Fayoum University
Faculty of Engineering
2nd Year Elect. & Commun. Dept.

Model Answer

Applied Statistics
Final Term Exam.
June, 22, 2011

Time: 3 Hrs

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Max. Points: 70

Answer of Q1

(20 Marks)

الرقم السري

ورقة الإجابة الإلكترونية

إختبار نهاية الفصل الدراسي الثاني
للعام الدراسي 2010 / 2011 م

Fayoum University

عزيزي الطالب يرجى مراعاة الآتي عند الإجابة على الأسئلة الموضوعية في ورقة الإجابة الالكترونية:

ظلل (سود) الإجابة الصحيحة في ورقة الإجابة الالكترونية, بحيث يركز تظليل الإجابة في مركز الدائرة هكذا :
لا يعتد بالإجابة عند اختيار إجابتين أو أكثر ما لم يطلب منك غير ذلك.
ممنوع استخدام الكوريكتور أو المزيل.
اكتب بياناتك بالقلم الحبر الجاف في المكان المخصص الموجود أعلى الورقة.
يستخدم القلم الرصاص أولا وبعد التأكد من الإجابة الصحيحة استخدم القلم الجاف لتأكيد الإجابة.
تتم الإجابة على الأسئلة المقالية في كراسة الإجابة العادية.
بعد الانتهاء من الامتحان ضع ورقة الإجابة الالكترونية داخل كراسة الأسئلة ثم ضعهما معا داخل كراسة الإجابة العادية وسلم الجميع الى الأستاذ الملاحظ.

نموذج إمتحان

ظلل أو (سود) الدائرة الدالة على رمز الإجابة الصحيحة من الإجابات المطبوعة في ورقة الأسئلة

	A	B	C	D
1				●
2			●	
3		●		
4		●		
5			●	
6				●
7	●			
8		●		
9			●	
10			●	

Answer of Q2

a) From the given data we find that:

$$\left.\begin{aligned}&P(D_1)=P(G_1)=5/10=1/2\\&P(D_2/D_1)=5/10=1/2,\quad P(D_2/G_1)=4/10=2/5\\&P(D_2)=P(D_2/D_1)P(D_1)+P(D_2/G_1)P(G_1)\\&=(1/2)(1/2)+(1/2)(2/5)=0.45\end{aligned}\right\}$$ ....**(2 Marks)**

Note that the required probability is $P(A/D_2)$ **. Where** A **is the deffective circuit that drawn from Box2 is the same circuit that drawn from Box1**

Then we find that:

$$\left.\begin{aligned}&P(A)=P(A/D_1)P(D_1)+P(A/G_1)P(G_1)\\&=(1/10)(1/2)+(0)(1/2)=1/20\end{aligned}\right\}$$ .... **(1 Mark)**

Then the required probability may be written as

$$P(A/D_2)=\frac{P(A\cap D_2)}{P(D_2)}=\frac{P(A)}{P(D_2)}$$ ....**(1 Mark)**

$$\boxed{P(A/D_2)=1/9\cong 0.111}$$ ....**(1 Mark)**

b) Assume that face number = i **and its pobability =** ci

$\sum_{i=1}^{6}ki=1 \Rightarrow c=1/21$. **Then, the face of number 6 will appear in one toss with probability** $p=6/21=2/7$ ....**(1 Mark)**

Tossing the die 9 times, yields a binomial distribution with

$$\left.\begin{aligned}&q=1-p=5/7,\ n=9,\\&p(x)={}^{n}C_x\ p^x q^{n-x}={}^{9}C_x\ (2/7)^x(5/7)^{9-x}\end{aligned}\right\}$$ ....**(1 Marks)**

The probrbility that the face of number 6 will appear at least twice is

$$P(X\geq 2)=1-P(X\leq 1)=1-p(0)-p(1)=1-\left(\frac{5}{7}\right)^9-9\left(\frac{2}{7}\right)\left(\frac{5}{7}\right)^8$$ ....**2 Mark)**

$$\boxed{P(X\geq 2)=0.777}$$ ....**(1 Mark)**

Answer of Q3

a)

$$P[(X<0.616)\cap(X>0.333)]=\int_{0.333}^{0.616}30\,x^2\,(1-x)^2\,dx$$

$$=30\int_{0.333}^{0.616}(x^2-2x^3+x^4)dx$$

............**(2 Marks)**

$$=30\left[\frac{x^3}{3}-\frac{x^4}{2}+\frac{x^5}{5}\right]_{0.333}^{0.616}$$

$$=30\left[\frac{.616^3}{3}-\frac{.616^4}{2}+\frac{.616^5}{5}\right]-30\left[\frac{.333^3}{3}-\frac{.333^4}{2}+\frac{.333^5}{5}\right]$$

..............**(2 Marks)**

$\boxed{P[(X<0.616)\cap(X>0.333)]=0.500}$ ....**(1 Mark)**

b)

$$E\left((X+m)^2-m_2\right)=E\left(X^2+2mX+m^2-m_2\right)=E\left(X^2\right)+2mE(X)+m^2-m_2$$

$$=E\left(X^2\right)+3m^2-\left(E\left(X^2\right)-m^2\right)=4m^2$$

....**(2 Marks)**

$$m=\int_{-\infty}^{\infty}x\,f(x)dx=30\int_0^1 x^3\,(1-x)^2\,dx=30\int_0^1\left(x^3-2x^4+x^5\right)dx$$

$$=30\left[\frac{x^4}{4}-2\frac{x^5}{5}+\frac{x^6}{6}\right]_0^1=30\left[\frac{1}{4}-\frac{2}{5}+\frac{1}{6}\right]=\frac{1}{2}$$

..............**(2 Marks)**

$\boxed{\Rightarrow E\left((X+m)^2-m_2\right)=\frac{4}{4}=1}$ ....**(1 Mark)**

Answer of Q4

a) Using the given table, we may construct the following table

(x, y)	(1, 5)	(1, 8)	(1, 12)	(3, 5)	(3, 8)	(3,12)
$p(x, y)$	0.05	0.08	0.12	0.15	0.24	0.36
$p_X(x)\ p_Y(y)$	0.05	0.08	0.12	0.15	0.24	0.36

……….**(2 Marks)**

Then the random variables X and Y are independent, since

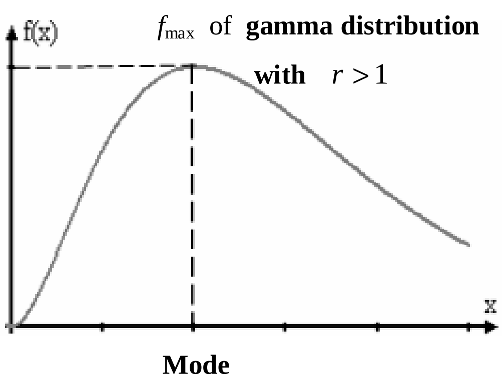
$p(x, y) = p_X(x)\, p_Y(y)$ for all points (x, y) ……………………………………...**(2 Marks)**

Thus, the correlation coefficient

$r(X, Y) = 0$ ……………………………..…………..…………...…………**(1 Marks)**

b) The gamma distribution is written as

$$f(x) = \frac{l}{\Gamma(r)}(l\,x)^{r-1} e^{-l\,x}, \quad \text{where,} \quad r\,,\, l\,,\, x > 0$$



……….……………………………..…..**(1 Mark)**

The critical points is calculated by solving the equation

$f'(x) = 0$ …………………………………………………………………..…..**(1 Mark)**

$$\Rightarrow \frac{l^2}{\Gamma(r)}\left[(r-1) - l\,x\right](l\,x)^{r-2} e^{-l\,x} = 0$$

$$\Rightarrow \left[(r-1) - l\,x\right] = 0$$

……………………..……………..**(2 Marks)**

$$\Rightarrow x = (r-1)/\,l$$

$\Rightarrow \text{Mode} = (r-1)/\,l$ ……………………………..……………….……..**(1 Mark)**

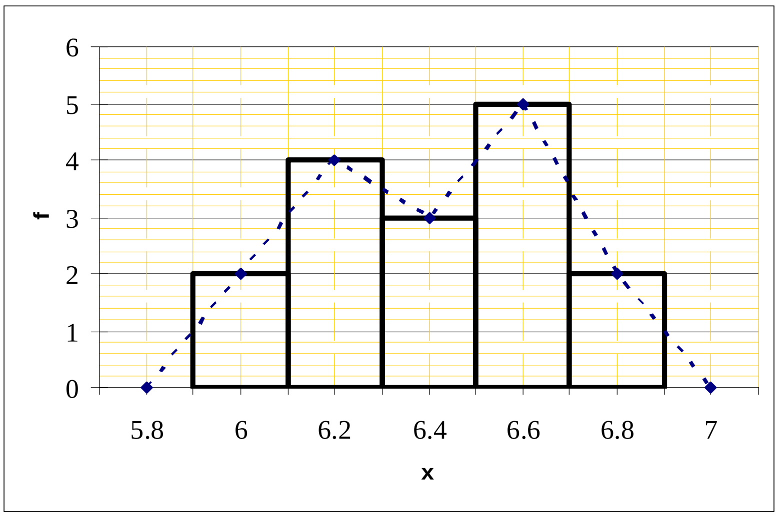
Answer of Q5

a) Using the given data, we may construct the following frequency table

x	6	6.2	6.4	6.6	6.8
f	2	4	3	5	2

………..………….(1 Mark)

The required Histogram and frequency polygon are shown in the following chart



…………...(4 Marks)

b) The given (raw) data result:

$$\bar{x} = \frac{1}{16}\sum_{i=1}^{16} x_i = \frac{1}{16}\left(6+6.6+\ldots+6.8\right) = 6.4125,$$

$$s = \frac{1}{15}\sum_{i=1}^{16}(x_i-\bar{x})^2 = \frac{1}{15}\left((6-6.4125)^2+\ldots+(6.6-6.4125)^2\right)$$

$$= 0.25787593$$

……………..(2 Marks)

In this case, $\bar{X}$ is t distributed (Since n is small and s is unknown but X is normally distributed). Thus

$$P(\bar{X} < 6.387) = P\left(t_{15,a} < \frac{6.387-6.5}{0.25787593/4}\right) = P\left(t_{15,a} < \frac{6.387-6.5}{0.25787593/4}\right)$$

$$= P\left(t_{15,a} < -1.753\right) = P\left(t_{15,a} > 1.753\right) = a$$

…….(2 Marks)

$$= P\left(\bar{X} < 6.387\right) = 0.05$$

……………………………………..……….…….(1 Mark)

Answer of Q6

a) In this case, $\overline{X}$ is normally distributed (Since s is known but X is normally distributed). Thus

The 90% confidence interal on m is

$$\left(\overline{x} - z_{a/2}\left(s/\sqrt{n}\right), \ \overline{x} + z_{a/2}\left(s/\sqrt{n}\right)\right) = \overline{x} \pm z_{a/2}\left(s/\sqrt{n}\right)$$

$$40.31 \pm z_{0.05}\left(10/4\right) = 40.31 \pm 1.645 * \left(10/4\right)$$

……………….…..….(4 Marks)

So, the 90% confidence interal on m is

$\left(36.198, 44.423\right)$ **………………………….……………………...……….…….(1 Mark)**

b) Note that the test,

$H_0 : m = 37$,

$H_A : m > 37$,

is right-tailed, then the criteria is written as

$C = m_0 + z_a \left(s/\sqrt{n}\right)$ **………….…………………………………………….……………(1 Mark)**

The given data are: n=16, $s = 10$, $\overline{x} = 40.31$ and $a = 0.05$. Then

$C = 37 + z_{0.05}\left(10/4\right) = 37 + 1.645 * (10/4) = 41.113$ **…………….……………...……..(2 Marks)**

Since $\overline{x} = 40.31 \Rightarrow \overline{x} < C$. Then,

We accept H_0 ……….…………………………………………………………..(2 Marks)

دعـــواتــي بالــتــفـــوق .... د/ كـــارم محمـــود عويـــس