2004), The behaviour of structures composed of composite materials, New York, Boston, Dordrecht, Moscow.

Yildirim V., (2000), "Effect of the longitudinal to transverse moduli ratio on the in-plane natural frequencies of symmetric cross-ply laminated beams by the stiffness method" Composite Structures, Vol.50, pp. 319–26.

Yõldõrõm V, Kõral E., (2000), "Investigation of the rotary inertia and shear deformation effects on the out-of-plane bending and torsional natural frequencies of laminated beams", Composite Structures,Vol.49, pp. 313-320.

Zapfe JA, Lesieutre GA., (1997) "Vibration analysis of laminated beams using an iterative smeared laminated model", Journal of Sound and Vibration, Vol. 199, pp. 275-84.