Fayoum University	**Engineering Mathematics (2A) Final Exam.**
Faculty of Engineering	**Jan., 16, 2010**
First Year Electrical Eng. Department	**Time Allowed: 3 Hours**

Attempt All Questions: Total Mark = 85

Q1. (a) Find $\lim\limits_{(x,y)\to(0,2)} \dfrac{\sin(xy)}{x}$

(b) Find partial derivatives of the function and its total differential $\mathbf{u} = \mathbf{x}^{yz}$

(c) Find the interval of convergent of the series $\sum\limits_{n=1}^{\infty} \dfrac{1}{n(2^n)}(x+5)^n$ (18M)

Q2. (a) Find the absolute maximum and minimum of the function $f(x,y) = x^2 - y^2$ on the closed circle $x^2 + y^2 \leq 1$

(b) Find the points on the paraboloid $z = \dfrac{x^2}{25} + \dfrac{y^2}{4}$ that is closed to the point (0,5,0). (18M)

(c) Find the value of the integral $\int_0^1 \cosh x^2 dx$ approximately up to 4 d.p.

Q3. (a) Find the center of the mass of the region bounded by the curves $y = \sec x$, $y = 1/2$, $x = -\pi/4$ and $x = \pi/4$, where the density at any point (x, y) = $2y$.

((b) Evaluate the integral $\iint_R (x^2 + 2y^2) dxdy$, where R is the region bounded by the curves $xy = 1$, $xy = 2$, $y = |x|$ **and** $y = 2x$. (16M)

Q4. (a) Find the surface area of the region cut from the upper half of the sphere $x^2+y^2+z^2 = 9$ by the cylinder $x^2+y^2 = 9$.

(b) Evaluate $\iint_S \mathbf{F} \bullet \mathbf{n}\, dS$, where $\mathbf{F} = (6x^2 + 2xy)\mathbf{i} + (2y + x^2 z)\mathbf{j} + 4x^2 y^3 \mathbf{k}$, and S is the solid in the first octant bounded by the coordinate planes and the cylinder $x^2 + y^2 = 4$ and the plane $z = 4$. (16M)

Q5. (a) Evaluate $\oint_C [(x^4 + 4)dx + xydy]$; C is the cardioid $r = 1 + \cos\theta$.

(b) Evaluate $\int_{(0,0)}^{(1,\pi/2)} e^x [\sin y dx + \cos y dy]$, where C is the curve $x = \sin y$.

(c) Find the volume of the region bounded by the surfaces $z = x^2 + 3y^2$ and

Model Answer of Mathematics 2 (a) Exam (23-1-2010)

First Year of Electrical Engineering Department

Q1 (a) $\lim\limits_{(x,y)\to(0,2)} \dfrac{\sin(xy)}{y}$

$$\lim_{(x,y)\to(0,2)} \frac{\sin(xy)}{x} = \frac{0}{0},$$

Let the general path $y = xf(x) + 2$, for any arbitrary function $f(x)$

$$\therefore \lim_{(x,y)\to(0,2)} \frac{\sin(xy)}{x} = \lim_{x\to 0} \frac{\sin(x^2 f(x) + 2x)}{x} = \frac{0}{0},$$

$$= \lim_{x\to 0}\left[(xf(x)+2)\frac{\sin(x^2 f(x)+2x)}{x(xf(x)+2)}\right]$$

$$= \lim_{x\to 0}(xf(x)+2) \lim_{x(xf(x)+2)\to 0} \frac{\sin(x^2 f(x)+2x)}{x^2 f(x)+2x} = (2)(1) = 2$$

Q1 (b) $u=x^{yz}$,

$\dfrac{\partial u}{\partial x} = yz(x)^{yz-1} \to$ since y and z are constants, $\dfrac{\partial u}{\partial y} = z(x)^{yz} \ln x \to$ since x and z are constants,

$\dfrac{\partial u}{\partial z} = y(x)^{yz} \ln x \to$ since x and y are constants,

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz = yz(x)^{yz-1}dx + z(x)^{yz}\ln x dy + y(x)^{yz}\ln x dz$$

Q1 (c)

$$\sum_{n=1}^{\infty}\frac{1}{n(2^n)}(x+5)^n \Rightarrow \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{n(2^n)(x+5)^{n+1}}{(n+1)(2^{n+1})(x+5)^n}\right| = \frac{1}{2}\lim_{n\to\infty}\left|\frac{n(x+5)}{(n+1)}\right|$$

$$= \frac{1}{2}|(x+5)|\lim_{n\to\infty}\frac{n}{(n+1)} = \frac{1}{2}|(x+5)| < 1 \text{ for convergent} \Rightarrow -7 < x < -3.$$

For x = - 7, we get the series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n} \Rightarrow$ conditionally converegent,

For x = - 3, we get the series $\sum_{n=1}^{\infty}\frac{1}{n} \Rightarrow$ the series is divergent,

then the interval for convergent is $[-7,-3[$, i.e. $-7 \le x < -3$.

Q2 (a).

$f_x = 2x = 0 \Rightarrow x = 0, \quad f_y = -2y = 0 \Rightarrow y = 0,$

The point (0,0) is a critical point and $f(0,0) = 0$.

The points on the boundary (circle) are $(0,\pm1)$, $(\pm1,0)$, then

$f(0,\pm1) = -1$ and $f(\pm1,0) = 1$,

therefore, the two points $(0,\pm1)$ are absolute minmum and

the two points $(\pm1,0)$ are absolute maximum.

Q2 (b).

$$g(x,y,z) = z - \frac{x^2}{25} - \frac{y^2}{4} = 0, \ f(x,y,z) = d^2 = x^2 + (y-5)^2 + z^2$$

$$\because \frac{f_x}{g_x} = \frac{f_y}{g_y} = \frac{f_z}{g_z} = \lambda \to \text{Lagrange Multiplier,}$$

$$\therefore \frac{2x}{(-2x/25)} = \frac{2(y-5)}{(-y/2)} = \frac{2z}{1} = \lambda$$

For solution, $x = 0$, $-yz = 2y - 10$, or $y = 10/(z+2)$

$$\therefore z = \frac{100}{4(z+2)^2} \Rightarrow z(z+2)^2 = 25 \Rightarrow z^3 + 4z^2 + 4z - 25 = 0.$$

$\therefore z \approx 1.77$ and $y = \pm 2.66$, then the rquired point is (0,2.66,1.77).

Q2(c).

$$\int_0^1 \cosh x^2 dx = \int_0^1 \sum_{n=0}^{\infty} \frac{(x^2)^{2n}}{2n!} dx = \sum_{n=0}^{\infty} \int_0^1 \frac{(x^2)^{2n}}{2n!} dx$$

$$= \sum_{n=0}^{\infty} \frac{x^{4n+1}}{2n!(4n+1)} \Bigg|_0^1 = \sum_{n=0}^{\infty} \frac{1}{2n!(4n+1)}$$

$$= 1 + \frac{1}{102} + \frac{1}{(24)9} + \frac{1}{(6!)(13)} + \ldots = 1 + 0.1 + 0.00463 + 0.000107 + \ldots \cong 1.10474$$

Q3(a). The density $\sigma = 2y$

$$M = \int_{-\pi/4}^{\pi/4} \left[\int_{1/2}^{\sec x} 2y\,dy \right] dx = \int_{-\pi/4}^{\pi/4} (y^2)\Big|_{1/2}^{\sec x} dx = \int_{-\pi/4}^{\pi/4} (\sec^2 x - \frac{1}{4})dx = 2(\tan x - \frac{1}{4}x)\Big|_0^{\pi/4} = 2(1 - \frac{\pi}{8}$$

$$I_2 = \int_{-\pi/4}^{\pi/4} \left[\int_{1/2}^{\sec x} 2y(y)\,dy \right] dx = \frac{2}{3} \int_{-\pi/4}^{\pi/4} (y^3)\Big|_{1/2}^{\sec x} dx = \frac{2}{3} \int_{-\pi/4}^{\pi/4} (\sec^3 x - \frac{1}{8})dx = \frac{4}{3} \int_0^{\pi/4} (\sec^3 x - \frac{1}{8})dx$$

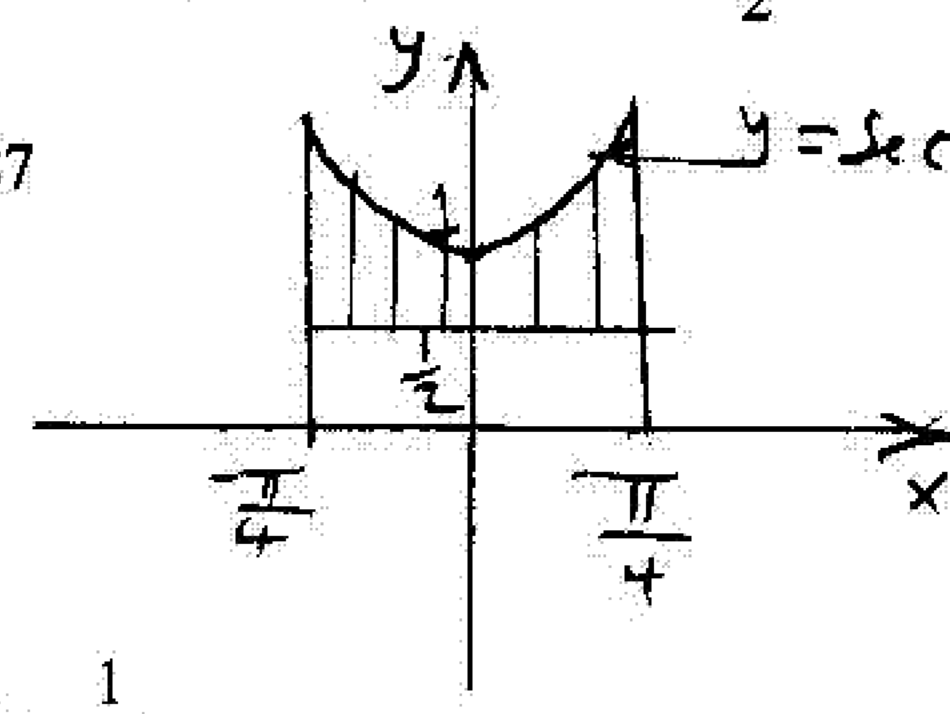
Since the gemetry and the density are symmetric about y - axis, then $\bar{x} = 0, \bar{y} = \frac{I_2}{M}$

$$I = \int_0^{\pi/4} \sec^3 x\,dx = \int_0^{\pi/4} \sec^2 x \sec x\,dx = (\tan x \sec x)\Big|_0^{\pi/4} - \int_0^{\pi/4} \tan x(\sec x \tan x)dx =$$

$$= \sqrt{2} - \int_0^{\pi/4} (\sec^3 x - \sec x)dx$$

$$\therefore 2I = \sqrt{2} + \int_0^{\pi/4} \sec x\,dx = \sqrt{2} + (\ln(\sec x + \tan x))\Big|_0^{\pi/4} = \sqrt{2} + \ln(\sqrt{2}+1) \to I = \frac{\sqrt{2} + \ln(\sqrt{2}+1)}{2}$$

$$\therefore I_2 = \frac{4}{3}[(\frac{\sqrt{2} + \ln(\sqrt{2}+1)}{2}) - \frac{\pi}{32}) \cong 1.4 \to \bar{y} = \frac{I_2}{M} \cong 0.87$$



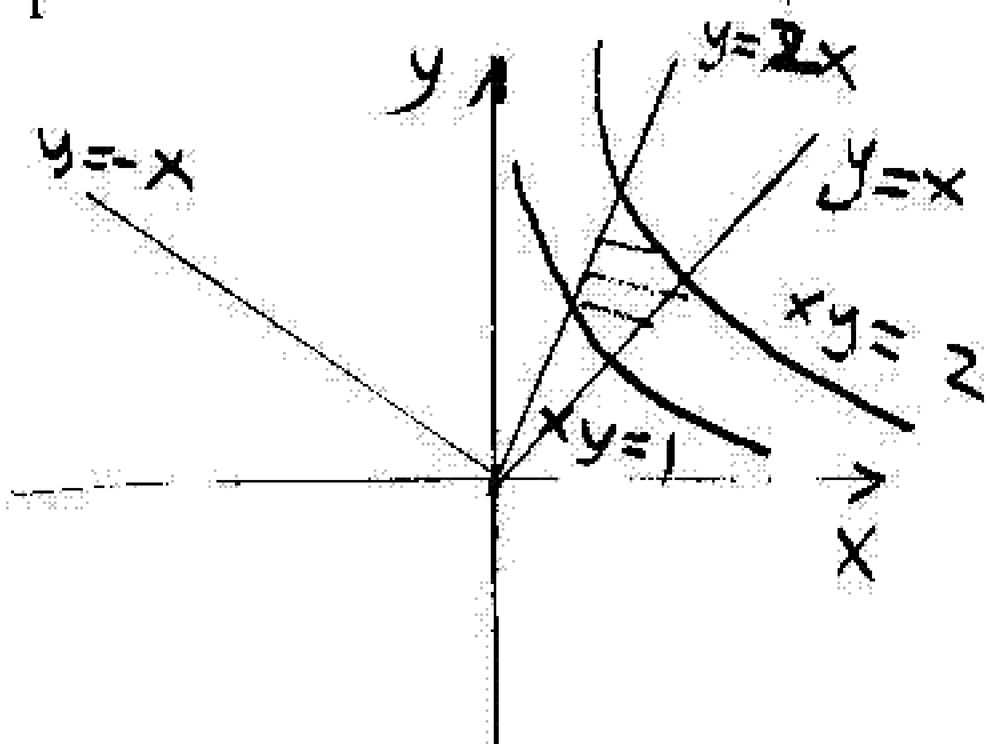
Q3 (b).

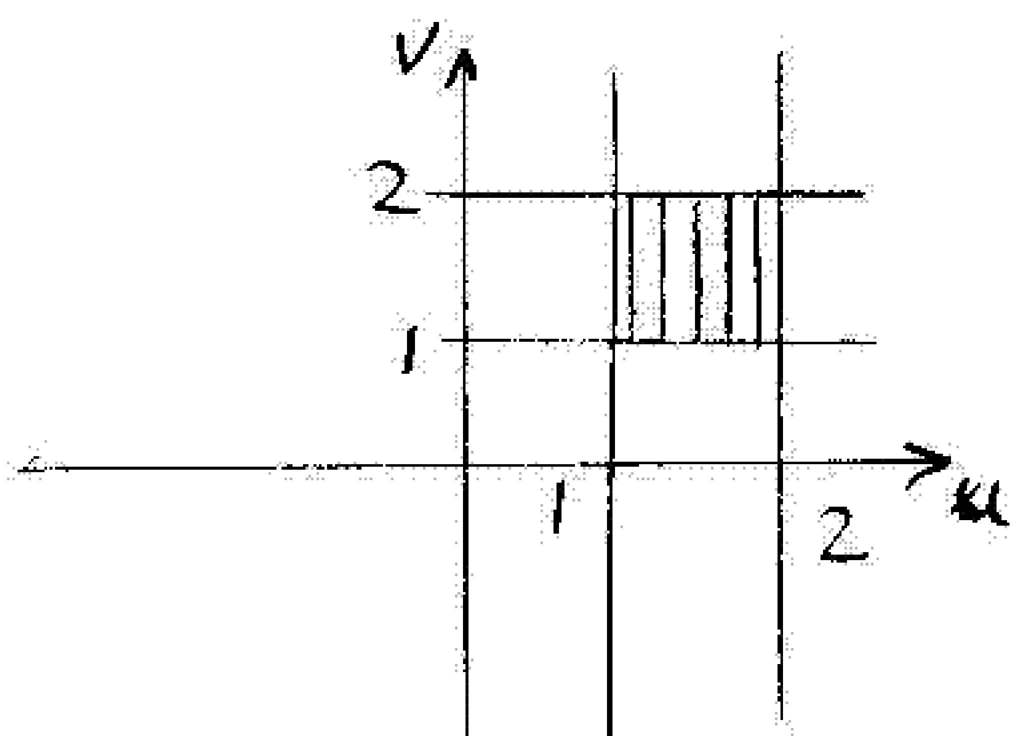
Let $u = xy$ and $v = y/x$,

$$\Delta = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial x} \end{vmatrix} = \begin{vmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x} \end{vmatrix} = 2\frac{y}{x} = 2v, \therefore J = \frac{1}{\Delta} = \frac{1}{2v}$$

$$I = \int_1^2\int_1^2 \frac{1}{2v}(\frac{u}{v} + 2uv)du\,dv = \frac{1}{2}\int_1^2\int_1^2 (\frac{u}{v^2} + 2u)du\,dv = \frac{1}{2}\int_1^2 (\frac{u^2}{2v^2} + u^2)\Big|_1^2 dv$$

$$= \frac{1}{2}\int_1^2 (\frac{3}{2}v^{-2} + 3)dv = \frac{1}{2}(-\frac{3}{2v} + 3v)\Big|_1^2 = \frac{1}{2}[(-\frac{3}{4} + 6) - (-\frac{3}{2} + 3)] = \frac{15}{8}$$



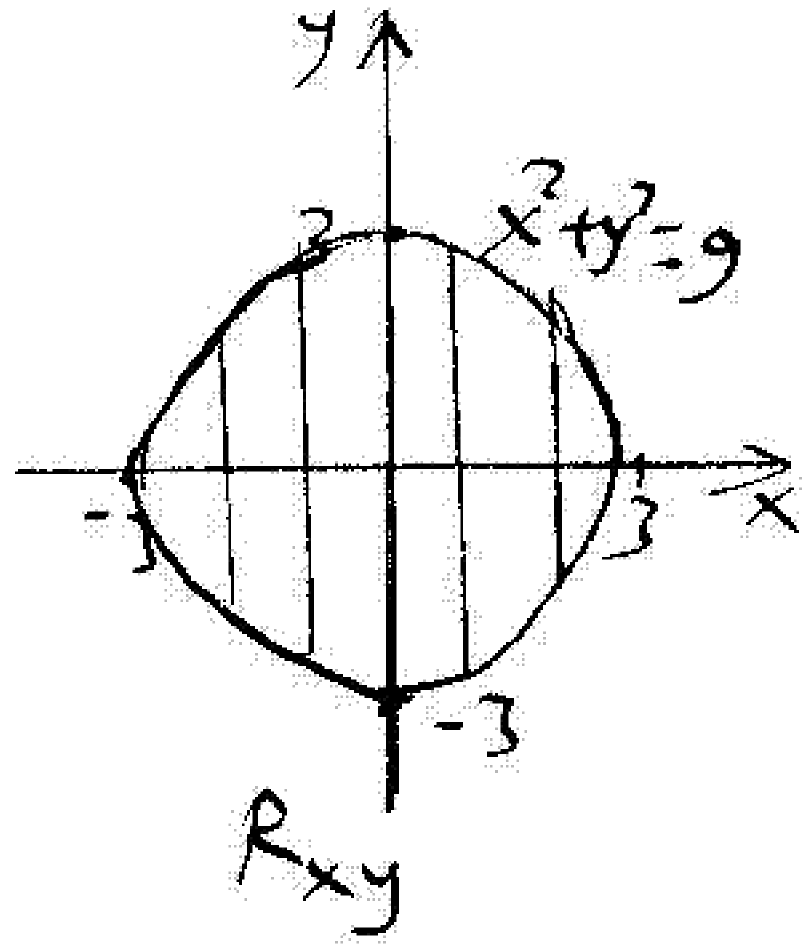


Q4 (a)

$\because F(x,y,z) = x^2 + y^2 + z^2 - 9 = 0,$

$\therefore \mathbf{n} = \frac{x}{3}\mathbf{i} + \frac{y}{3}\mathbf{j} + \frac{z}{3}\mathbf{k},$

$$S = \iint_{R_{xy}} \frac{3}{z} dA = \int_0^{2\pi}\int_0^3 \frac{3}{\sqrt{9-r^2}} r\,dr\,d\theta = 18\pi.$$

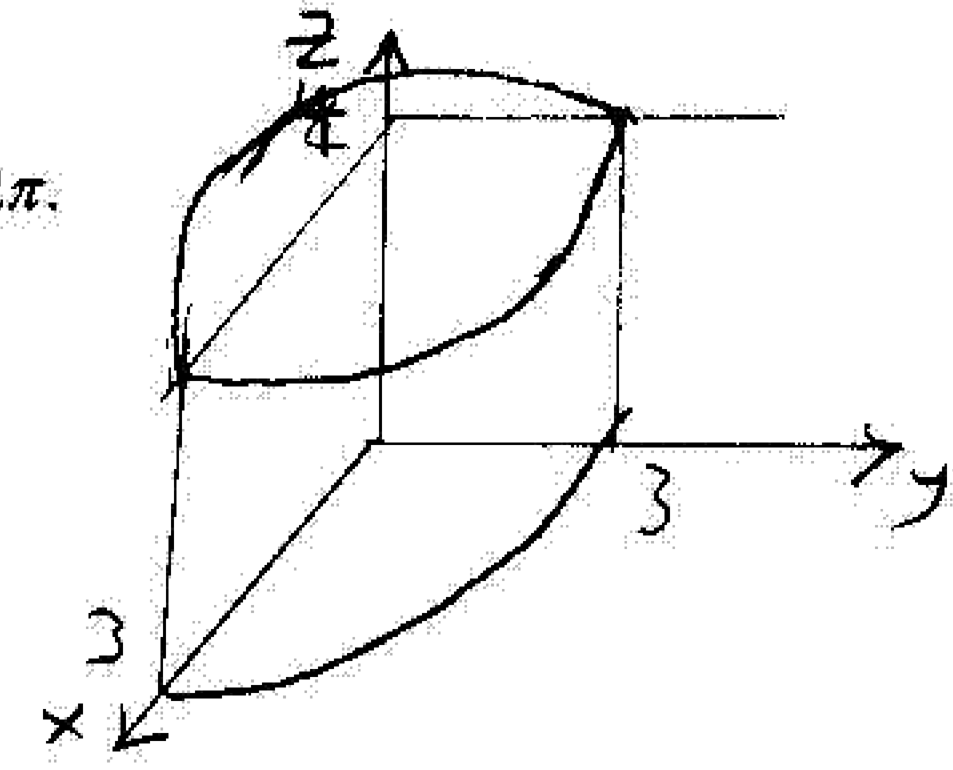


Q4 (b).

$$\iint_S (\mathbf{F} \bullet \mathbf{n}) dS = \iiint_R (\nabla \bullet \mathbf{F}) dV$$

$$= \iiint_{\mathrm{R}_{xyz}} (12x + 2y + 2) dV = 2\int_0^{2\pi}\int_0^3\int_0^4 (6r\cos\theta + r\sin\theta + 1) r\,dz\,dr\,d\theta$$

$$= 8\int_0^{2\pi}\int_0^3 (6r^2\cos\theta + r^2\sin\theta + r) dr\,d\theta = 72\pi.$$



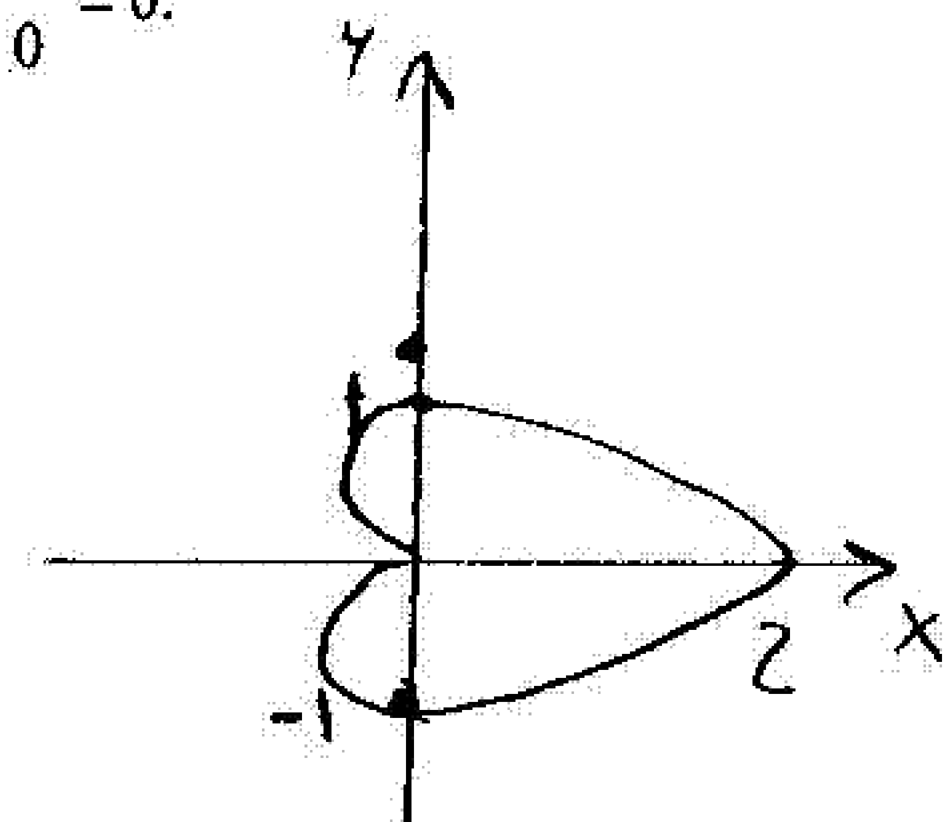
Q5 (a).

$$I = \oint_C y^2(x^2+4)dx + xy\,dy$$

$$= \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_R (y) dA \Rightarrow \text{ using polar coordinates}$$

$$\therefore I = \int_0^{2\pi}\int_0^{1+\cos\theta} r^2 \sin\theta\,dr\,d\theta = \int_0^{2\pi} \left(\frac{r^3}{3}\right)\Bigg|_0^{1+\cos\theta} \sin\theta\,d\theta$$

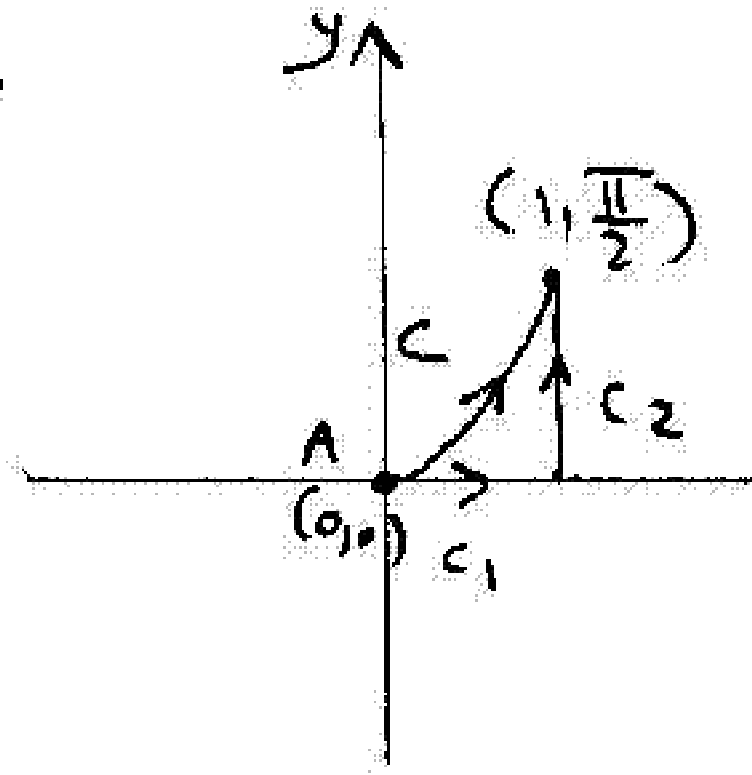
$$= \frac{1}{3}\int_0^{2\pi} (1+\cos\theta)^3 \sin\theta\,d\theta = -\frac{1}{12}(1+\cos\theta)^4 \Bigg|_0^{2\pi} = 0.$$



Q5(b).

$$I = \int_{(0,0)}^{(1,\pi/2)} e^x \sin y dx + e^x \cos y dy, \quad \because \frac{\partial P}{\partial y} = e^x \cos y, \ \frac{\partial Q}{\partial x} = e^x \cos y,$$

Since, $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$, and are continuous , then The line integral is path independent, ther



$$I = \int_{C_1} e^x \sin y dx + e^x \cos y dy + \int_{C_2} e^x \sin y dx + e^x \cos y dy$$

$C_1 : y = 0, x \to 0 \text{ to } 1, C_2 : x = 1, y \to 0 \text{ to } \pi/2,$

$$I = 0 + \int_0^{\pi/2} e(\cos y) dy = e(\sin y)\Big|_0^{\pi/2} = e.$$

Q5(c).

$$\text{Volume} = V = \iint_{R_{xy}} \int_{x^2+3y^2}^{8-x^2-y^2} dz \ dA = \iint_{R_{xy}} (8 - 2x^2 - 4y^2) dA,$$

where R_{xy} is the elipose given by $\frac{x^2}{4} + \frac{y^2}{2} = 1$

Using the transformation $x = 2r\cos\theta, y = \sqrt{2} r \sin\theta$

$$J = \frac{\partial(x,y)}{\partial(r,\theta)} = 2\sqrt{2} r$$

$$\therefore V = 2\sqrt{2} \int_0^{2\pi} \int_0^1 (8\text{-}8r^2\cos^2\theta - 8r^2\sin^2\theta) r dr d\theta$$

$$= 16\sqrt{2} \int_0^{2\pi} \int_0^1 (1\text{-}r^2) r dr d\theta = 16\sqrt{2}\left(\frac{r^2}{2} - \frac{r^4}{4}\right)\Big|_0^1 (2\pi) = 8\sqrt{2}\ \pi.$$