Solution for the final Exam for the subject of Electric Circuits "ECE 102"

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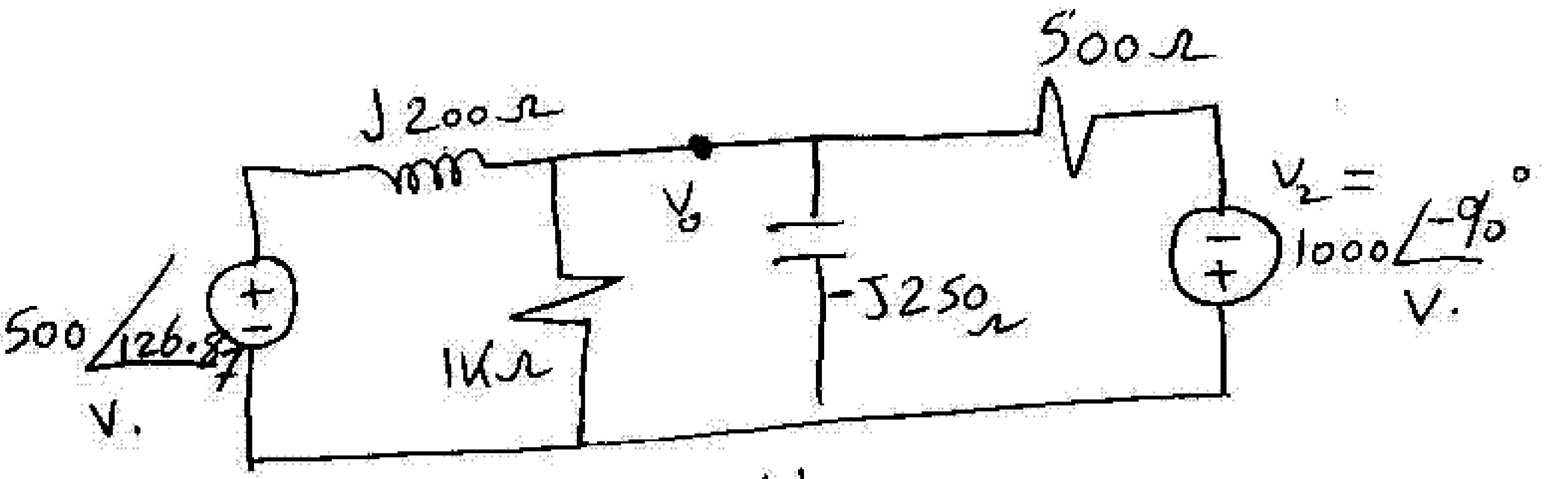
Q.1. (16 points)

L in $\Omega = j\omega L = j200\,\Omega$

C in $\Omega = \frac{1}{j\omega C} = -j250\,\Omega$



$\Downarrow$

Using Nodal-Voltage method

$$\Rightarrow \frac{V_o - V_1}{j200} + \frac{V_o}{1K} + \frac{V_o + V_2}{500} + \frac{V_o}{-j250} = 0$$

$$\Rightarrow \frac{V_o - 500\angle 126.87}{j200} + \frac{V_o}{1K} + \frac{V_o + 1000\angle -90}{500} + \frac{V_o}{-j250} = 0$$

$$\Rightarrow \frac{V_o + 3000 - j400}{j200} + \frac{V_o}{1000} + \frac{V_o + j1000}{500} - \frac{V_o}{j250} = 0$$

$$\Rightarrow V_o\left\{\frac{1}{j200} + \frac{1}{1000} + \frac{1}{500} - \frac{1}{j250}\right\} = \frac{j1000}{500} - \frac{3000}{j200} + 2$$

$$V_o\left\{0.003 - j0.001\right\} = 2 + j17$$

$$\Rightarrow V_o = \frac{2 + j17}{0.003 - j0.001} = \frac{17.11\angle 83°}{0.0032\angle -18.4} \simeq 5346.9\angle 101.7° \text{ volt}$$

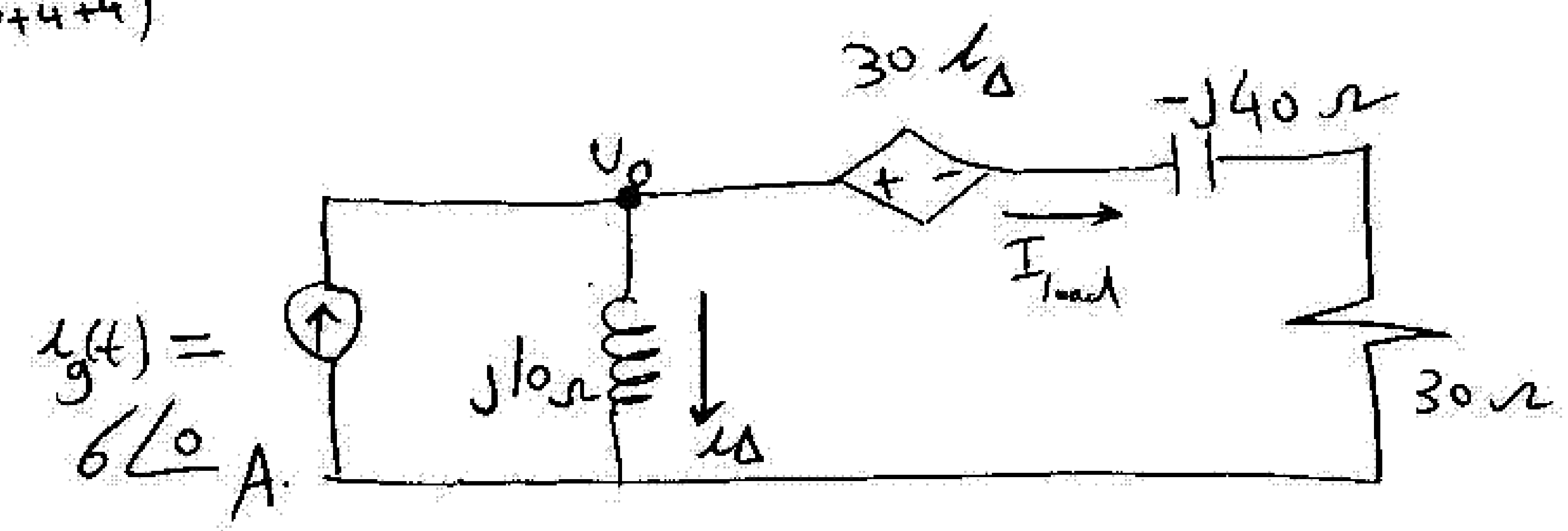
$$\rightarrow V_o = 5346.9 \cos(8000t + 101.7) \text{ volt}$$

Q.2 (18 Points)(10+4+4)

$\omega = 20{,}000 \ \frac{rad}{sec.}$

$L = j\omega L = j10 \ \Omega$

$C = \frac{1}{j\omega C} = -j40$



Using the nodal-voltage method

$$\Rightarrow \frac{V_o - 30 i_\Delta}{30 - j40} + \frac{V_o}{j10} - 6\angle 0 = 0 \quad \text{—①}$$

$$\& \quad V_o = j10\, i_\Delta \quad \text{—②}$$

sub. ② in ①

$$\Rightarrow \frac{j10\, i_\Delta - 30 i_\Delta}{30 - j40} + \frac{j10\, i_\Delta}{j10} - 6\angle 0 = 0$$

$$\Rightarrow i_\Delta \frac{\{-30 + j10\}}{\{30 - j40\}} + i_\Delta = 6\angle 0$$

$$\Rightarrow i_\Delta \left\{\frac{-j30}{30 - j40}\right\} = 6\angle 0$$

$$\Rightarrow i_\Delta = 8 + 6j = 10\angle 36.87 \text{ A.} \quad \& \quad V_o = 100\angle 126.87 \text{ V.}$$

(a) real & reactive power associated with "L" $= S_L = V_L I_L^*$

$= 100\angle 126.87 \cdot 10\angle -36.87 = 1000\angle 90 = j1000$ VA.

$\Rightarrow$ real power $= 0$ & reactive power $= 1000$ VAR.

& real & reactive power associated with the load $(30 - j40)$ is

$S_{load} = V_{load} I_{load}^*$ $\quad I_{load} = 6\angle 0 - i_\Delta = 6\angle 0 - 10\angle 36.87 = \underbrace{6.324}_{\sqrt{40}}\angle -71.565°$ A

$V_{load} = V_o - 30 i_\Delta = 100\angle 126.87 - 300\angle 36.87 = -300 - j100 = 316.227\angle -161.434$ V.

$\Rightarrow S_{load} = 2000.141\angle -53.131° = 1200.056 - j1600.131$ VA

power associated with the source $i_g(t) = -V_o\, i_g^*(t) = -100\angle 126.87 \times 6\angle 0 = -600\angle 126.87$

$= +360 - j480$ VA

power associated with the $30 i_\Delta$ source $= -(30 i_\Delta) \cdot I_{load}^* = -1897.366\angle -34.695$

$= -1560 + j1079.998$ VA

Cont.

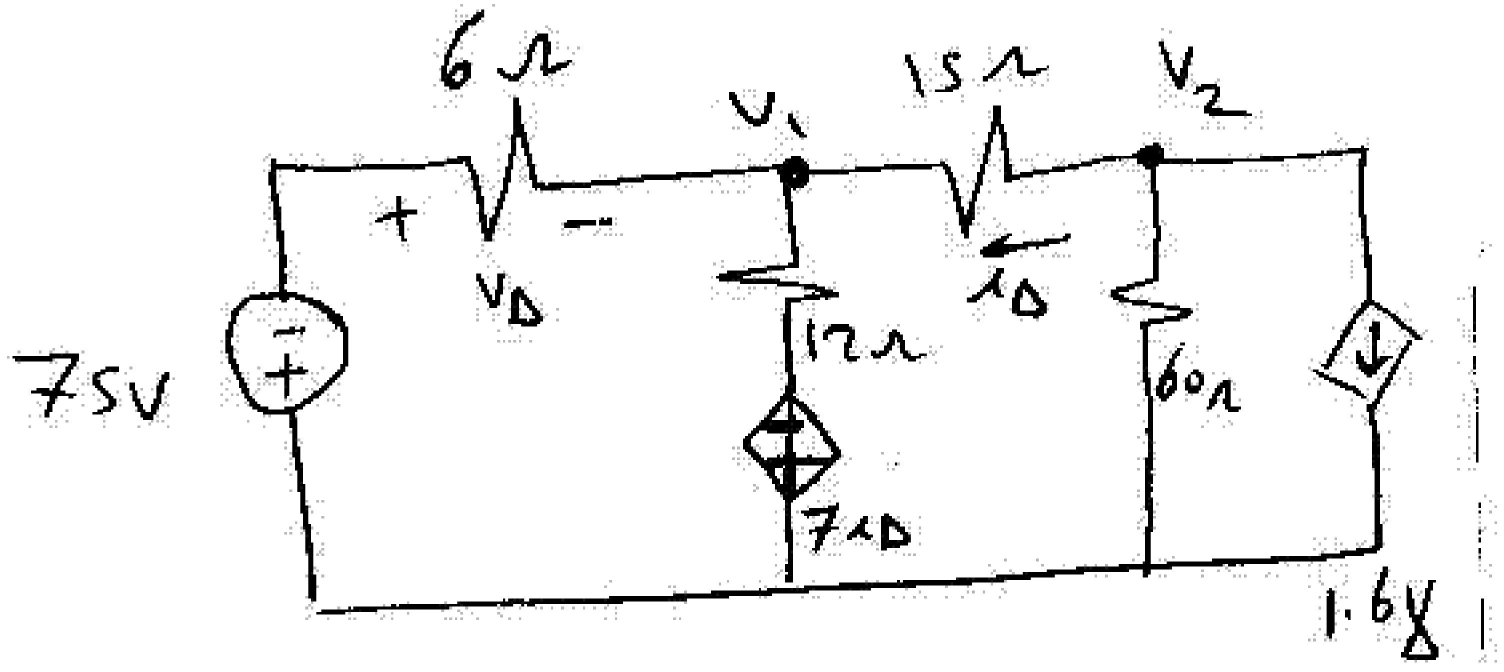
(b) real power generated = ~~reactive~~ real power absorbed

as seen from part (a) $\Rightarrow$ the real termes add up to zero.

(c) reactive power generated = reactive power absorbed

as seen from part (a) $\Rightarrow$ the imaginary terms add up to zero.

Q.3. (14 points)



using nodal- voltage method

$$\frac{V_1+75}{6} + \frac{V_1+7i_\Delta}{12} + \frac{V_1-V_2}{15} = 0 \quad (1)$$

$$\frac{V_2-V_1}{15} + \frac{V_2}{60} + 1.6\,V_\Delta = 0 \quad (2)$$

$$V_\Delta = V_1 + 75 \quad (3)$$

sub. (3) & (4) in (1) and (2)

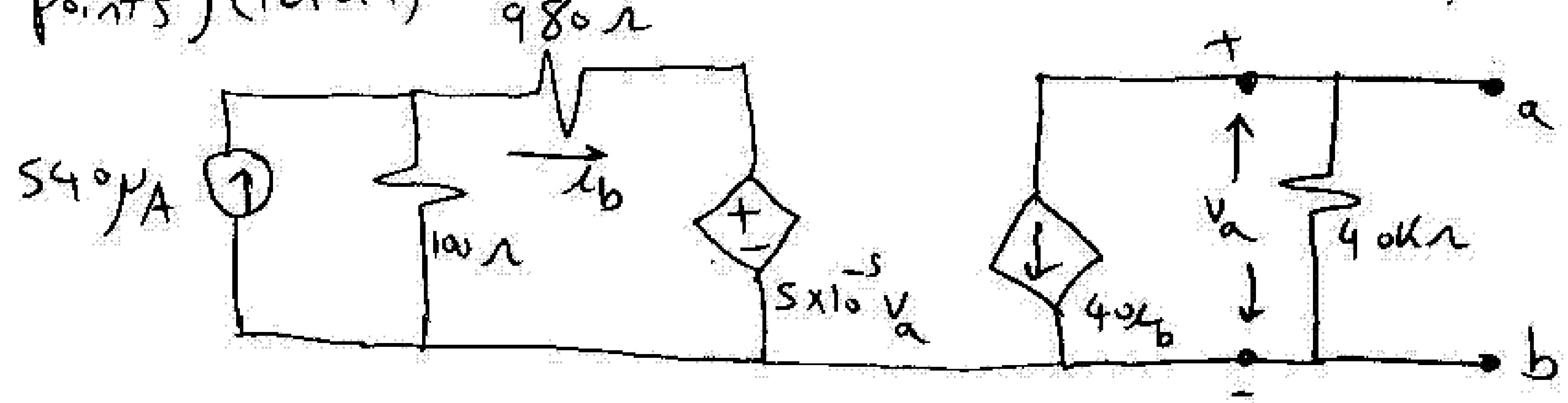
$$i_\Delta = \frac{V_2 - V_1}{15} \quad (4)$$

get V_1 & V_2 then get V_Δ & i_Δ

$\Rightarrow$ total power dissipated $= I_{6\Omega}^2 \times 6 + I_{12\Omega}^2 \times 12 + I_\Delta^2 \times 15 + \frac{V_2^2}{60}$

$I_{6\Omega} \to \left(\frac{V_\Delta}{6}\right)^2$ $\quad I_{12\Omega} \to \left(\frac{V_1+7i_\Delta}{12}\right)^2$

Q.4 (18 points) (12+2+4) 4



to get V_{th}

$V_{th} = V_{ab} = -40i_b \times 40K\Omega = -1600\times10^3 \, i_b \text{ V.} = V_a$ (1)

$i_b = \dfrac{54\times10^{-3} - 5\times10^{-5} V_a}{1080} \Longrightarrow$ sub in (1)

$\Longrightarrow \dfrac{-1600\times10^3}{1080}\left\{54\times10^{-3} - 5\times10^{-5} V_a\right\} = V_a$

$\Longrightarrow V_a = -0.864$ Volt $= V_{th}$

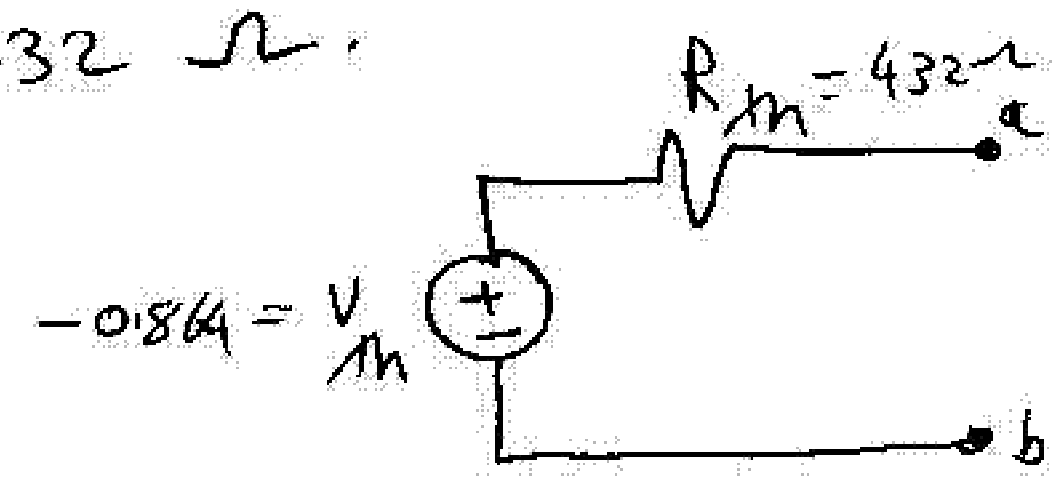
for R_{th} short circuit the output and get $I_{s.c.}$

$\Longrightarrow V_a = 0 \Longrightarrow I_{s.c.} = -40i_b$

i_b in this case $= \dfrac{540\mu A \times 100}{100+980} = 50\mu A.$

$\Longrightarrow I_{s.c.} = -40\times50\mu = -2 \text{ mA}.$

$\Longrightarrow R_{th} = \dfrac{V_{th}}{I_{s.c.}} = \dfrac{-0.864}{-2m} = 432\ \Omega.$

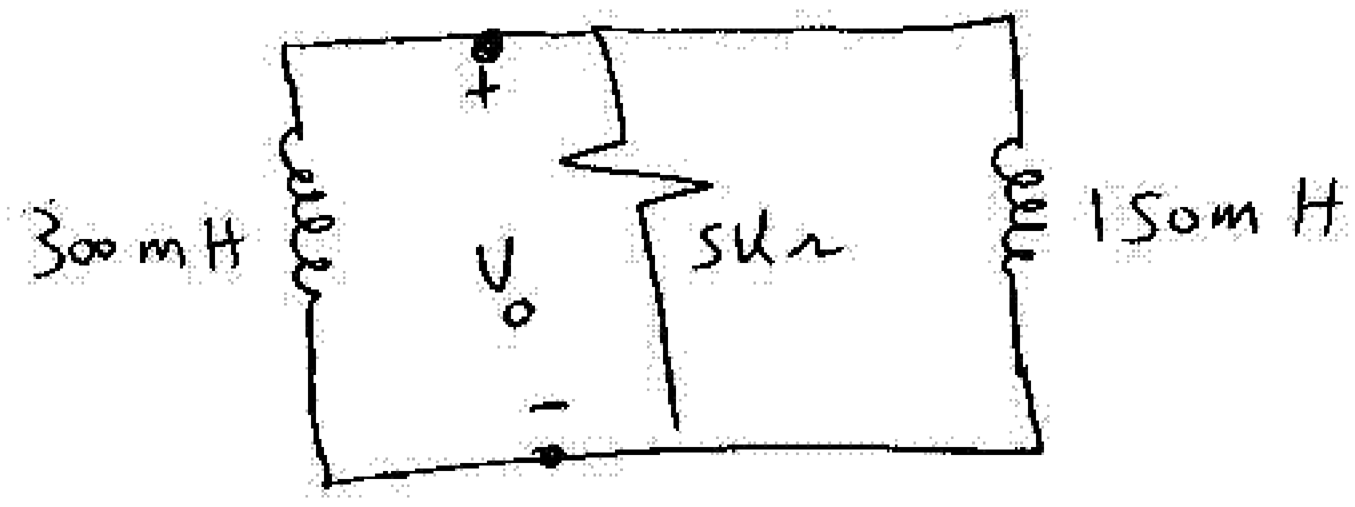


(b) $R_L = R_{th} = 432\ \Omega$

(c) max. power $= \dfrac{V_{th}^2}{4R_L} = \dfrac{(-0.864)^2}{4\times432} = 0.432$ mW.

Q.5 (16 points) (12+4)

(a) for $t \geq 0$



The initial Current in 300mH is

$$I_o = \frac{6m \times 3.2K\Omega}{3.2K + 1.6K} = 4\,mA.$$

$$\Rightarrow v_o(t) = I_o R e^{-t/\tau} \quad \text{+} \quad \tau = \frac{L}{R} = \frac{(300m // 150m)}{5K}$$

$$= \frac{\frac{300 \times 150}{450}\,m}{5\,K} = \frac{100}{5} = 25 \text{ Sec}$$

$$\Rightarrow v_o(t) = (4 \times 5)\, e^{-0.04t}$$

$$= 20\, e^{-0.04t}\ V.$$

(b) $$w = \int_0^{\infty} p(t)\, dt = \int_0^{\infty} \frac{v_o^2(t)}{R}\, dt = 80m \int_0^{\infty} e^{-0.0016t}\, dt$$

$$= 80m \left. \frac{e^{-0.0016t}}{-0.0016} \right|_0^{\infty} = \frac{80\,m}{0.0016} = 50\,J.$$

Q.6 (18 points) (2+2+12+2).

(a) initial value of $v_c = -\frac{40 \times 60}{60+20} = -30V.$

final value of $v_c = 90\ V.$

(b) $\tau = RC = 400K. \times 0.5\mu F = 0.2$ Sec.

(c) for $t \geq 0$ $v_c(t) = 90 + (-30 - 90)\, e^{-5t}$ volt.

$$i_c(t) = C \frac{dv_c(t)}{dt} = 0.5\mu F\, (-120)\, e^{-5t} \times (-5)$$

$$= 300\, e^{-5t}\ \mu A.$$

(d) $90 - 120\, e^{-5t} = 0 \Longrightarrow t = \frac{1}{5} \ln\left(\frac{4}{3}\right) = 57.54\ m\,s.$