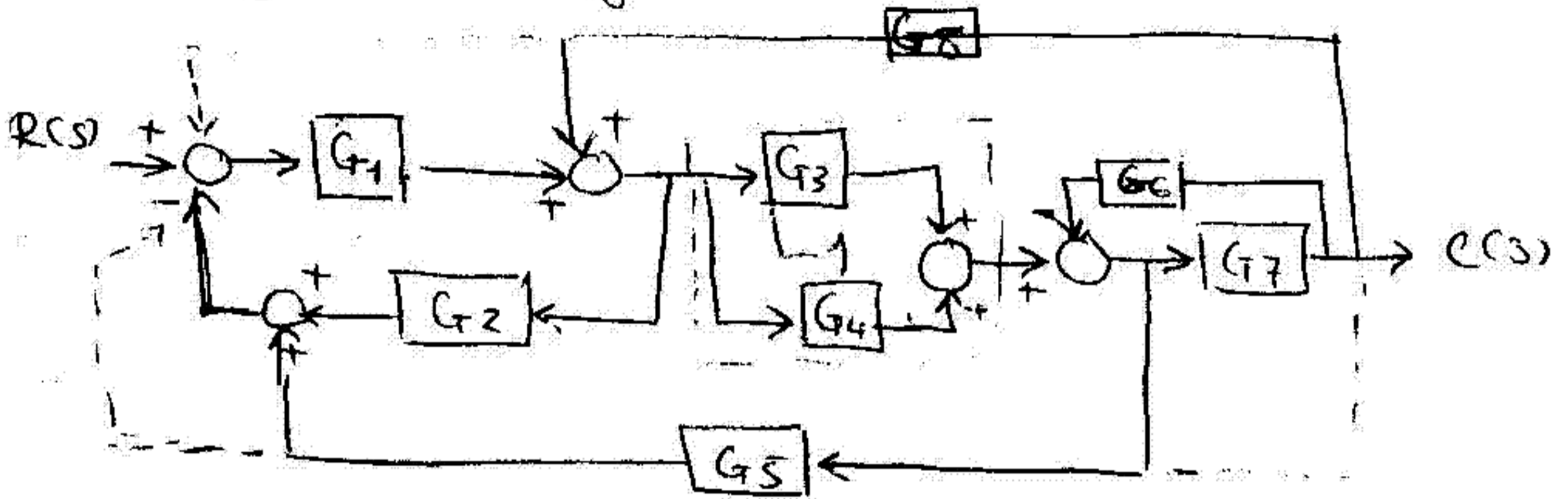
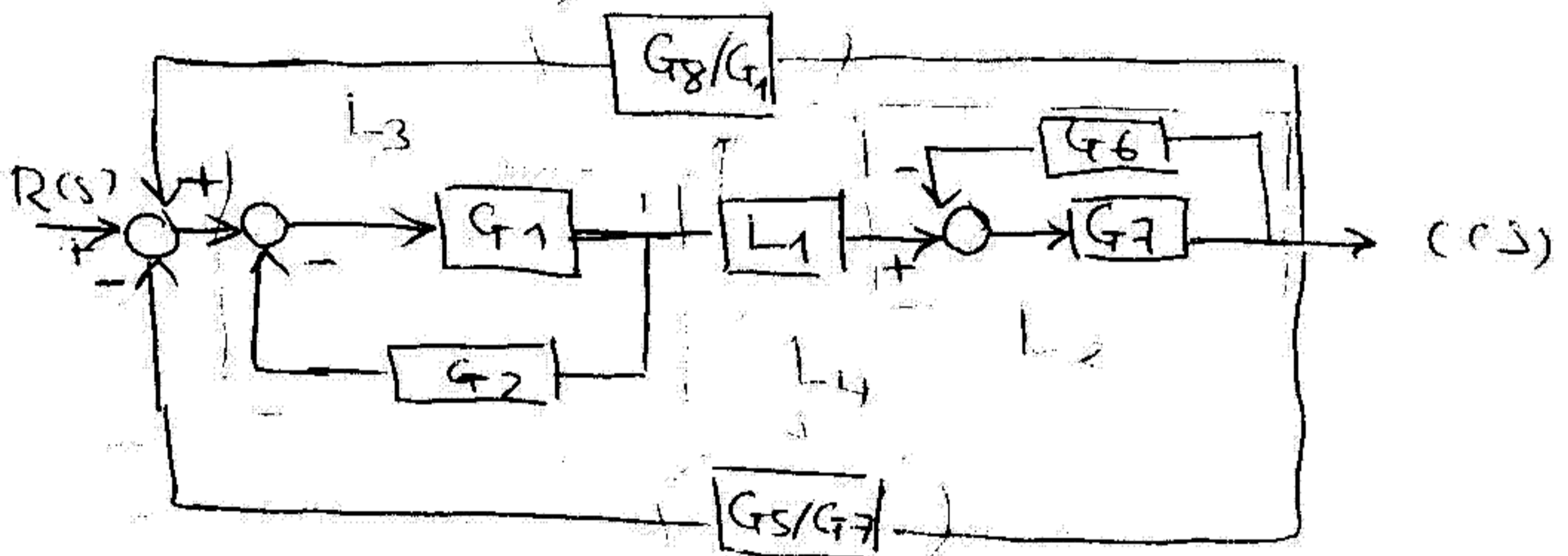


محمد ال
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1. a) Using Block diagram reduction

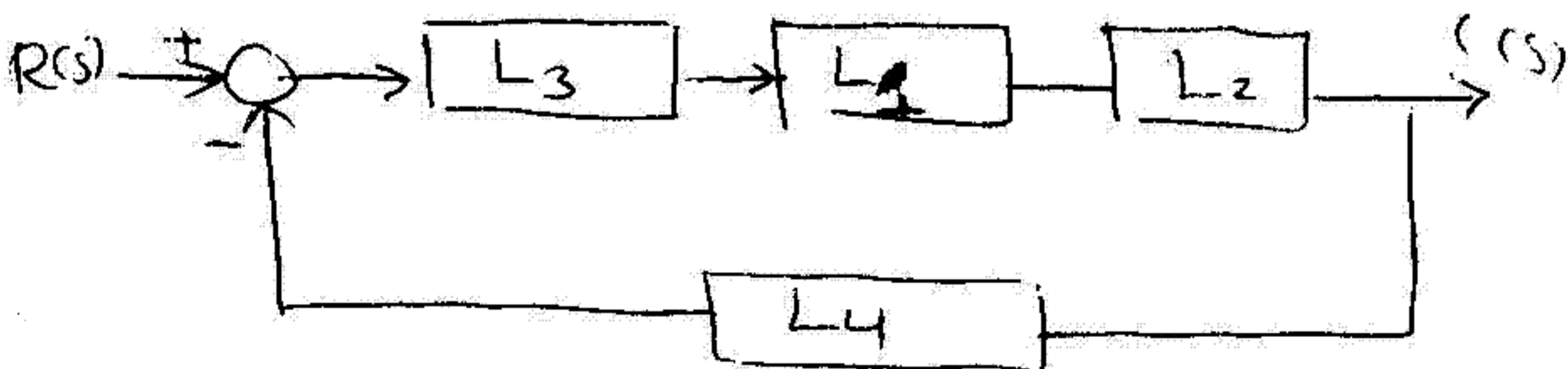


$$L_1 = G_3 + G_4$$



$$L_3 = \frac{G_1}{1 + G_1 G_2}, \quad L_2 = \frac{G_7}{1 + G_6 G_7}$$

$$L_4 = \frac{G_5}{G_7} - \frac{G_8}{G_1}$$

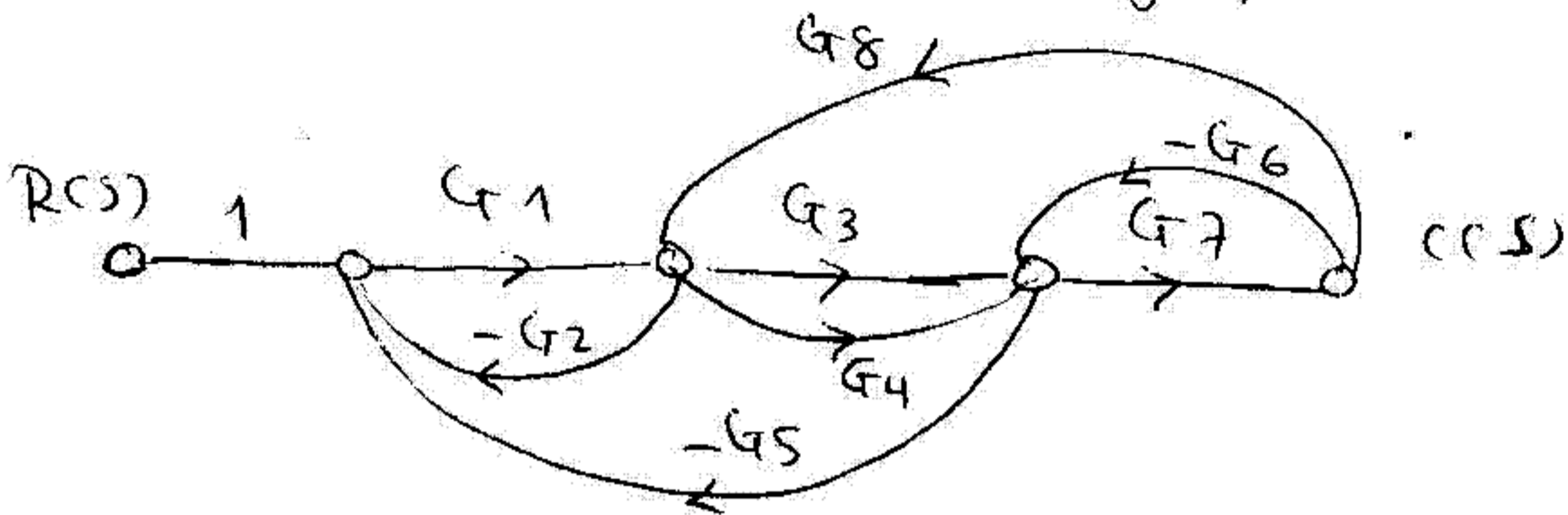


$$G(s) = \frac{L_1 L_2 L_3}{1 + L_1 L_2 L_3 L_4}$$

$$= \frac{(G_3 + G_4) \left(\frac{G_7}{1 + G_6 G_7} \right) \left(\frac{G_1}{1 + G_1 G_2} \right)}{1 + (G_3 + G_4) \left(\frac{G_7}{1 + G_6 G_7} \right) \cdot \left(\frac{G_1}{1 + G_1 G_2} \right) \cdot \frac{G_1 G_5 - G_7}{(G_1 G_7)}}$$

$$\therefore G(s) = \frac{G_1 G_3 G_7 + G_1 G_4 G_7}{1 + G_1 G_2 + G_6 G_7 + G_1 G_2 G_6 G_7 + G_1 G_3 G_5 + G_1 G_4 G_5 - G_3 G_7 G_8 - G_4 G_7}$$

10 b) using signal flow graph & Mason's rule



Paths :
 $P_1: G_1 G_3 G_7$
 $P_2: G_1 G_4 G_7$

Loops
 $L_1 = -G_1 G_2$
 $L_2 = -G_1 G_3 G_5$
 $L_3 = -G_1 G_4 G_5$
 $L_4 = G_3 G_7 G_8$
 $L_5 = -G_6 G_7$
 $L_6 = G_4 G_7 G_8$

non-touching loops L_1 & L_5

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6) + L_1 L_5$$

$$\Delta_1 = 1 \quad ; \quad (\text{all loops touching Path 1})$$

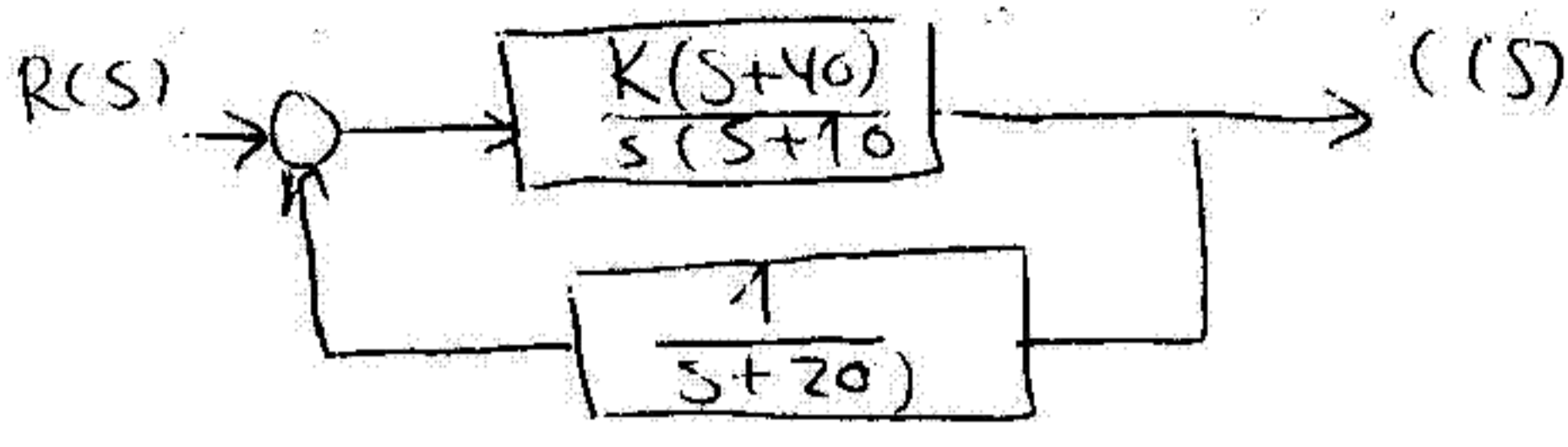
$$\Delta_2 = 1 \quad ; \quad (\text{all loops touching Path 1})$$

$$G(s) = \frac{\sum_{i=1}^2 \Delta_i P_i}{\Delta}$$

$$= \frac{G_1 G_3 G_7 + G_1 G_4 G_7}{\Delta}$$

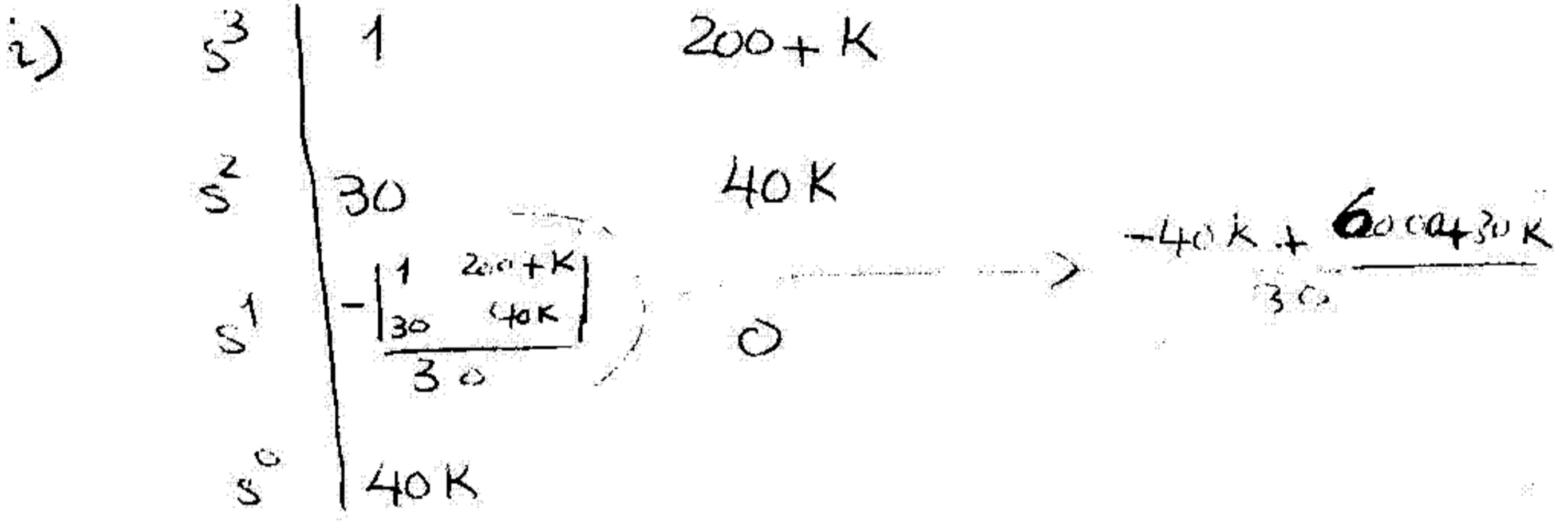
$$\Delta = 1 + G_1 G_2 + G_1 G_3 G_5 + G_1 G_4 G_5 - G_3 G_7 G_8 - G_6 G_7 + G_4 G_7 G_8$$

2.



$$1 + GH = 1 + \frac{K(s+40)}{s(s+10)} \cdot \frac{1}{(s+20)} = 0$$

$$\therefore s^3 + 30s^2 + (200+K)s + 40K = 0$$



For a stable system ;

$$6000 - 10K > 0 \Rightarrow K < 600$$

$$40K > 0 \quad K > 0$$

∴ range for stable system

$$0 < K < 600$$

2.

ii) For a marginally stable system $K=600$
 ω_n for s^2

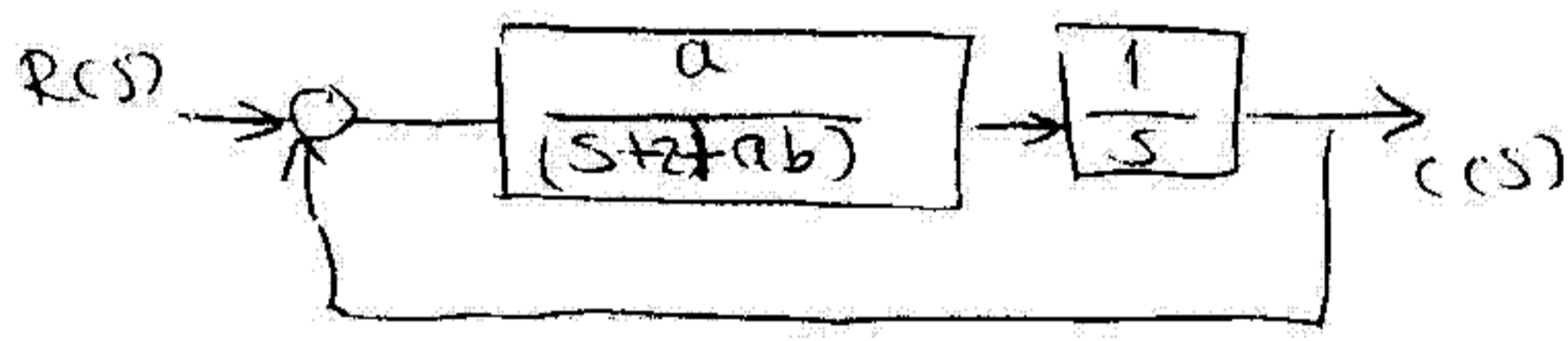
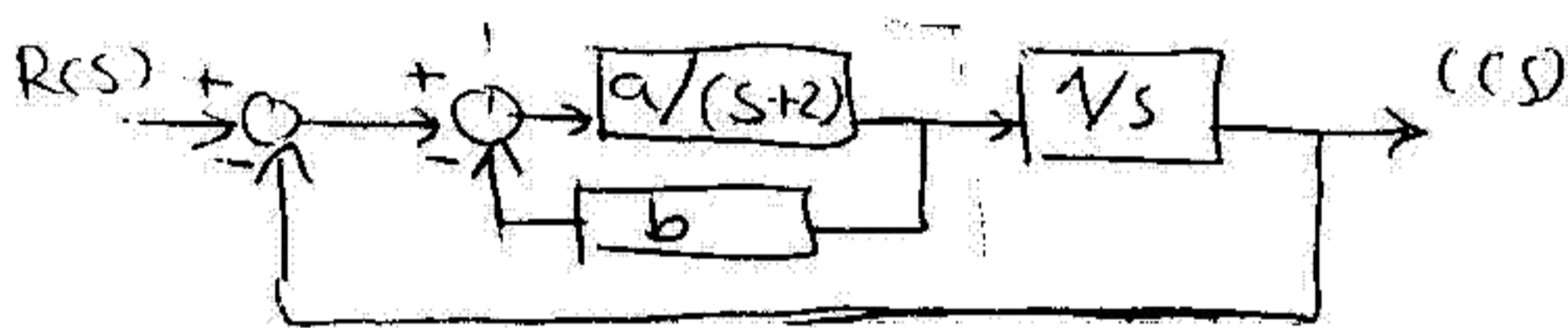
$$30s^2 + 40 \times 600 = 0$$

$$\therefore s^2 = 800 \Rightarrow s_{1,2} = \pm j\sqrt{800}$$

$$= \pm j 20\sqrt{2} ; \omega_n =$$



3 a)



$T_s = 1.4 \text{ sec}$ $\%OS = 4.6\%$

$$1 + GH = 0 \Rightarrow 1 + \frac{a}{s(s+2+ab)} = 0$$

ω_n $s^2 + (2+ab)s + a = 0$
 standard form $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$\omega_n^2 = a$, $2\zeta\omega_n = (2+ab)$

$$T_s = \frac{4}{z\omega_n} \Rightarrow z\omega_n = \frac{4}{1.43}$$

$$\%OS = 4.6\% \Rightarrow z = \frac{\ln(\%OS)^2}{\sqrt{\pi^2 + (\ln \%OS)^2}} = 0.7$$

$$\therefore \omega_n = \frac{4}{1.43 \times 0.7} = 4$$

$$\therefore a = \omega_n^2 = 16$$

$$2+ab = 2z\omega_n = 2 \times 0.7 \times 4 = 5.6$$

$$\therefore b = \frac{3.6}{16} = 0.225$$

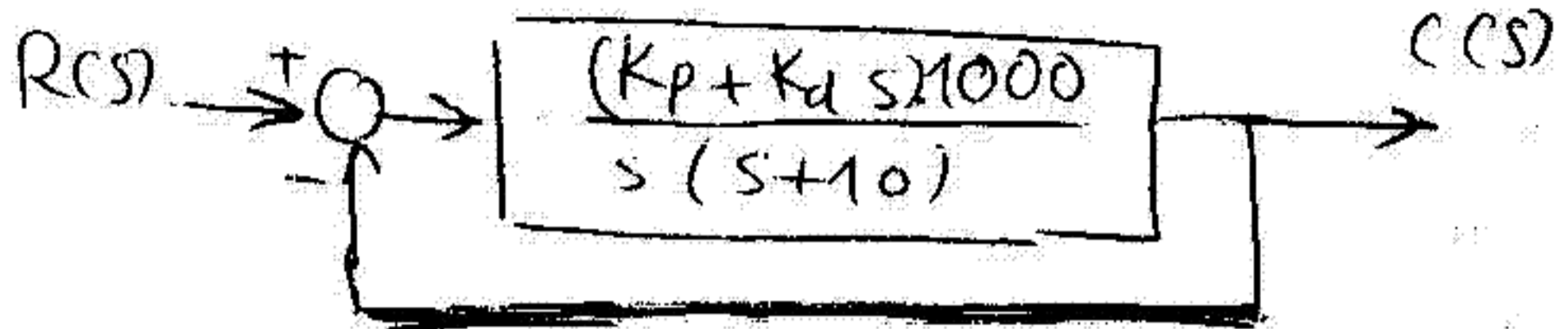
• Since the system is type 1, then

$$e_{ss} \text{ (for unit step) } = \text{zero}$$

• The steady state error for unit ramp input:

$$\begin{aligned} e_{ss} &= \lim_{s \rightarrow 0} \frac{s \cdot \frac{1}{s^2}}{1 + G(s)} = \lim_{s \rightarrow 0} \frac{1}{s + \frac{a}{s + (2+ab)}} \\ &= \frac{(2+ab)}{a} \\ &= \frac{2 + 16 \times 0.225}{16} \\ &= 0.35 \end{aligned}$$

3b).



$$G(s) = \frac{C(s)}{1 + G(s) \cdot H(s)} = \frac{(K_p + K_d s) \cdot 1000}{s(s+10)}$$

$$= \frac{1000(K_p + K_d s)}{s^2 + (10 + 1000K_d)s + 10000}$$

Since the system is type 1, then the steady state error constant is K_v
 $\therefore K_v = 1000$

$$K_v = \lim_{s \rightarrow 0} sG(s) = \frac{1000K_p}{10} = 1000$$

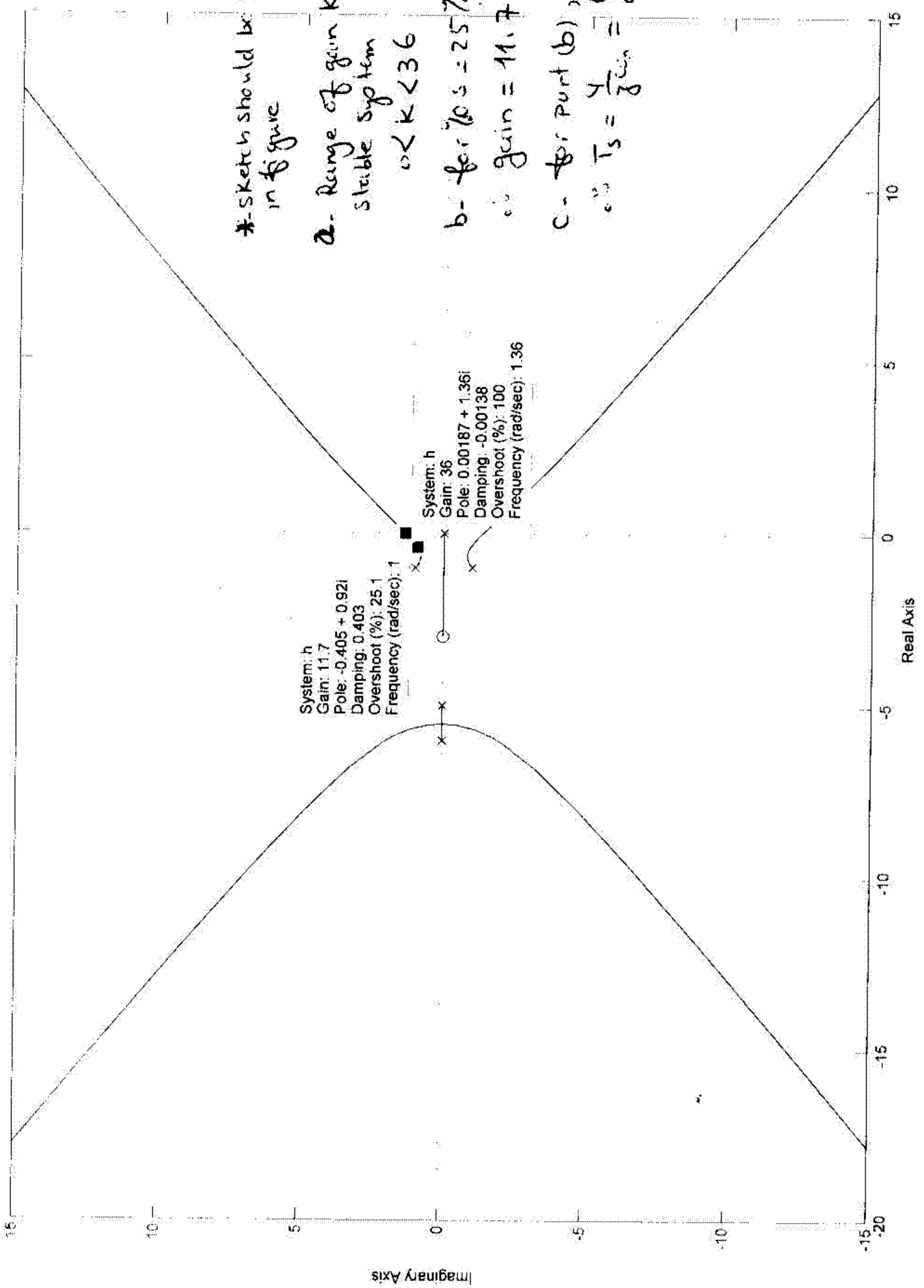
$$\therefore K_p = 10$$

for $T_s = 0.08 = \frac{4}{\zeta\omega_n} \Rightarrow \zeta\omega_n = 50$

$$\therefore (10 + 1000K_d) = 2\zeta\omega_n = 100$$

$$\therefore K_d = \frac{90}{1000} = 0.09$$

Root Locus



* Sketch should be as shown in figure

a- Range of gain k for stable system
 $0 < k < 36$

b- for $\zeta = 0.403$ ($\zeta = 0.403$)
 $\omega_n = 11.7$

c- for part (b), $\zeta \omega_n = 0.403 \times 11.7$
 $\omega_n T_s = \frac{4}{\zeta \omega_n} = \frac{4}{0.403 \times 11.7} \approx 1.5s$

Solution of Prob. 5 for Final Exam of EPM 308 (Autom Control course), Fall 2009/2010.

$$KGH = K \frac{(s+50)}{s(s+10)} \cdot \frac{1}{(s+20)}$$

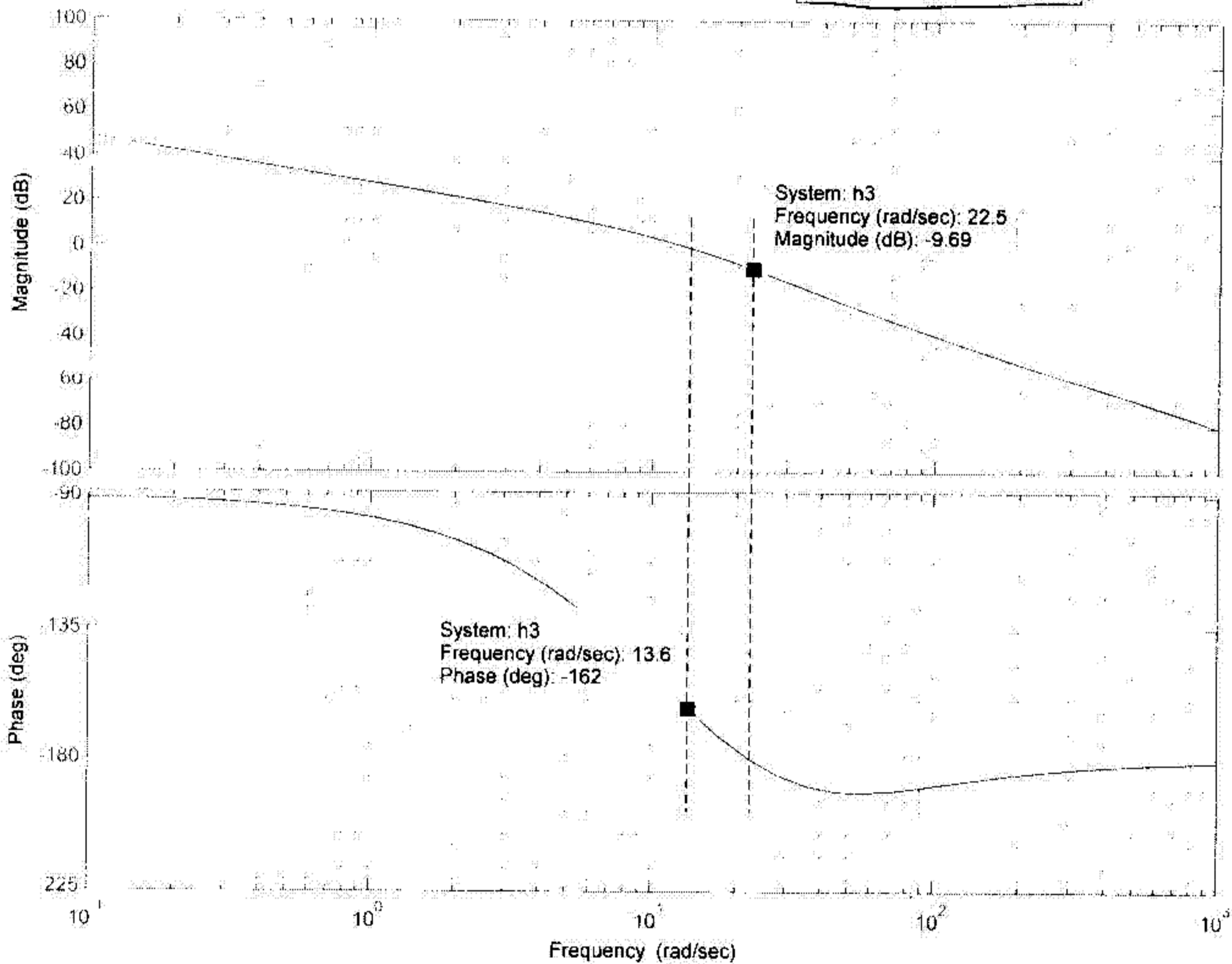
$$= \frac{K \cdot 50}{10 \times 20} * \frac{(s/50 + 1)}{s(\frac{s}{10} + 1)(\frac{s}{20} + 1)}$$

for $K = 100$

$$\text{or } \frac{K \times 50}{200} = 25 \Rightarrow 20 \log 25 = 27.96$$

$$GM = 9.69 \text{ dB}$$

Bode Diagram



$$\text{Phase Margin} = 180 - 162 = 18^\circ$$