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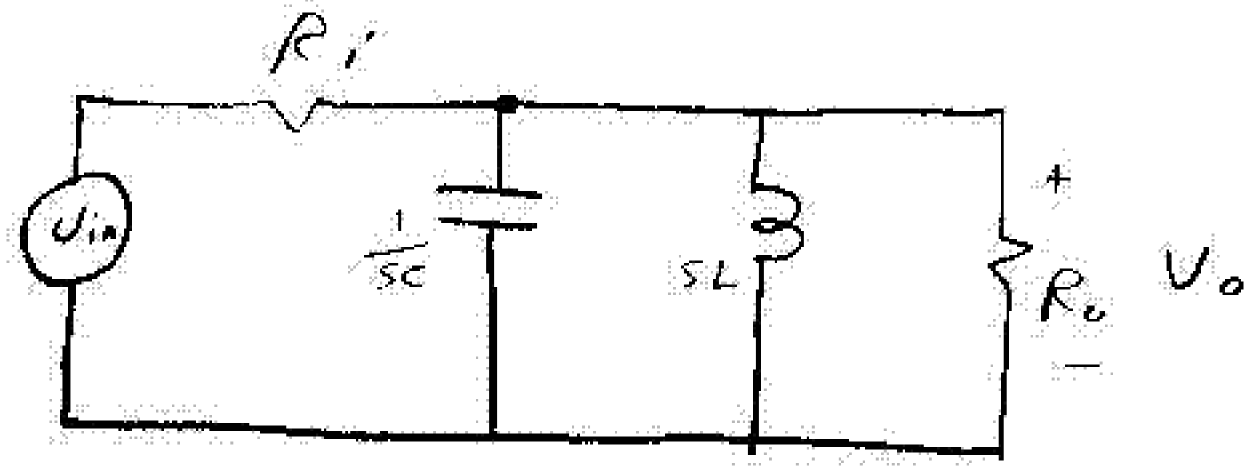
Circuits II 2nd electric Communication

2009-2010

(1) to get the TF, $\frac{V_o}{V_i}$



$$\frac{V_o - V_{in}}{R_i} + V_o sC + \frac{V_o}{sL} + \frac{V_o}{R_o} = 0$$

$$V_o\left(\frac{1}{R_i} + \frac{1}{R_o} + sC + \frac{1}{sL}\right) = \frac{V_{in}}{R_i}$$

$$\frac{V_o}{V_i} = \frac{1/R_i}{\frac{1}{R_i} + \frac{1}{R_o} + sC + \frac{1}{sL}} = \frac{s/CR_i}{s^2 + \frac{s}{C}\left(\frac{1}{R_i} + \frac{1}{R_o}\right) + \frac{1}{CL}} \quad \boxed{3}$$

(a) The function is Band Pass Filter. $\boxed{2}$

(b)
$$\left|\frac{V_o}{V_i}\right| = \frac{\omega/CR_i}{\sqrt{\left(\frac{1}{CL} - \omega^2\right)^2 + \left(\frac{\omega}{C}\left(\frac{1}{R_i} + \frac{1}{R_o}\right)\right)^2}} \longrightarrow \# \quad \boxed{3}$$

(c) $\boxed{\omega_o = \frac{1}{\sqrt{CL}}}$ $\boxed{2}$, to get ω_{c_1} & ω_{c_2}. first compute the magnitude of $\left(\frac{V_o}{V_i}\right)$ at $\omega = \omega_o$

$$\left|\frac{V_o}{V_i}\right|\Bigg|_{\omega=\omega_o} = \frac{\omega_o/CR_i}{\sqrt{\left(\frac{1}{CL} - \omega_o^2\right)^2 + \left(\frac{\omega_o}{C}\left(\frac{1}{R_i} + \frac{1}{R_o}\right)\right)^2}}$$

$$\left|\frac{V_o}{V_i}\right|\Bigg|_{\omega=\omega_o} = \frac{1/R_i}{1/R_i + 1/R_o} = \frac{R_o}{R_o + R_i} \quad \boxed{1}$$

at ω_{c_1} & $\omega_{c_2} \rightarrow$ the $\left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{2}}\left|\frac{V_o}{V_i}\right|\Bigg|_{\omega=\omega_o}$

$$\frac{(1/R_i)^2}{2\left(\frac{1}{R_i} + \frac{1}{R_o}\right)^2} = \frac{\frac{\omega^2}{C^2 R_i^2}}{\left(\frac{1}{CL} - \omega^2\right)^2 + \left(\frac{\omega}{C}\left(\frac{1}{R_i} + \frac{1}{R_o}\right)\right)^2}$$

$$\frac{2\omega^2}{C^2}\left(\frac{1}{R_i} + \frac{1}{R_o}\right)^2 = \left(\frac{1}{CL} - \omega^2\right)^2 + \frac{\omega^2}{C^2}\left(\frac{1}{R_i} + \frac{1}{R_o}\right)^2$$

$$\frac{\omega^2}{C^2}\left(\frac{1}{R_i} + \frac{1}{R_o}\right)^2 = \left(\frac{1}{CL} - \omega^2\right)^2$$

$$\pm \frac{\omega}{C}\left(\frac{1}{R_i} + \frac{1}{R_o}\right) = \frac{1}{CL} - \omega^2$$

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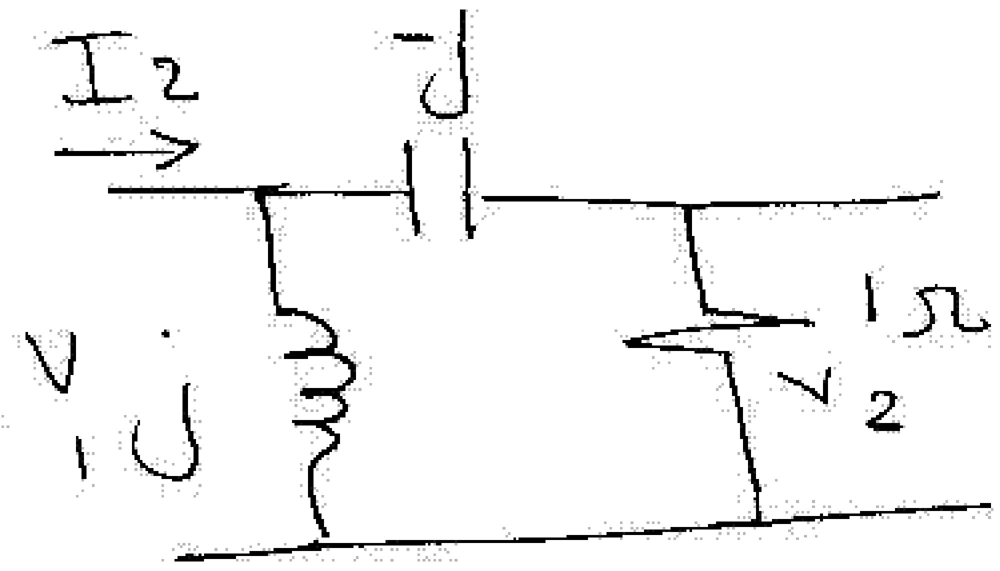

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$Z_{11} = \frac{j(1-j)}{j+(1-j)} = 1+j$$

$$V_2 = \frac{V_1}{(1-j)} \times 1$$

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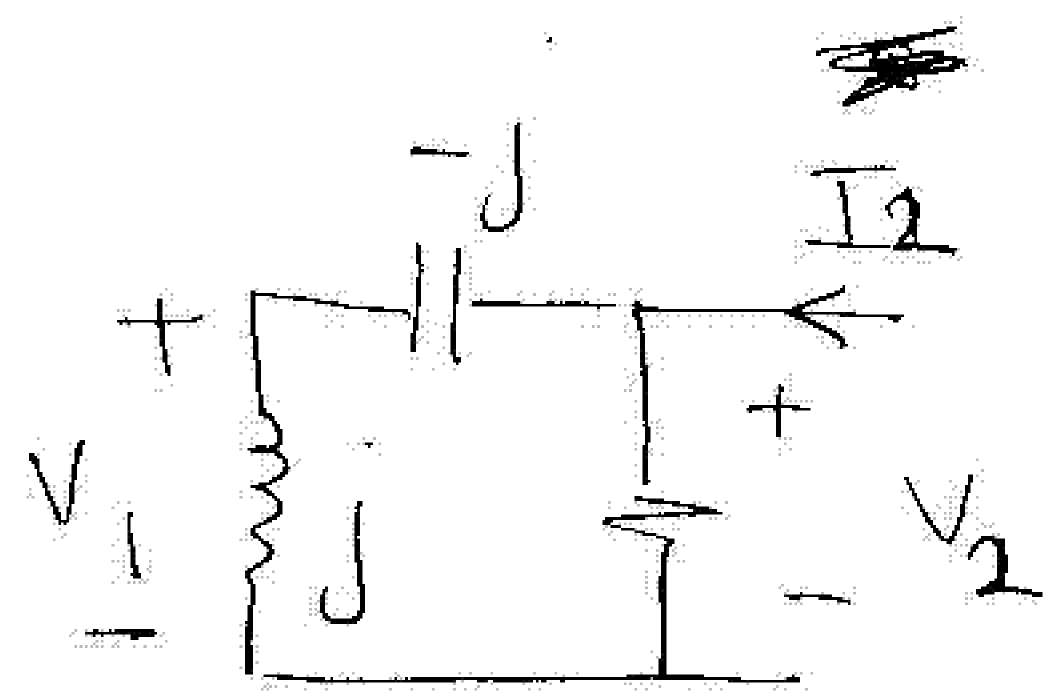
$$I_1 = \frac{V_1}{Z_{11}} = \frac{V_1}{1+j}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{V_1}{(1-j)}\frac{(1+j)}{V_1} = j$$



$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$Z_{22} = \frac{(j-j)*1}{j-j+1} = 0$$

$$Z_{12} = \frac{V_1}{I_2}$$

$$Z_{12} = \frac{I_2 j}{I_2}$$

the equivalent Z in this branch $= j - j = 0 \to$ S.C

So the total current I_2 will flow in the branch.

$$Z = \begin{bmatrix} 1+j & j \\ j & 0 \end{bmatrix}$$

$$\omega_{c_1}^2 + \frac{\omega_{c_1}}{C}\left(\frac{1}{R_i} + \frac{1}{R_o}\right) - \frac{1}{CL} = 0$$

$$\omega_{c_1} = \frac{-\frac{1}{C}\left(\frac{1}{R_i} + \frac{1}{R_o}\right) + \sqrt{\frac{1}{C^2}\left(\frac{1}{R_i} + \frac{1}{R_o}\right)^2 + \frac{4}{CL}}}{2}$$

$$\omega_{c_1} = -\frac{1}{2C}\left(\frac{1}{R_i} + \frac{1}{R_o}\right) + \sqrt{\frac{1}{4C^2}\left(\frac{1}{R_i} + \frac{1}{R_o}\right)^2 + \frac{1}{CL}} \quad \boxed{1}$$

$$\omega_{c_2} = +\frac{1}{2C}\left(\frac{1}{R_i} + \frac{1}{R_o}\right) + \sqrt{\frac{1}{4C^2}\left(\frac{1}{R_i} + \frac{1}{R_o}\right)^2 + \frac{1}{CL}} \quad \boxed{1}$$

(d) $\omega_o = \frac{1}{\sqrt{5*10^{3}*200*10^{-12}}} = 1M \text{ rad/sec}$ $\boxed{2}$

$\omega_{c_1} = 0.969238162 * 10^6$ rad/sec

$\omega_{c_2} = 1.031738162 * 10^6$ rad/sec

$BW = \omega_{c_2} - \omega_{c_1} = 62.5$ krad/sec $\boxed{1}$

$Q = \frac{\omega_o}{BW} = \frac{1*10^6}{62.5*10^3} = 16$ $\boxed{1}$

(e) at $\omega = \omega_o$ $\left|\frac{v_o}{v_i}\right|_{\omega=\omega_o} = \frac{R_o}{R_i + R_o}$

$\therefore v_o = \frac{400*10^3}{100*10^3 + 400*10^3} * 250 \cos\omega_o t$ mV

$\boxed{v_o = 200 \cos\omega_o t \text{ mV}}$ $\boxed{3}$

(2) $M = K\sqrt{L_1 L_2} = 0.5\sqrt{1\times1} = 0.5\,H$

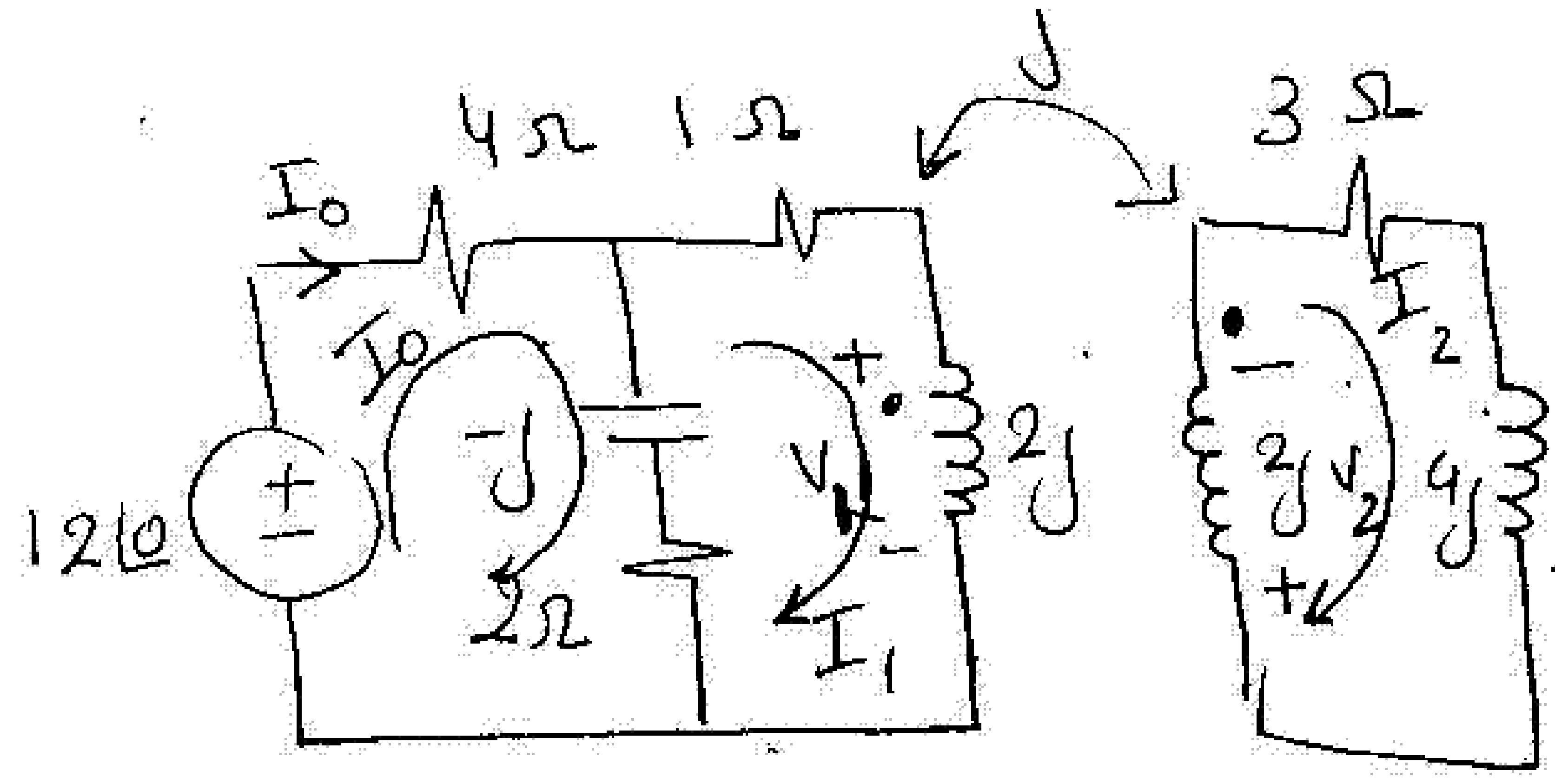
$\omega = 2$

$0.5F \rightarrow \frac{1}{j\omega C} = -j$

$1\,H \rightarrow j\omega L = 2j$

$2\,H = 4j$

$0.5H = j$



loop (1)

$12\angle 0 - 4I_0 - (-j+2)(I_0 - I_1) = 0$

$12\angle 0 + (+j-6)I_0 + (2-j)I_1 = 0 \rightarrow$ eq 1

loop (2)

$(2-j)(I_1 - I_0) + I_1 + V_1 = 0$

$V_1 = 2jI_1 - jI_2$

$(3-j)I_1 - (2-j)I_0 + 2jI_1 - jI_2 = 0$

$-(2-j)I_0 + (3+j)I_1 - jI_2 = 0 \longrightarrow$ eq (2)

loop (3)

$V_2 + 4jI_2 + 3I_2 = 0$

$V_2 = 2jI_2 - jI_1$

$2jI_2 - jI_1 + 4jI_2 + 3I_2 = 0$

$-jI_1 + (6j+3)I_2 = 0 \longrightarrow$ eq (3) eq 1

$$-(2-j)\, I_1 = 12 + (j-6)\, I_o$$

$$I_1 = \frac{12 + (j-6) I_o}{(j-2)} = (-4.8 - 2.4j) + (2.6 + 0.8j) I_o$$

eq (3)

$$I_2 = \frac{j I_1}{(6j+3)} = (0.133 + j0.066j) I_1$$

$$= (-0.48 - 0.636j) + (0.293 + j0.278) I_o$$

eq (2)

$$(j-2) I_o + (3+j) \left[(-4.8-2.4j) + (2.6+0.8j) I_o\right) - j \left[(-0.48-0.636j) + (0.293 + j\,0.278) I_o\right] = 0$$

$$(j-2) I_o - (12+12j) + (7+5j) I_o + (-0.636 + j0.48) + (0.278 - 0.293j) I_o = 0$$

$$(5.278 + j5.707) I_o + (-12.636 - 11.52j) = 0$$

$$I_o = 2.19 - 0.187j = 2.197 \underline{|-4.88^\circ} \text{ A}$$

$$I_1 = 1.043 - j1.134j = 1.54 \underline{|-47.4}$$

$$I_2 = 0.21 - 0.082j = 0.225 \underline{|-21.3}$$

max

$$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 - M I_1 I_2$$

$$I_1 = 1.54 \angle{-47.4} = 1.54 \cos(2t - 47.4)$$

$$= 1.54 \cos\left(2 * \frac{180}{\pi} \times 2 \times 10^{-3} - 47.4\right) = 1.047\ A$$

$$I_2 = 0.225 \angle{-21.3} = 0.225 \cos(2t - 21.3)$$

$$= 0.225 \cos\left(2 \times \frac{180}{\pi} \times 2 \times 10^{-3} - 21.3\right) = 0.21A \quad A$$

$$W = \frac{1}{2} 1 (1.047)^2 + \frac{1}{2} 1 (0.21)^2 - 0.5(1.047)(0.21)$$

$$= 0.46\ j$$

$$I_1 = 1.54 \sin\left(2 \times \frac{180}{\pi} \times 2 \times 10^{-3} - 47.4\right) = -1.13\ A$$

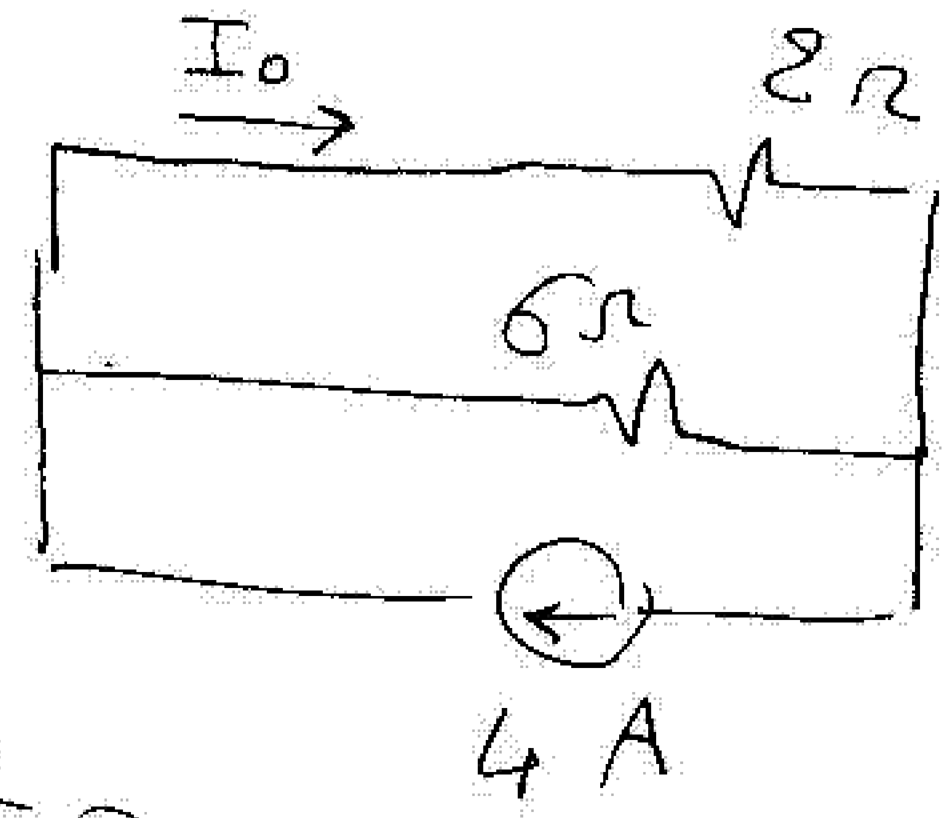
$$I_2 = 0.225 \sin\left(2 \times \frac{180}{\pi} \times 2 \times 10^{-3} - 21.3\right) = -0.081$$

$$W = \frac{1}{2} 1 (-1.13)^2 + \frac{1}{2} 1 \times (-0.081)^2 - 0.5(-1.13)(-0.081)$$

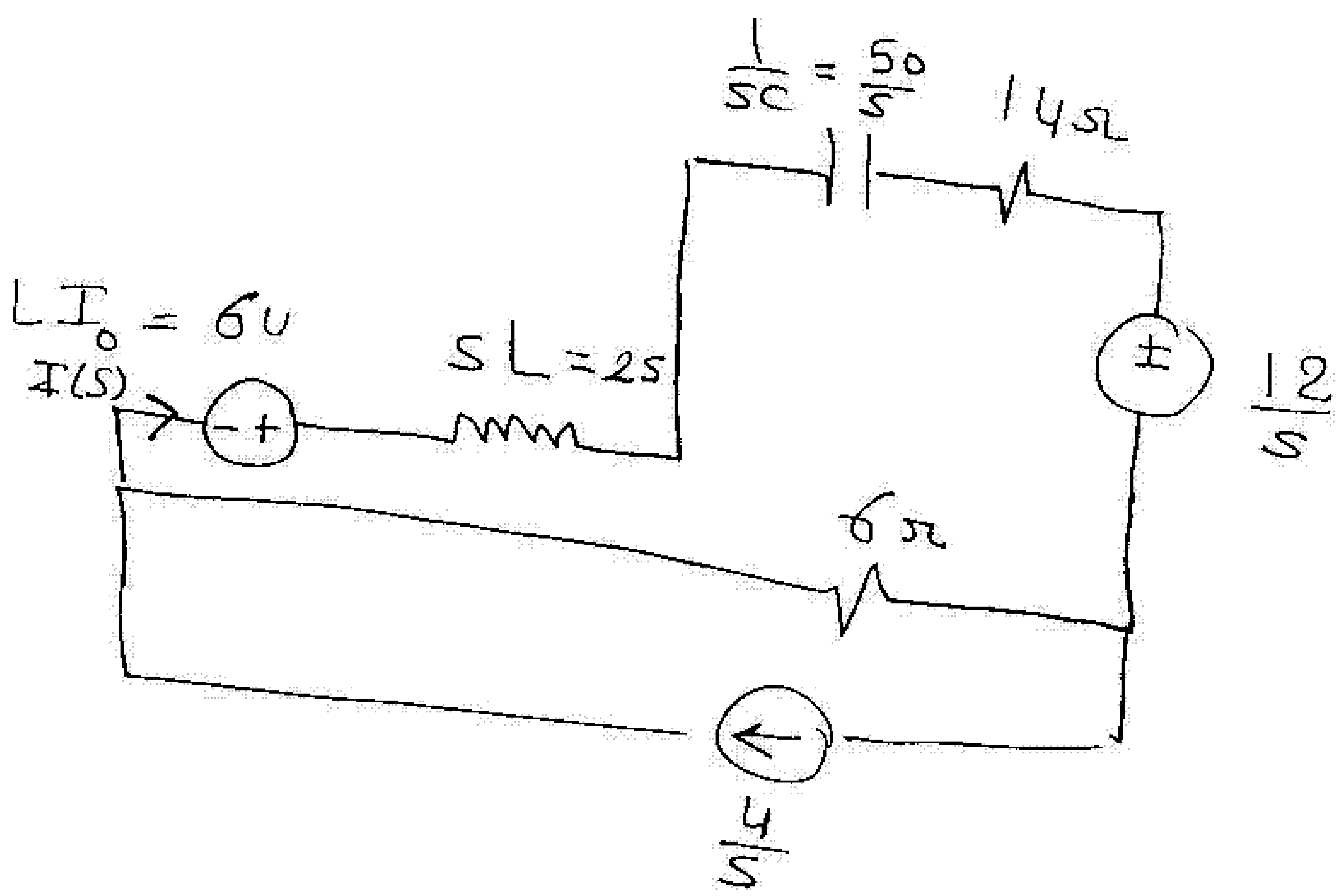
$$= 0.59\ j$$

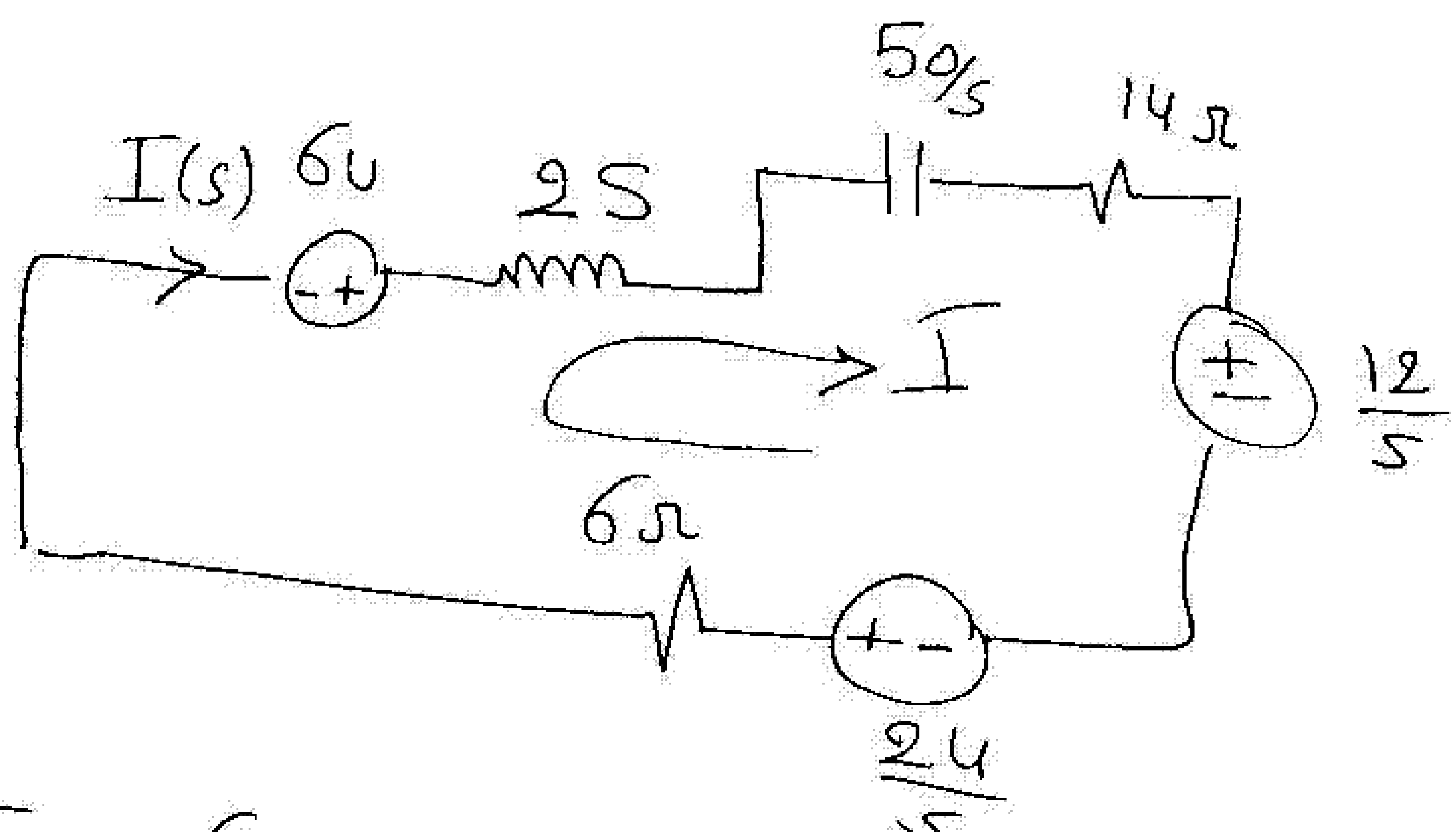
③ at $t < 0$ switch at pos. a

$$I_0 = \frac{4 \times 6}{8} = 3A$$



at $t = 0$ switch at pos. b





$$\frac{24}{s} - 6I + 6 - 2sI - \frac{50}{s}I - 14I - \frac{12}{s} = 0$$

$$\frac{12}{s} + 6 + I\left(-6 - 2s - 14 - \frac{50}{s}\right) = 0$$

$$\frac{12}{s} + 6 = I\left(2S + \frac{50}{S}\right) + 20$$

$$I = \frac{\frac{12}{S} + 6}{20 + 2S + \frac{50}{S}} = \frac{6S + 12}{2S^2 + 20S + 50}$$

$$= \cancel{\frac{6}{S}\left(\frac{S+2}{2S+10}\right)} = \frac{3\,(S+2)}{S^2 + 10S + 25}$$

$$= 3\left[\frac{K_1}{(S+5)^2} + \frac{K_2}{S+S}\right]$$

$$I(t) = 3\left(k_1\, t\, e^{-St} + k_2\, e^{-St}\right)$$

(4) $f(t) = f(-t)$ even

$b_n = 0$

$a_v = \frac{2}{T}\int_0^{T/2} f(t)\,dt$

$T = 5 \text{ sec} \longrightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{5}$

$$f(t) = \begin{cases} 0 & 0<t<1 \\ 5 & 1<t<2 \\ 10 & 2<t<2.5 \end{cases}$$

$$a_v = \frac{\cancel{2}}{2.5}\left[\int_0^1 0\,dt + \int_1^2 5\,dt + \int_2^{2.5} 10\,dt\right]$$

$$= \frac{\cancel{2}}{2.5}\left[5t\Big|_1^2 + 10t\Big|_2^{2.5}\right]$$

$$= \frac{\cancel{2}}{2.5}\left[5(2-1) + 10(2.5-2)\right] = \frac{\cancel{2}}{2.5}(5+5) = 4$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{4}{5}\left[\int_1^2 5\cos n\omega_0 t\,dt + \int_2^{2.5} 10\cos n\omega_0 t\,dt\right]$$

$$= \frac{4}{5}\left[\frac{5}{n\omega_0}\sin n\omega_0 t\Big|_1^2 + \frac{10}{n\omega_0}\sin n\omega_0 t\Big|_2^{2.5}\right]$$

$$= \frac{4}{5}\left[\frac{5}{n\left(\frac{2\pi}{5}\right)}\left(\sin n\frac{2\pi}{5}\times 2 - \sin n\frac{2\pi}{5}\times 1\right)\right.$$
$$\left. + \frac{10}{n\left(\frac{2\pi}{5}\right)}\left(\sin n\frac{2\pi}{\cancel{5}}\times\frac{\cancel{5}}{2} - \sin n\frac{2\pi}{5}\times 2\right)\right]$$

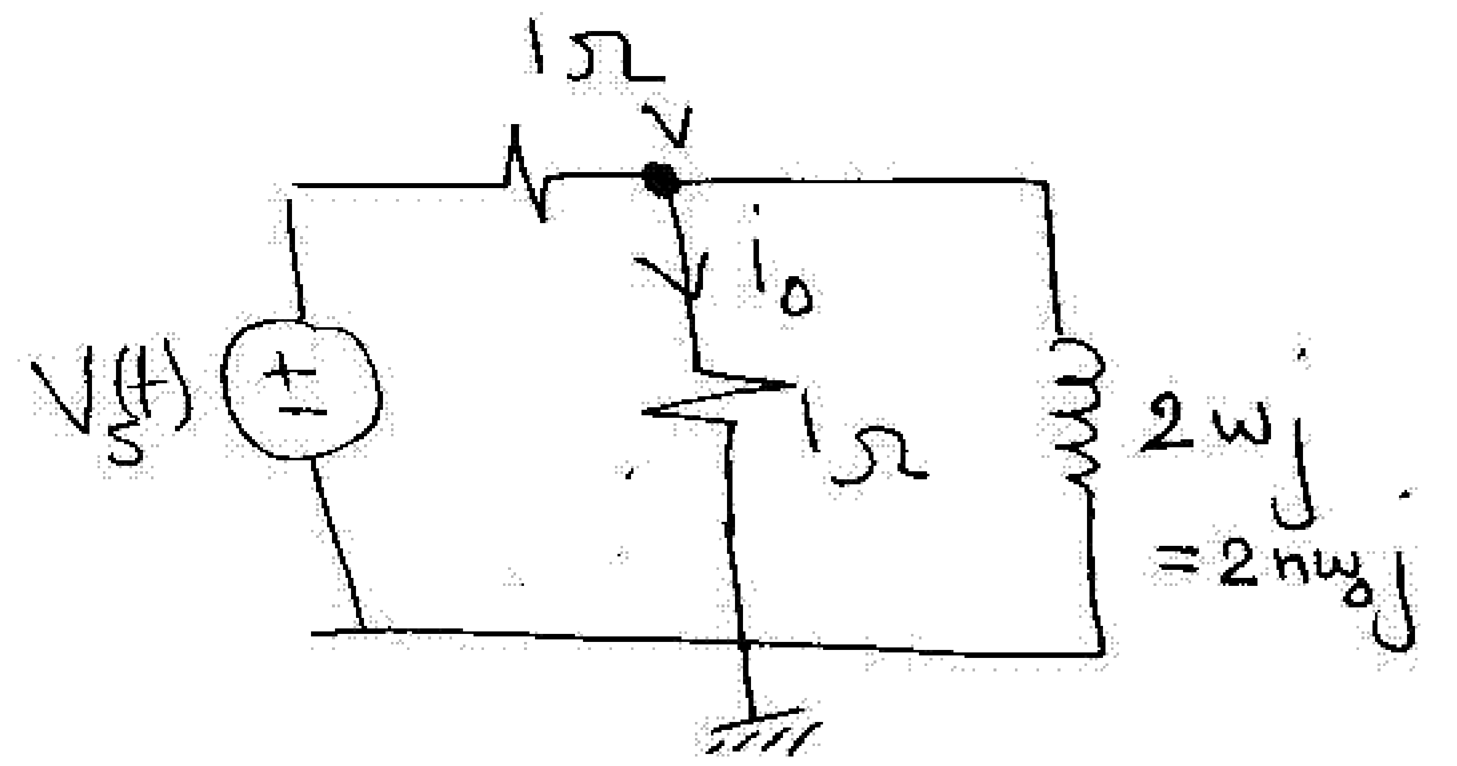
$$= \frac{4}{5}\left[\frac{25}{2n\pi}\left(\sin n\frac{4\pi}{5} - \sin\frac{2\pi}{5}n\right)\right.$$
$$+ \frac{\cancel{2}50}{\cancel{2}4 n\pi}\left(0 - \sin n\frac{4\pi}{5}\right)$$

$$a_n = \frac{4}{5}\left[\frac{-25}{2n\pi}\sin n\frac{4\pi}{5} - \frac{25}{2n\pi}\sin\frac{2\pi}{5}n\right]$$

$$= \frac{-10}{n\pi}\left(\sin n\frac{4\pi}{5} + \sin n\frac{2\pi}{5}\right)$$

$$V_s(t) = a_v + \sum_{n=1}^{\infty}\left(a_n \cos n\omega_0 t\right)$$

$$= 4 + \sum_{n=1}^{\infty} \frac{-10}{n\pi}\left(\sin\frac{4\pi}{5}n + \sin\frac{2\pi}{5}n\right)\cos n\omega_0 t$$



$$\frac{V - V_s(t)}{1} + \frac{V}{1} + \frac{V}{2\omega j} = 0$$

$$V\left(1 + 1 + \frac{1}{2\omega j}\right) = V_s(t)$$

$$V = \frac{V_s(t)}{\left(2 + \frac{1}{2j\omega}\right)}$$

$$i_o = \frac{V}{1} = \frac{V_s(t) \; 2j\omega}{(4j\omega + 1)}$$

a.C

$$d.C = \frac{\left[4 + \sum_{n=1}^{\infty} \frac{-10}{n\pi}\left(\sin\frac{4\pi}{5}n + \sin\frac{2\pi}{5}n\right)\right]2j\omega}{(4j\omega + 1)}$$

~~for dc $\omega = 0$~~ $= 0$

$$n\omega_0 = n\left(\frac{2\pi}{5}\right)$$