Fayoum University
Faculty of Engineering
Arch. Eng. Department
First Year

Mathematics & Statistics
Jan., 10, 2011
Final Exam - Final Mark = 75
Time Allowed : 3 hours

تخلف التخلف *Model Answer*

1-A) $$\frac{\mathrm{P(A) - P(A \cap B)}}{\mathrm{1 - P(B)}} = \frac{P(A) - P(A)P(B)}{1 - P(B)} = \frac{P(A)(1 - P(B))}{1 - P(B)} = \mathrm{P(A)}$$

1-B) We may use the Baye's theorem and combination, where,

The event *A* represents : drawing 5 black balls and 5 white balls

The event *I* represents : drawing the box *I*

The event *II* represents : drawing the box *II*

$$\mathbf{P(I/A)} = \frac{P(I)P(A/I)}{P(I)P(A/I) + P(II)P(A/II)} = \frac{1/2 * {}^{25}C_5 \; {}^{25}C_5 \; / \; {}^{50}C_{10}}{1/2 * {}^{25}C_5 \; {}^{25}C_5 \; / \; {}^{50}C_{10} + 1/2 * {}^{35}C_5 \; {}^{15}C_5 \; / \; {}^{50}C_{10}}$$

$$\Rightarrow \boxed{\mathbf{P(I/A) = 0.743}}$$

2-A) $$\int_{-\infty}^{\infty} f(x)\,dx = \int_{0}^{\infty} l \; e^{-l x}\,dx = \frac{l}{-l} e^{-l x}\Big|_0^{\infty} = \frac{l}{-l} e^{-l x}\Big|_0^{\infty} = -(0-1) = 1$$

2-B) This is a geometric distribution:

$$\Rightarrow \quad f(x) = p\,q^x \quad , \quad x = 0,\ 1,\ 2,\ 3,\ \ldots\ldots\ldots \quad \textbf{, with } \boldsymbol{p = 1/6} \Rightarrow \boldsymbol{q = 5/6}$$

$$f(x) = \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^x \quad , \quad x = 0,\ 1,\ 2,\ 3,\ \ldots\ldots\ldots$$

$$\Rightarrow \quad F(x) = 1 - q^{x+1} = 1 - \left(\frac{5}{6}\right)^{x+1} \quad , \quad x = 0,\ 1,\ 2,\ 3,\ \ldots\ldots\ldots$$

$$\Rightarrow \quad P(X+1 \le 4) = P(X \le 3) = F(3) = 1 - \left(\frac{5}{6}\right)^4 \qquad = \boxed{\mathbf{0.518}}$$

3-A) $F(x) = \int_{-\infty}^{x} f(X)\,dX = \frac{3}{2}\int_{0}^{x} \sqrt{X}\,dX = \frac{3}{2}\,\frac{2}{3}\,X^{3/2}\Big|_{0}^{X} = x^{3/2} \Rightarrow F(x) = \begin{cases} 0 & x < 0 \\ x^{3/2} & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$

3-B) $m'_k = E\left(x^k\right) = \int_{-\infty}^{\infty} x^k\, f(x)\,dx = \frac{3}{2}\int_{0}^{1} x^k\,\sqrt{x}\,dx = \frac{3}{2}\int_{0}^{1} x^{k+1/2}\,dx = \frac{3}{2}\,\frac{2}{2k+3}\,x^{k+3/2}\Big|_{0}^{1} = \frac{3}{2k+3}$

$$\Rightarrow \boxed{m'_k = \frac{3}{2k+3}}$$

$\Rightarrow$ **The mean** $m = m'_1 = \frac{3}{2(1)+3} \Rightarrow \frac{3}{5} \Rightarrow \boxed{m = 0.6}$

$\Rightarrow$ **The variance** $s^2 = m'_2 - m^2 = \frac{3}{2(2)+3} - \left(\frac{3}{5}\right)^2 \Rightarrow \boxed{s^2 = 0.069}$

3-C) $m'_k = \frac{3}{2k+3} \Rightarrow m'_4 = \frac{3}{2(4)+3} = \frac{3}{11} \Rightarrow \boxed{m'_4 = 0.273}$

4-A) $P\left(\overline{X} \le m + Z_{2a}\, s/\sqrt{n}\right) = P\left(\frac{\overline{X} - m}{s/\sqrt{n}} \le Z_{2a}\right)$

Using the transformation $Z = \frac{\overline{X} - m}{s/\sqrt{n}}$**, we find that**

$$P\left(\frac{\overline{X} - m}{s/\sqrt{n}} \le Z_{2a}\right) = P\left(Z \le Z_{2a}\right) = f\left(Z_{2a}\right) = 1 - 2a$$

4-B) $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{10}\sum_{i=1}^{10} x_i = 25.5$ **,** $s^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \overline{x})^2 = \frac{1}{9}\sum_{i=1}^{10}(x_i - 25.5)^2 = 10.5$

$\Rightarrow s = 3.24$

s **is unknown and *n* is small and** $X \sim N(m, s) \Rightarrow$ **use t-distribution**

(1-a) % confidence interval on m **is** $\left(\overline{X} - t_{n-1,a/2}\; s/\sqrt{n},\; \overline{X} + t_{n-1,a/2}\; s/\sqrt{n}\right)$

$\Rightarrow$ **95 % confidence interval on** m **is** $\left(25.5 - t_{9,0.025}\; 3.24/\sqrt{10},\; 25.5 + t_{9,0.025}\; 3.24/\sqrt{10}\right)$

$= \left(25.5 - 2.262 * 3.24/\sqrt{10},\; 25.5 + 2.262 * 3.24/\sqrt{10}\right)$

$\Rightarrow$ **95 % confidence interval on** m **is** $\boxed{(\,22.182,\quad 27.818\,)}$

5-A) s is unknown and n is small and $X \sim N(m, s) \Rightarrow$ using t-distribution, we may write:

$$H_0 : \quad m = m_0$$

$$H_A : \quad m \le m_0$$

$$P\left(t > -t_{n-1,a}\right) = 1 - a \quad = P\left(\frac{\bar{x} - m_0}{s/\sqrt{n}} > -t_{n-1,a}\,\frac{s}{\sqrt{n}}\right) \; = P\left(\bar{x} > m_0 - t_{n-1,a}\,\frac{s}{\sqrt{n}}\right) = P\left(\bar{x} > C\right)$$

Where, C is the required criteria $\Rightarrow \quad C = m_0 - t_{n-1,a}\,\frac{s}{\sqrt{n}}$

We accept H_0 if $\bar{X} > C$ and reject H_0 if $\bar{X} \le C$

5-B) The given data: m_0 =36 , m_A=34 , s =1.52 a =0.05 b =0.1

The required sample size is

$$n = \left(\frac{s\ (Z_a + Z_b)}{m_0 - m_A}\right)^2 = \left(\frac{1.52\ (1.645 + 1.28)}{36 - 34}\right)^2 \cong 5$$

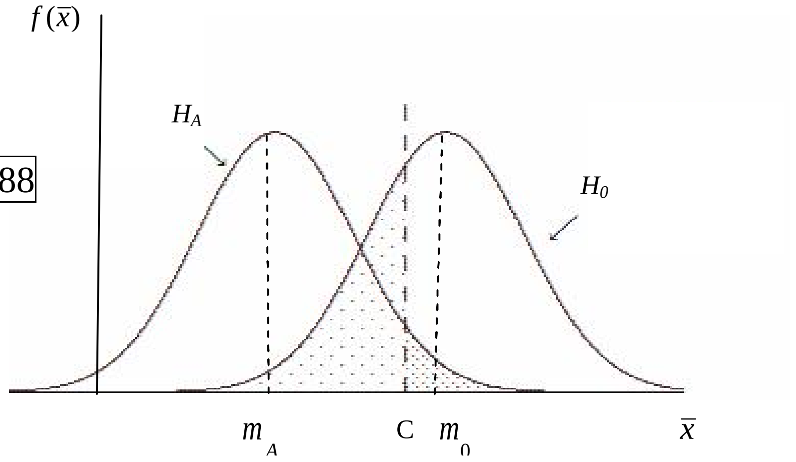
$$H_0 : \quad m = 36$$

$$H_A : \quad m \le 36$$

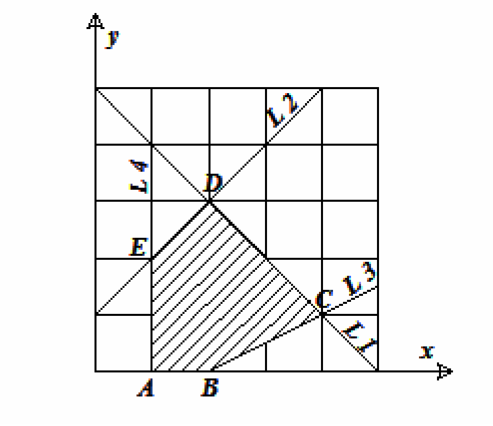
The required criteria,

$$C = m_0 - z_a\,\frac{s}{\sqrt{n}} \; = \; 36 - 1.645\frac{1.52}{\sqrt{5}} \; = \boxed{34.88}$$

Since $\bar{x} = 35 \Rightarrow \bar{x} > C$



$\Rightarrow$ $\boxed{\text{Decision : We accept } H_0}$.



6-A) A=(1,0) , B=(2,0)

C is computed from L1 and L3 $\Rightarrow$C =(4,1)

D is computed from L1 and L2 $\Rightarrow$ D=(2,3)

E is computed from L2 and L4 $\Rightarrow$ E=(1,2)

We may construct the following table

Point	A(1,0)	B(2,0)	C(4,1)	D(2,3)	E(1,2)
Z	12	24	44	12	4

$\Rightarrow \boxed{z_{\max} = 44}$

6-B) The objective function is $z = 10x + 5y$

$\Rightarrow z - 10x - 5y = 0$ ……………………………………….……..(1)

The constraints are : $-x + y \le 1$, $x + y \le 2$,

Using the ***simplex method***, the constraints are written in the form:

$-x + y + s_1 = 1$ ……………………………………………….……..(2)

$x + y + s_2 = 2$ ………………………………...………….……..(3)

Using equations (1, 2 and 3), we may construct the following table to compute $z_{\max}$

pivot column

$\downarrow$

	z	x	y	s_1	s_2	Solution	Ratio
z	1	-10	-5	0	0	0	----
s_1	0	-1	1	1	0	1	-1
s_2	0	1	1	0	1	2	2
z	1	0	5	0	10	20	----
s_1	0	0	2	1	1	3	----
x	0	1	1	0	1	2	----

$\leftarrow$ pivot row

Note: Stop calculations in case of all coefficients in *z* are non-negative

$\Rightarrow$ z is maximum $\Rightarrow \boxed{z_{\max} = 20}$

مع دعواتي بالتفوق د / كارم محمود عويس