Model Answer

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Final Exam - Final Mark = 75
Time Allowed : 3 hours

Answer of (Q1)

No.	(A)	(B)	(C)	(D)
1				▨
2				▨
3				▨
4			▨	
5		▨		
6			▨	
7		▨		
8			▨	
9				▨
10			▨	
11		▨		
12	▨			
13		▨		
14			▨	
15				▨

<u>Answer of (Q2)</u>

2-a) We may use the Baye's theorem and combination, where,

The event *A* represents : drawing 3 black balls and 2 white balls

The event *I* represents : drawing the box *I*

The event *II* represents : drawing the box *II*

$$\mathbf{P(I/A)} = \frac{P(I)P(A/I)}{P(I)P(A/I)+P(II)P(A/II)} = \frac{1/2 \; * \; {}^{4}C_{2}\; {}^{6}C_{3} \;/\; {}^{10}C_{5}}{1/2 \; * \; {}^{4}C_{2}\; {}^{6}C_{3} \;/\; {}^{10}C_{5} + 1/2 \; * \; {}^{3}C_{2}\; {}^{7}C_{3} \;/\; {}^{10}C_{5}}$$

$\Rightarrow$ $\boxed{\mathbf{P(I/A) = 0.533}}$

2-b) This is a geometric distribution:

$$\Rightarrow \quad f(x) = p\, q^{x} \quad , \quad x = 0,\ 1,\ 2,\ 3,\ \ldots\ldots\ldots \quad \textbf{,} \text{ with } \; p = 1/2 \;\Rightarrow\; q = 1/2$$

$$f(x) = \left(\frac{1}{2}\right)^{x+1} \quad , \quad x = 0,\ 1,\ 2,\ 3,\ \ldots\ldots\ldots$$

$$\Rightarrow \quad F(x) = 1 - q^{x+1} = 1 - \left(\frac{1}{2}\right)^{x+1} \quad , \quad x = 0,\ 1,\ 2,\ 3,\ \ldots\ldots\ldots$$

$$\Rightarrow \quad P(X+1 > 10) = P(X > 9) = 1 - P(X \le 9) = 1 - F(9) = \left(\frac{1}{2}\right)^{10} \; = \; \boxed{0.001}$$

Answer of (Q3)

3-a) $F(x) = \int_{-\infty}^{x} f(X)\,dX = \frac{10}{7}\int_{0}^{x} X^{3/7}\,dX = \frac{10}{7}\,\frac{7}{10}\,X^{10/7}\Big|_{0}^{X} = x^{10/7}$

$\Rightarrow$ The cumulative probability distribution function is written as:

$$F(x) = \begin{cases} 0 & x < 0 \\ x^{10/7} & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

Note: We can solve (b, c) using the non-central moment:

$$m'_k = E(x^k) = \int_{-\infty}^{\infty} x^k\ f(x)\,dx\ \frac{10}{7}\int_{0}^{1} x^k\ x^{3/7}\ dx = \frac{10}{7}\int_{0}^{1} x^{k+3/7}\ dx = \frac{10}{7}\ \frac{7}{7k+10}\ x^{k+10/7}\Big|_{0}^{1} \Rightarrow m'_k = \frac{10}{7k+10}$$

3-b) $m = m'_1 = \frac{10}{7(1)+10} = \frac{10}{17}$

The mean is written as: $m = 0.588$

$s^2 = m'_2 - m^2 = \frac{10}{7(2)+10} - \left(\frac{10}{17}\right)^2$

The variance is written as: $s^2 = 0.071$

3-c) $m'_k = \frac{10}{7k+10} \Rightarrow m'_4 = \frac{10}{7(4)+10} = \frac{10}{38} = \frac{5}{19}$

The fourth non central moment is written as: $m'_4 = 0.263$

Answer of (Q4)

4-a) The given data: s =1.52, a =0.10, n=5, $\bar{x} = 35$

100(1-α) % confidence interval on m is

$$\left(\bar{X} - z_{a/2}\ s/\sqrt{n},\ \bar{X} + z_{a/2}\ s/\sqrt{n}\right)$$

$\Rightarrow$ 90 % confidence interval on m is

$$\left(35 - z_{0.05}\ 1.52/\sqrt{5},\ 35 + z_{0.05}\ 1.52/\sqrt{5}\right)$$

$$= \left(35 - 1.645 * 1.52/\sqrt{5},\ 35 + 1.645 * 1.52/\sqrt{5}\right)$$

$\Rightarrow$ 90 % confidence interval on m is $\boxed{(\ 33.882,\ \ 36.118\ )}$

4-b) The given data: m_0 =36 , s =1.52, a =0.05, n=5, $\bar{x} = 35$

$H_0: \quad m = 36$

$H_A: \quad m \leq 36$

The required criteria is written as:

$$C = m_0 - z_a \frac{s}{\sqrt{n}} = 36 - 1.645 \frac{1.52}{\sqrt{5}} = \boxed{34.88}$$

Since $\bar{x} = 35 \quad \Rightarrow \quad \bar{x} > C$

$\Rightarrow$ $\boxed{\text{Decision : We accept } H_0}$.

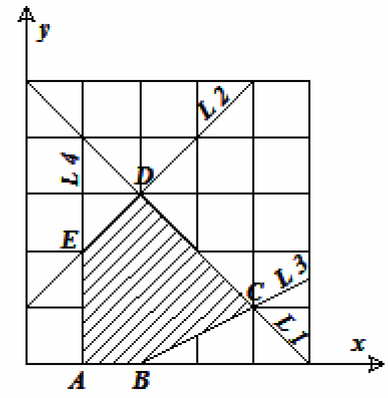
Answer of (Q5)

5-a) A=(1,0) , B=(2,0)

C is computed from L1 and L3 $\Rightarrow$ C =(4,1)

D is computed from L1 and L2 $\Rightarrow$ D=(2,3)

E is computed from L2 and L4 $\Rightarrow$ E=(1,2)



Point	A(1,0)	B(2,0)	C(4,1)	D(2,3)	E(1,2)
z	7	14	26	8	3

$\Rightarrow$ $z_{min} = 3$

5-b) The objective function is $z = 8x + 5y$

$\Rightarrow z - 8x - 5y = 0$ (1)

The constraints are : $-x + y \le 1$, $x + y \le 2$,

Using the *simplex method*, the constraints are written in the form:

$-x + y + s_1 = 1$ (2)

$x + y + s_2 = 2$ (3)

Using equations (1, 2 and 3), we may construct the following table

pivot column (x) ↓

	z	x	y	s_1	s_2	Solution	Ratio
z	1	-8	-5	0	0	0	----
s_1	0	-1	1	1	0	1	-1
s_2	0	1	1	0	1	2	2 ← pivot row
z	1	0	3	0	8	16	----
s_1	0	0	2	1	1	3	----
x	0	1	1	0	1	2	----

Note: All coefficients in z are non-negative $\Rightarrow$ Stop calculations

$\Rightarrow$ z is maximum $\Rightarrow$ $z_{max} = 16$