Fayoum University Faculty of Engineering 2nd year Prwer & Elect. Mach. Dept. Time: 3 Hrs	**ModelAnswer**	Applied Statistics Final Term Exam. June, 16, 2010

Answer of Q1 (20 Marks)

No.	Completion	
i	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	$P(B_k/A) = \dfrac{P(B_k)P(A/B_k)}{\sum_{i=1}^{n} P(B_i)P(A/B_i)}$
ii	0	
iii	$f(x) = {}^{x+r-1}C_x\, p^r\, q^x, \quad x = 0, 1, 2, \dots$	$f(x) = {}^{n}C_x\, p^x\, q^{n-x}, \quad x = 0, 1, 2, \dots, n$
iv	0.1	
v	0	1
vi	216	216
vii	0	0.2
viii	$100 - 600\alpha$	
ix	21.920	
x	2.500	

Answer of Q2 (20 Marks)

No.	Correct choice
i	d
ii	c
iii	a
iv	b
v	c
vi	d
vii	c
viii	c
ix	b
x	b

Answer of Q3 **(10 Marks)**

a) We may use the Baye's theorem and combination, where,

The event *A* represents : drawing 4 black balls and 2 white balls

The event *I* represents : drawing the box *I*

The event *II* represents : drawing the box *II*

$$\boldsymbol{P(II/A)} = \frac{P(II)P(A/II)}{P(I)P(A/I)+P(II)P(A/II)} = \frac{1/2 * {}^{7}C_{4}\ {}^{3}C_{2}\ /\ {}^{10}C_{6}}{1/2 * {}^{5}C_{4}\ {}^{5}C_{2}\ /\ {}^{10}C_{6} + 1/2 * {}^{7}C_{4}\ {}^{3}C_{2}\ /\ {}^{10}C_{6}}$$

$\Rightarrow$ $\boxed{\boldsymbol{P(II/A) = 0.677}}$

b) Success: getting number 3 ,

Failure: getting a number different from 3 (i.e. 1 or 2 or 4 or 5 or 6)

$\Rightarrow p(\text{one trial}) = 1/5 \Rightarrow q = 5/6$

The random variable *X* follows the negative binomial distribution

$\Rightarrow f(x) = {}^{x+r-1}C_{x}\ p^{r}\ q^{x},\ \ x = 0, 1, 2, \ldots \ \ \Rightarrow\ r = 3,\ \ x = 10-3, = 7$

$\Rightarrow P = f(7) = {}^{9}C_{7}\ (1/6)^{3}\ (5/6)^{7}$

$\Rightarrow$ $\boxed{P \cong 0.047}$

Answer of Q4 **(10 Marks)**

a) $f(x,y)=\dfrac{4}{\pi}\exp\left[-\left(x^2+y^2\right)\right]=\dfrac{2}{\sqrt{\pi}}\exp\left(-x^2\right)\dfrac{2}{\sqrt{\pi}}\exp\left(-y^2\right)$

$$\Rightarrow f(x,y)=f_X(x)\,f_Y(y)\Rightarrow f_X(x)=\frac{2}{\sqrt{\pi}}\exp\left(-x^2\right)$$

$$\Rightarrow \mu'_k=E\left[x^k\right]=\frac{2}{\sqrt{\pi}}\int_0^\infty x^k\exp\left(-x^2\right)dx$$

Let $u=x^2\Rightarrow du=2x\,dx\Rightarrow \mu'_k=\dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^\infty u^{k/2}\exp(-u)\frac{du}{2u^{1/2}}=\frac{1}{\sqrt{\pi}}\int_0^\infty u^{(k+1)/2-1}e^{-u}du$

$$\Rightarrow \boxed{\mu'_k=\frac{1}{\sqrt{\pi}}\Gamma\left(\frac{k+1}{2}\right)}$$

Since $f_Y(y)$ **is similar to** $f_X(x)$**, The Kurtosis of Y equals the the Kurtosis of X**

$$\Rightarrow \mu_4=\mu'_4-4\mu\mu'_3+6\mu^2\mu'_2-3\mu^4$$

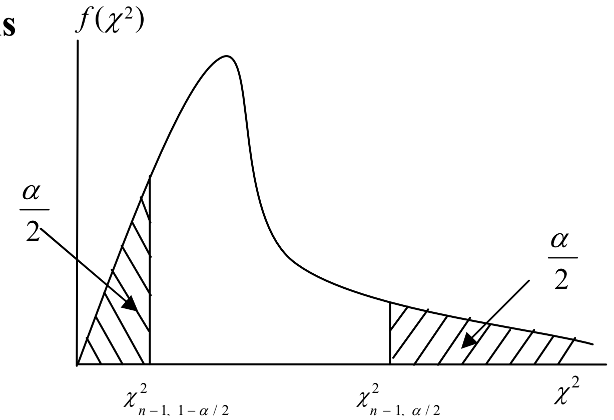
$$\Rightarrow \mu_4=\frac{1}{\sqrt{\pi}}\Gamma\left(\frac{4+1}{2}\right)-\frac{4}{\pi}\Gamma\left(\frac{1+1}{2}\right)\Gamma\left(\frac{3+1}{2}\right)+\frac{6}{\pi\sqrt{\pi}}\Gamma^2\left(\frac{1+1}{2}\right)\Gamma\left(\frac{2+1}{2}\right)-\frac{3}{\pi^2}\Gamma^4\left(\frac{1+1}{2}\right)$$

$$=\frac{3}{4}-\frac{4}{\pi}+\frac{3}{\pi}-\frac{3}{\pi^2}\Rightarrow \qquad \boxed{\mu_4\cong 0.128}$$

b) **(1-α) % confidence interval on** σ^2 **is** $\left(\dfrac{(n-1)s^2}{\chi^2_{n-1,\,\alpha/2}}\ ,\ \dfrac{(n-1)s^2}{\chi^2_{n-1,\,1-\alpha/2}}\right)$

$\Rightarrow$ **(1-α) % confidence interval on** σ **is**

$$\left(s\sqrt{\frac{(n-1)}{\chi^2_{n-1,\,\alpha/2}}}\ ,\ s\sqrt{\frac{(n-1)}{\chi^2_{n-1,\,1-\alpha/2}}}\right)$$



$\Rightarrow$ **95 % confidence interval on** σ **is**

$$\left(3.24\sqrt{\frac{9}{\chi^2_{9,\,0.025}}}\ ,\ s\sqrt{\frac{9}{\chi^2_{9,\,0975}}}\right)$$

$$=\qquad \boxed{(\ 2.229,\quad 5.915\ )}$$

Answer of Q5 **(10 Marks)**

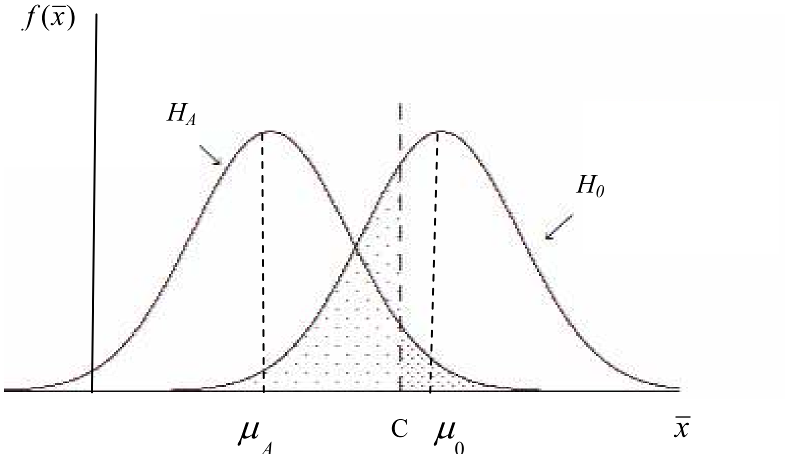
a) μ_0 **=36 ,** σ **=1.52,** α **=0.05,** n**=5**

$H_0: \quad \mu = 36$

$H_A: \quad \mu \leq 36$

The required criteria, $C = \mu_0 - z_\alpha \dfrac{\sigma}{\sqrt{n}}$

= $36 - 1.645\dfrac{1.52}{\sqrt{5}}$ **=34.88**



Since $\bar{x} = 35 \quad \Rightarrow \quad \bar{x} > C$

Decision : $\Rightarrow$ **We accept** H_0

b) We may calculate the following data from the given table

X data	Y data
$\sum_{i=1}^{10} x_i = 13005$	$\sum_{i=1}^{10} y_i = 9757$
$\bar{x} = 1300.5$	$\bar{y} = 975.7$
$\sum_{i=1}^{10} x_i^2 = 1.7434 * 10^7$	$\sum_{i=1}^{10} y_i^2 = 9.8895 * 10^6$
$\sum_{i=1}^{10} x_i \; y_i = 1.311 * 10^7$	

$$\Rightarrow b_1 = \frac{\sum_{i=1}^{10}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{10}(x_i - \bar{x})^2} = \frac{\left(\sum_{i=1}^{10} x_i \; y_i\right) - n\,\bar{x}\,\bar{y}}{\left(\sum_{i=1}^{10} x_i^2\right) - n\bar{x}^2} \cong 0.808 \Rightarrow b_0 = \bar{y} - b_1 \bar{x} \cong -75.556$$

$y = b_0 + b_1 x \qquad \Rightarrow \qquad y \cong 0.808x - 75.556$

At x=1500 $\qquad \Rightarrow \qquad y \cong 1136.444$

مع دعواتي بالتفوق .... د.م / كـارم محـمـود