Solution to Final Exam June 2010
Measurements 1st Elect.

(1) (b) $R_t = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \dfrac{1}{\frac{1}{250} + \frac{1}{500} + \frac{1}{375}}$

$= 115.4\ \Omega$

$\Delta R_1 = 0.025 \times 25 = 6.25\ \Omega$

$\therefore R_1 = 250 + 6.25 = 256.25\ \Omega$

Similarly:

$\Delta R_2 = -0.036 \times 500 = -18\ \Omega$

$R_2 = 500 - 18 = 482\ \Omega$

$\Delta R_3 = 0.014 \times 375 = 5.25\ \Omega$

$R_3 = 375 + 5.25 = 380.25\ \Omega$

Therefore:

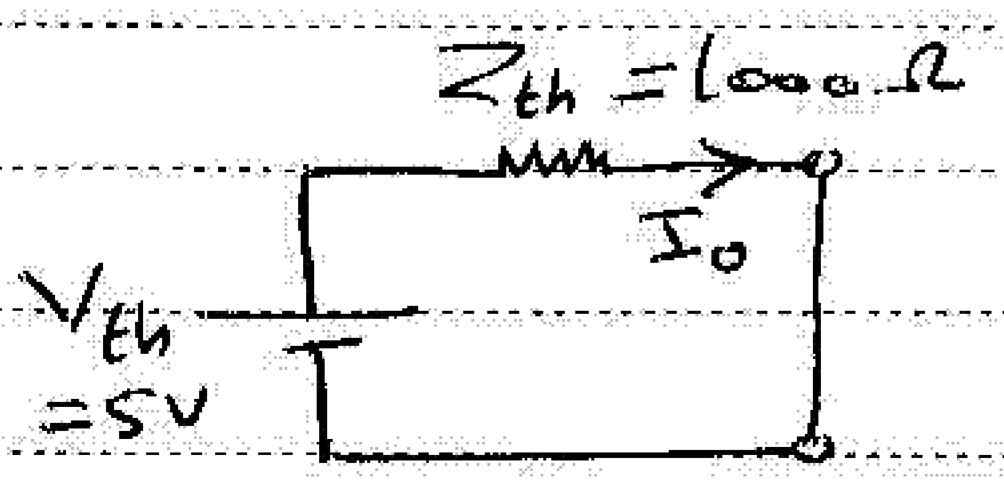
$R_t = \dfrac{1}{\frac{1}{256.25} + \frac{1}{482} + \frac{1}{380.25}} = 116.3\ \Omega$

$\dfrac{\Delta R_t}{R_t} = \dfrac{116.3 - 115.4}{115.4} = 0.00776$

(2) (a) Reduce the actual circuit to an equivalent Thevenin's.

$$V_{th} = V_{oc} = 10 - \frac{10}{1000+1000} \times 1000 = 5 \text{ V}$$

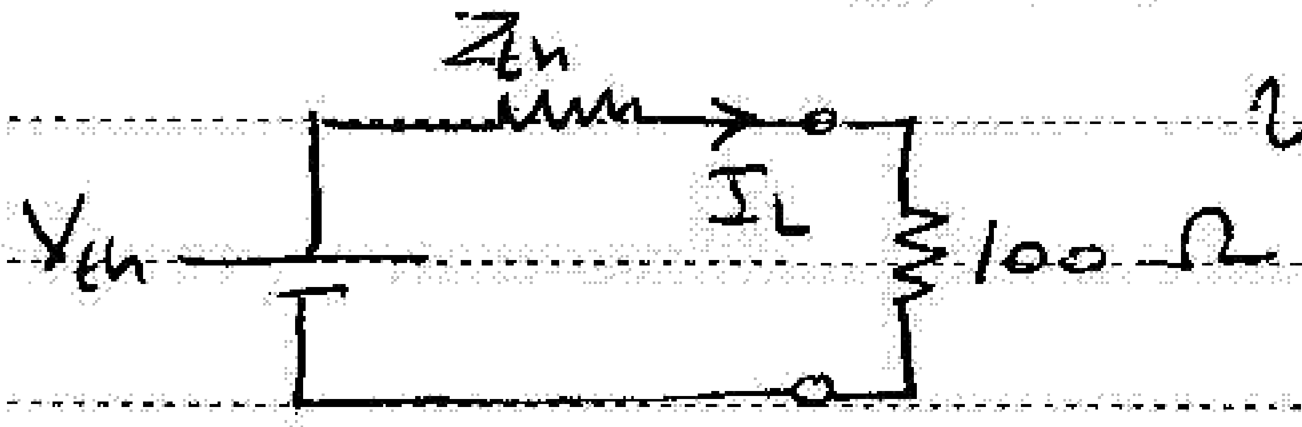
$$Z_{th} = \frac{1000 \times 1000}{1000+1000} + 500 = 1000 \ \Omega$$



Actual current $I_o = \frac{V_{th}}{Z_{th}}$

$$= \frac{5}{1000} \text{ A} = 5 \text{ mA}$$

When the ammeter is introduced, then measured value is:



$$I_L = \frac{V_{th}}{Z_{th} + Z_L} = \frac{5}{1000+100} = 4.55 \text{ mA}$$

$$\text{Error} = \frac{4.55 - 5}{5} \times 100 = -9\%$$

$$= 9\% \text{ low}$$

Accuracy of measurement $= 100 - 9 = 91\%$

(b) The total shunt resistance R_{sh} is:

$$R_{sh} = \frac{R_m}{n-1} = \frac{1\times10^3}{100-1} = \frac{1000}{99} = 10.1\ \Omega$$

This is R_{sh} for the 10-mA range.

When the meter is set on the 100-mA range

$$R_b + R_c = \frac{I_m(R_{sh}+R_m)}{I_2}$$

$$= \frac{100\times10^{-6}\times(10.1+1\times10^3)}{100\times10^{-3}} = 1.01\ \Omega$$

The resistance R_c, which provides the shunt resistance on the 1-A range is:

$$R_c = \frac{I_m(R_{sh}+R_m)}{I_3}$$

$$= \frac{100\times10^{-6}\times(10.1+1\times10^3)}{1} = 0.101\ \Omega$$

$$\therefore R_b = (R_b+R_c) - R_c = 1.01 - 0.101 = 0.909\ \Omega$$

$$R_a = R_{sh} - (R_b + R_c)$$
$$= 10.1 - (0.909 + 0.101) = 9.09\ \Omega$$

check:

$$R_{sh} = R_a + R_b + R_c$$
$$= 9.09 + 0.909 + 0.101 = 10.1\ \Omega$$

(3) (b) To Calculate V_{th}:

$$V_{th} = E\left(\frac{R_3}{R_3+R_1} - \frac{R_4}{R_4+R_2}\right)$$

$$= 6\times\left(\frac{3.5}{3.5+1} - \frac{7.5}{7.5+1.6}\right)$$

$$= 6\times(0.778 - 0.824) = 0.276 \text{ V}$$

To Calculate R_{th}:

$$R_{th} = \frac{R_1R_3}{R_1+R_3} + \frac{R_2R_4}{R_2+R_4}$$

$$= \frac{1\times3.5}{1+3.5} + \frac{1.6\times7.5}{1.6+7.5} = 2.097 \text{ k}\Omega$$

$$I_g = \frac{V_{th}}{R_{th}+R_g} = \frac{0.276}{2.097\times10^{3}+200} = 120\ \mu A$$

Deflection of the galvanometer θ

$$\theta = S' \times I_g = 1\times120 = 120 \text{ mm}$$

Sensitivity of bridge $= \dfrac{\theta}{\Delta R}$

R_4 for balanced condition $= \dfrac{1.6\times3.5}{1} = 5.6$

$$\Delta R = 7.5 - 5.6 = 1.9 \text{ k}\Omega$$

$\therefore$ Sensitivity $= \dfrac{120}{1.9} = 63.16 \text{ mm/k}\Omega$

(4) (b) Reduce the circuit to its Thevenin's equivalent

$$V_{th} = 100 \times \frac{200}{400} = 50 \text{ V}$$

$$R_{th} = \frac{200 \times 200}{200 + 200} = 100 \text{ k}\Omega$$

Voltage appearing under loading conditions E is

$$E = \frac{50}{(100 + 1000) \times 10^3} = 45.45 \text{ V}$$

$$\text{Loading error} = \frac{45.45 - 50}{50} \times 100 = -9.1\%$$
$$= 9.1\% \text{ low}$$

$$\text{Accuracy} = 100 - 9.1 = 90.9\%$$

(c) The equivalent resistance of the voltmeter on its 50 V scale is

$$R_V = 100 \times 50 = 5 \text{ k}\Omega$$

Let R_p = the parallel resistance of R_x and R_V

$$R_p = \frac{V_p}{V_s} \times R_s = \frac{4.65}{95.35} \times 100 = 4.878 \text{ k}\Omega$$

Then

$$R_x = \frac{R_p \times R_V}{R_V - R_p} = \frac{4.878 \times 5}{0.122} = 200 \text{ k}\Omega$$