Comm.

$L_1 = 1414 - j1414 = 2000\angle{-45}$

$L_2 = 1200 - j800$

$L_3 = 4000 + j1937$

$P_1 = I_1 V_1 \cos\phi \rightarrow (\theta_v - \theta_i)$

$1414 = I_1\, 100 \cos(90 - \theta_i)$

$S_1 = I_1^* V_1$

$2000\angle{-45} = I_1\, 100\angle{90}$

$I_1^* = 20\angle{-135^\circ}$ A

$I_1 = 20\angle{135^\circ}$

$L_2 + L_3 = 5200 + 1139j$

$S = 5323.28\angle{12.35^\circ}$

$S = I_2^* V_2$

$5323.28\angle{} = I_2^*\, 100\angle{90}$

$I_2^* = 53.2328\angle{-77.6^\circ}$

$I_2 = 53.2328\angle{77.6^\circ}$ A

$I_0 = I_1 + I_2 = 20\angle{135} + 53.2\angle{77.6}$

$= 66.19\angle{94.4^\circ}$ A

$S_T = 6614 - j277.13$ VA

② $I_1 = 3\angle 90$ A

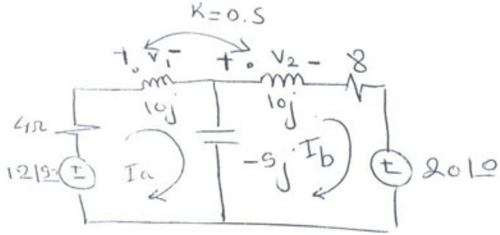
$X_L = \omega L$

$M = k\sqrt{L_1 L_2}$

$= 0.5 * \sqrt{\frac{10}{1000} \times \frac{10}{1000}}$

$= 5 \times 10^{-3}$ H

$\omega M = 5j$



$$12\angle 90 - 4 I_a - 10j\, I_a - (-5j)(I_a - I_b) - V_1 = 0$$

$$V_1 = + + 5j\, I_b$$

$$-5j(I_b - I_a) + 10j\, I_b + 8 I_b + 20\angle 0 + V_2 = 0$$

$$V_2 = + + 5j\, I_a$$

solve eq(1) and eq(2)

$$12j + (-4 - 10j + 5j) I_a + (-5j + 5j) I_b = 0$$

$$12j + (-4 - 5j) I_a = 0 \rightarrow I_a = \frac{-12j}{-4-5j}$$

$$I_a = \frac{12\angle 90}{6.4\angle 51.3} = 1.875\angle 38.7^\circ \text{ A}$$

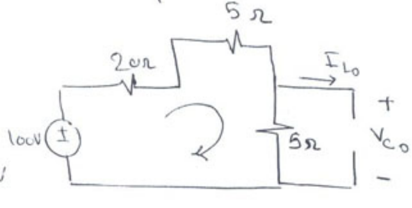
$I_2 = I_b$ 5 $I_3 = I_a - I_b$

$$E = \frac{1}{2} L_1 I_a^2 + \frac{1}{2} L_2 I_b^2 + M I_a I_b$$

b) at $t<0$ switch opened

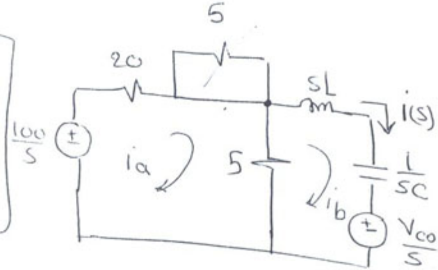
$I_{L_0} = 0$

$V_{c_0} = \frac{100 * 5}{30} = \left(\frac{50}{3}\right) V$



at $t \geq 0$ switch closed

$$\begin{cases} \frac{100}{s} - 20 i_a - 5(i_a - i_b) = 0 \\ 5(i_b - i_a) + sL i_b + \frac{1}{sc} i_b + \frac{V_{c_0}}{s} = 0 \end{cases}$$



solve 2 equations

$\frac{100}{s} - 25 i_a + 5 i_b = 0 \rightarrow i_a = \frac{\frac{100}{s} + 5 i_b}{25}$

$\frac{50}{3s} + \left(5 + sL + \frac{1}{sc}\right) i_b - 5 i_a = 0$

$s\left[\frac{50}{3s} + \left(5 + sl + \frac{1}{sc}\right) i_b - \cancel{5}\left(\frac{\frac{100}{s} + 5 i_b}{\cancel{25}}\right)\right] = 0$

$\frac{50}{3} + \left(s^2 L + 5s + \frac{1}{c}\right) i_b - (20 + s i_b) = 0$

$-\frac{10}{3} + i_b\left(s^2 L + 4s + \frac{1}{c}\right) = 0$

$$i_b = \frac{\frac{10}{3}}{s^2 L + 4s + \frac{1}{c}} = \frac{10/3}{s^2 + 4s + 25}$$

$$s = -2 \pm j\sqrt{21}$$

$$i_b = \frac{10/3}{(s + 2 + j\sqrt{21})(s + 2 - j\sqrt{21})} = \frac{k}{(\ \ )} + \frac{k^*}{(\ \ )}$$

$$i(t) = \cancel{\tfrac{10}{3}}\ 2|K|\, e^{-2t} \cos(\sqrt{21}\, t + \theta)$$

$$\theta = \tan^{-1} \frac{y}{x} \text{ of } k$$

(4) $f(t) = \begin{cases} 7.5 & 0 < t < 1 \\ 2.5 & 1 < t < 2 \end{cases}$

$$a_v = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{2}\left[\int_0^1 7.5\,dt + \int_1^2 2.5\,dt\right]$$

$$= \frac{1}{2}\left[7.5(1-0) + 2.5(2-1)\right] = \underline{\underline{5}}$$

$$\boxed{\begin{aligned} w_0 &= \frac{2\pi}{T} \\ &= \frac{2\pi}{2} = \pi \end{aligned}}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n w_0 t\,dt$$

$$= \frac{2}{2}\left[\int_0^1 7.5\cos n w_0 t\,dt + \int_1^2 2.5\cos n w_0 t\,dt\right]$$

$$= \left[7.5\left.\frac{\sin n w_0 t}{n w_0}\right|_0^1 + 2.5\left.\frac{\sin n w_0 t}{n w_0}\right|_1^2\right.$$

$$= \frac{7.5}{n\cancel{\pi}}\left(\frac{\sin n\pi}{\cancel{n}} - 0\right) + \frac{2.5}{n\pi}\left(\sin 2n\pi - \sin n\pi\right)$$

$$= \frac{5}{n\pi}\sin n\pi \qquad = \underline{\underline{0}}$$

$$b_n = \frac{2}{T}\int_0^t f(t)\sin n w_0 t\,dt$$

$$= \frac{2}{2}\left[\int_0^1 7.5\sin n w_0 t\,dt + \int_1^2 2.5\sin n w_0 t\,dt\right]$$

$$= 7.5\left.\left(-\frac{\cos n w_0 t}{n w_0}\right)\right|_0^1 + \frac{2.5}{n w_0}\left.\left(-\cos n w_0 t\right)\right|_1^2$$

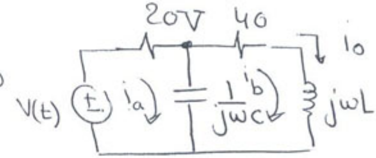
$$= -\frac{7.5}{n\pi}\left(\cos n\pi - 1\right) - \frac{2.5}{n\pi}\left(\cos 2n\pi - \cos n\pi\right)$$

$$b_n = -\frac{5}{n\pi}\cos n\pi + \frac{\cancel{5}\,5}{n\pi} = \frac{5}{n\pi}(1-\cos n\pi)$$

$$V(t) = a_v + \sum_{n=1}^{\infty}\left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)$$

$$V(t) = 5 + \sum_{n=1}^{\infty}\frac{5}{n\pi}(1-\cos n\pi)\sin n\omega_0 t$$

$$\frac{V - V(t)}{20} + \frac{V}{\frac{1}{j\omega C}} + \frac{V}{40+j\omega L} = 0$$



$$V\left(\frac{1}{20} + j\omega C + \frac{1}{40+j\omega L}\right) = \frac{V(t)}{20}$$

$$V = \frac{V(t)}{1 + 20j\omega C + \frac{20}{40+j\omega L}} = \frac{\cancel{j\omega t}}{\cancel{j\omega t} -}$$

$$i_0 = \frac{V}{40+j\omega L} = \frac{V(t)}{(40+j\omega L)\left[1 + 20j\omega C + \frac{20}{40+j\omega L}\right]}$$

$$i_0 = \frac{5 + \sum_{n=1}^{\infty}\frac{5}{n\pi}(1-\cos n\pi)\sin n\omega_0 t}{40 + j\omega L + 20j\omega C(40+j\omega L) + 20}$$

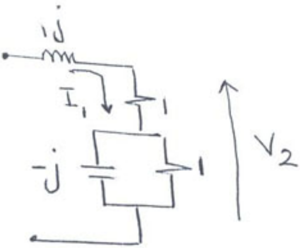
$L \to 100\times10^{-3}$, $C \to 50\times10^{-3}$, $L \to 100\times10^{-3}$

$(60 + 40.1j\omega - 0.1\omega^2)$

(5) $Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$ $\quad Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$

$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$ $\quad Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$

to get Z_{11}



$$Z_{11} = 1 + j + \frac{1(-j)}{1-j}$$

$$= \frac{-j + 1 - j^2}{1-j} = \frac{2-j}{1-j}$$

$I_2 = 0$

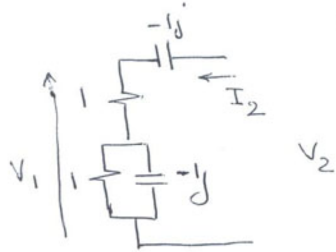
$$V_2 = \frac{V_1}{\left(\frac{2-j}{1-j}\right)} * \left(1 + \frac{-j}{1-j}\right) = V_1\left[\frac{1-j}{2-j} - \frac{j}{2-j}\right]$$

$$= V_1\left(\frac{1-2j}{2-j}\right)$$

$$I_1 = \frac{V_1}{Z_{11}} = \frac{V_1}{(2-j)}(1-j)$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{V_1\,(1-2j)\,(2-j)}{V_1(1-j)\,(2-j)} = \frac{1-2j}{1-j}$$

to get Z_{22}



$$Z_{22} = -j + 1 - \frac{j}{1-j}$$

$$= \frac{-j + 1 - 2j + j^2}{1-j} = \frac{-3j}{1-j}$$

$I_1 = 0$

$$V_1 = \frac{V_2}{\left(\frac{-3j}{1-j}\right)} \left[1 + \frac{-j}{1-j}\right] = V_2 \left[\frac{1-j}{-3j} + \frac{-j}{-3j}\right]$$

$$= V_2 \left[\frac{1-2j}{-3j}\right]$$

$$I_2 = \frac{V_2}{Z_{22}} = \frac{V_2}{\frac{-3j}{1-j}} = \frac{V_2}{-3j}(1-j)$$

$$Z_{12} = \frac{V_1}{I_2} = \frac{V_2}{V_2} \frac{(1-2j)(-3j)}{(-3j)(1-j)} = \frac{1-2j}{1-j}$$

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \left(\frac{2-j}{1-j}\right) & \left(\frac{1-2j}{1-j}\right) \\ \left(\frac{1-2j}{1-j}\right) & \left(\frac{-3j}{1-j}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 + 0.5j & 1.5 - 0.5j \\ 1.5 - 0.5j & 1.5 - 1.5j \end{bmatrix}$$