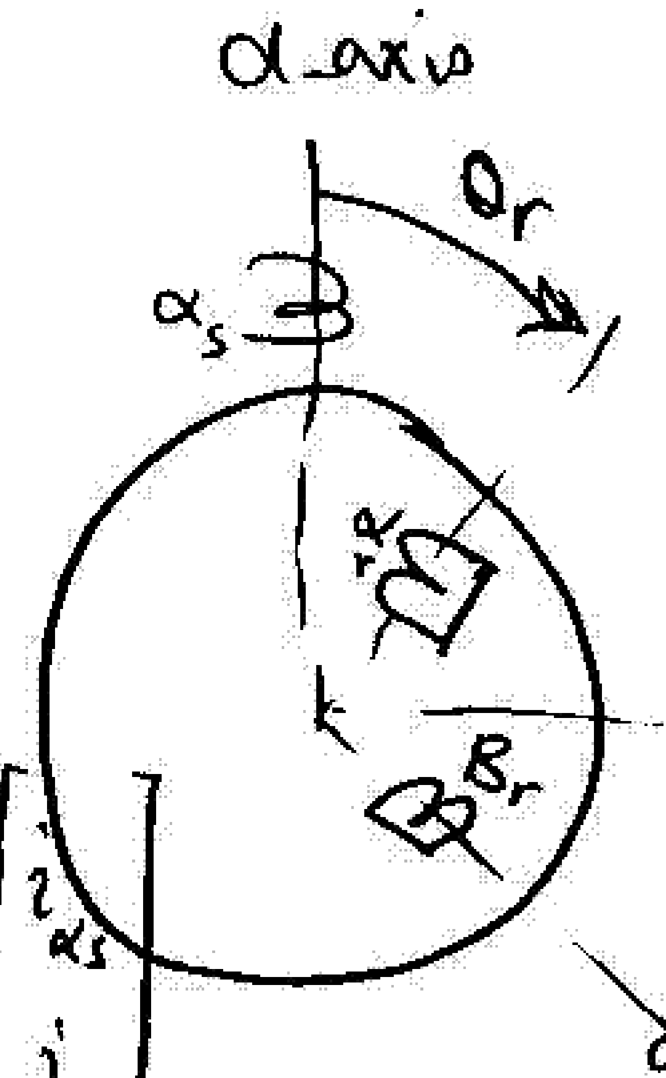
Electric machines III EPM 405 A
Final exam 2009/2010
Solution

الزتكويم (٣١)



1. Voltage equations

$$\begin{bmatrix} v_{\alpha s} \\ v_{\beta s} \\ \hline v_{\alpha r} \\ v_{\beta r} \end{bmatrix} = \left[\begin{array}{cc|cc} R_s+PL_s & 0 & PL_o\cos\theta_r & -PL_o\sin\theta_r \\ 0 & R_s+PL_s & PL_o\sin\theta & PL_o\cos\theta_r \\ \hline PL_o\cos\theta_r & PL_o\sin\theta_r & R_r+PL_r & 0 \\ -PL_o\sin\theta_r & PL_o\cos\theta_r & 0 & R_r+PL_r \end{array}\right] \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{\alpha r} \\ i_{\beta r} \end{bmatrix}, \quad L_s = L_{ls} \ldots \quad L_r = L \ldots$$

$$T_e = P\,\frac{\partial W_{field}}{\partial \theta_r}, \quad P: \text{no. of pair poles}$$

$$W_{field} = \frac{1}{2}\, I^T \cdot L_{\alpha\beta} \cdot I$$

$$I = \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{\alpha r} \\ i_{\beta r} \end{bmatrix}, \quad L_{\alpha\beta} = \begin{bmatrix} L_s & 0 & L_o\cos\theta_r & -L_o\sin\theta_r \\ 0 & L_s & L_o\sin\theta_r & L_o\cos\theta_r \\ L_o\cos\theta_r & L_o\sin\theta_r & L_r & 0 \\ -L_o\sin\theta_r & L_o\cos\theta_r & 0 & L_r \end{bmatrix}$$

$$\therefore W_{Field} = \frac{1}{2} \times 2L_o \left\{ \begin{array}{l} i_{\alpha s}\left(\cos\theta_r\, i_{\alpha r} - \sin\theta_r\, i_{\beta r}\right) \\ + i_{\beta s}\left(\sin\theta_r\, i_{\alpha r} + \cos\theta_r\, i_{\beta r}\right) \\ + \text{other terms independent on } \theta_r \end{array}\right.$$

$$\therefore T_e = P\,\frac{\partial W_{field}}{\partial \theta_r} = PL_o \Big[i_{\alpha s}\left(-\sin\theta_r\, i_{\alpha r} - \cos\theta_r\, i_{\beta r}\right)$$

For a reference frame attached to stator;

$\therefore \; \dot{I}_{ds} = \dot{I}_{\alpha s}$ $\qquad \dot{I}_{qs} = \dot{I}_{\beta s}$

$$\begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \end{bmatrix}$$

Subsitute in the torque expression, then

$$T_e = P L_o \left[i_{ds}(-i_{qr}) + i_{qs}(i_{dr}) \right]$$

$$= P L_o \left[i_{qs} i_{dr} - i_{ds} i_{qr} \right]$$

2. State space model

$$\overset{\circ}{X} = AX + Bu$$

For a stator reference frame, then

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|cc} R_s + PL_s & 0 & PL_o & 0 \\ 0 & R_s + PL_s & 0 & PL_o \\ \hline PL_o & \dot{\theta}_r L_o & R_r + PL_r & \dot{\theta}_r L_r \\ -\dot{\theta}_r L_o & PL_o & -\dot{\theta}_r L_r & R_r + PL_r \end{array}\right] \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

2) $$[R] = \left[\begin{array}{cc|cc} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{array}\right],$$

$$[G_R] = \left[\begin{array}{cc|cc} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ \hline 0 & \dot{\theta}_r L_o & R_r & \dot{\theta}_r L_r \\ -\dot{\theta}_r L_o & 0 & -\dot{\theta}_r L_r & R_r \end{array}\right], \quad [L] = \left[\begin{array}{cccc} L_s & 0 & L_o & 0 \\ 0 & L_s & 0 & L_o \\ \hline L_o & 0 & L_r & 0 \\ 0 & L_o & 0 & L_r \end{array}\right]$$

$$[v] = \begin{bmatrix} v_{ds} \\ v_{qs} \\ 0 \\ 0 \end{bmatrix}, \quad [i] = \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

$$[v] = [G_R][i] + P[L][i] \qquad * \, L^{-1}$$

$$\therefore P[i] = [L^{-1}][G_R]\, i + [L^{-1}][v]$$

The mechanical equations;

$$T_e - T_m = J \frac{d\omega_m}{dt}$$

$$= \frac{J}{P} \frac{d\omega_r}{dt}$$

, where ω_r is the rotor speed in elect. rad/sec.
P: no. of pair pol

$$\therefore p\omega_r = \frac{P}{J}(T_e - T_m)$$

also

$$\frac{d\theta_r}{dt} = \omega_r$$

$$\therefore p\,\theta_r = \omega_r$$

now we have

$$\begin{bmatrix} \dot{i}_{ds} \\ \dot{i}_{qs} \\ \dot{i}_{dr} \\ \dot{i}_{qr} \\ \dot{\omega}_r \\ \dot{\theta}_r \end{bmatrix} = \underbrace{\left[\begin{array}{cccc|cc} \multicolumn{4}{c|}{[L^{-1}G_R]_{4\times4}} & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]}_{A_{New}} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \\ \omega_r \\ \theta_r \end{bmatrix} +$$

$$\left[\begin{array}{cccc|cc} \multicolumn{4}{c|}{[L^{-1}]_{4\times4}} & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & P/J & -P/J \end{array}\right] \begin{bmatrix} v_{ds} \\ v_{qs} \\ 0 \\ 0 \\ T_e \end{bmatrix}$$

2ii) For steady state voltage equations,

$$\dot{\theta} = 0$$

$$P = J\omega_s \quad , \quad \dot{\theta}_r = (1-s)\omega_s$$

$$\therefore \; V_{ds} = (R_s + J\omega_s L_s) I_{ds} + J\omega_s L_o I_{dr}$$

$$V_{qs} = (R_s + J\omega_s L_s) I_{qs} + J\omega_s L_o I_{qr}$$

$$\therefore \; V_{ds} + J V_{qs} = [R_s + J\omega_s L_s][I_{ds} + J I_{qs}]$$
$$+ J\omega_s L_o [I_{dr} + J I_{qr}]$$

$$\therefore \; \bar{V}_s = (R_s + J\omega_s L_s)\bar{I}_s + J\omega_s L_o \bar{I}_r$$

also

$$V_{dr} = J\omega L_o I_{ds} + (1-s)\omega_s L_o I_{qs}$$
$$+ (R_r + j\omega_s L_r) I_{dr} + (1-s)\omega_s L_r I_{qr}$$

and

$$V_{qr} = -(1-s)\omega_s L_o I_{ds} + J\omega_s L_o I_{qs}$$
$$- (1-s)\omega_s L_r + (R_r + J\omega_s L_r) I_{qr}$$

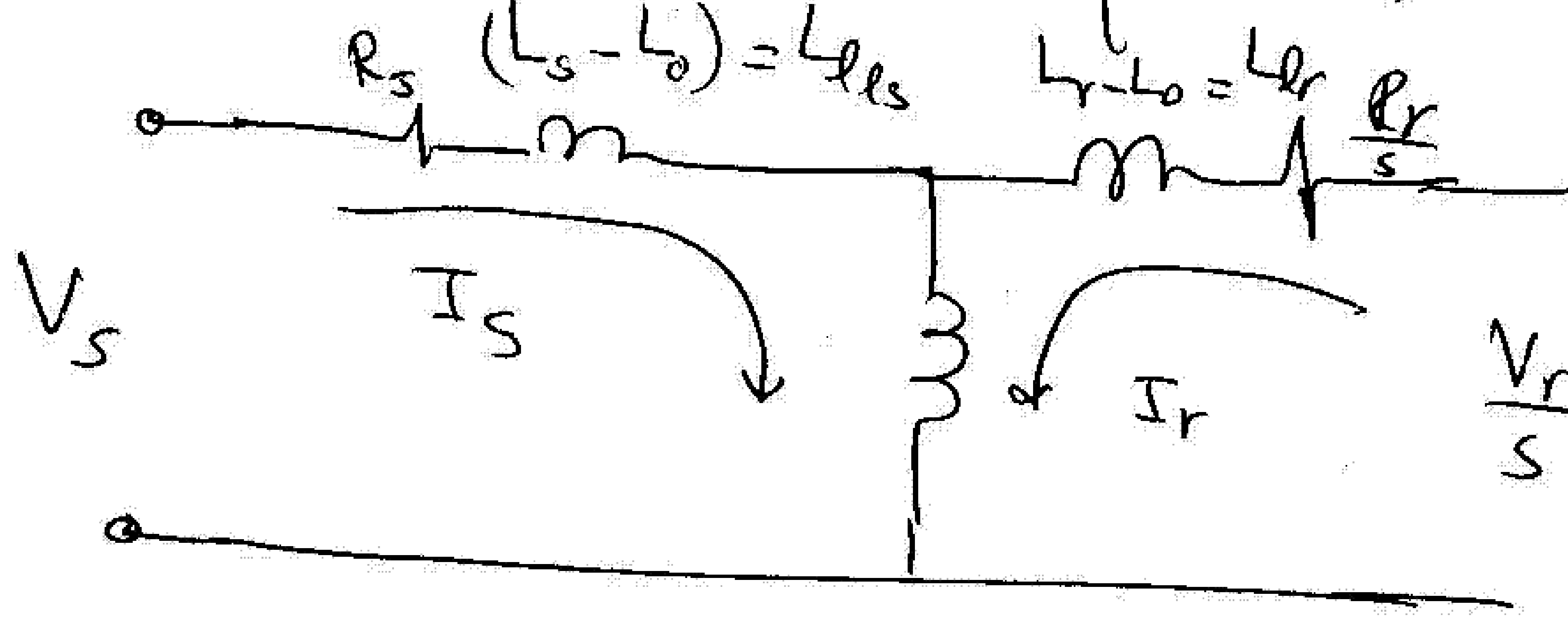
$$\therefore \; \bar{V}_{dr} + J\bar{V}_{qr} = s\omega_s L_o J(I_{ds} + J I_{qs})$$

from which

$$\frac{\bar{V}_r}{s} = \left(\frac{R_r}{s} + j\omega_s L_r\right)\bar{I}_r + j\omega_s L_o \bar{I}$$

2ii) Equivalent Circuit.

from the steady state voltage equations,



for squi[rrel]
cage $V_{[r]}$

3 i) Voltage equations

$$\begin{bmatrix} V_{\alpha s} \\ V_{\beta s} \\ V_{\alpha r} \\ V_{\beta r} \end{bmatrix} = \begin{array}{c} \begin{array}{cc} \qquad Z_{ss\alpha\beta} \qquad & \qquad Z_{sr\alpha\beta} \qquad \end{array} \\ \left[\begin{array}{cc|cc} R_s + pL_s & 0 & pL_o\cos\theta_r & -pL_o\sin\theta_r \\ 0 & R_s + pL_s & pL_o\sin\theta_r & pL_o\cos\theta_r \\ \hline pL_o\cos\theta_r & pL_o\sin\theta_r & R_r + pL_r & 0 \\ -pL_o\sin\theta_r & pL_o\cos\theta_r & 0 & R_r + pL_r \end{array}\right] \\ \begin{array}{cc} \qquad Z_{rs\alpha\beta} \qquad & \qquad Z_{rr\alpha\beta} \qquad \end{array} \end{array} \begin{bmatrix} i_{\alpha s} \\ i_{\beta s} \\ i_{\alpha r} \\ i_{\beta r} \end{bmatrix}$$

3 ii)

$$C_r^{*T} = \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix}$$

$$C_s^{*T} = \begin{bmatrix} \cos(\theta_r + \delta) & \sin(\theta_r + \delta) \\ -\sin(\theta_r + \delta) & \cos(\theta_r + \delta \end{bmatrix}$$

3iii)

$$Z_{sr} = C_s^{*T} * Z_{sr\alpha\beta}\, C_r$$

$$= C_s^{*T} * pL_o \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix}$$

$$= C_s^{*T} * pL_o \begin{bmatrix} \cos(\theta_r + \delta) & -\sin(\theta_r + \delta) \\ \sin(\theta_r + \delta) & \cos(\theta_r + \delta \end{bmatrix}$$

$$\therefore Z_{sr} = C_s^{*T} L_o \left(\begin{bmatrix} \cos(\theta_r+\delta) & -\sin(\theta_r+\delta) \\ \sin(\theta_r+\delta) & \cos(\theta_r+\delta) \end{bmatrix} \cdot P + \dot{\theta}_r \begin{bmatrix} -\sin(\theta_r+\delta) & -\cos(\theta_r+\delta) \\ \cos(\theta_r+\delta) & -\sin\theta_r+\delta \end{bmatrix} \right) X$$

$$= L_o \begin{bmatrix} \cos(\theta_r+\delta) & \sin(\theta_r+\delta) \\ -\sin(\theta_r+\delta) & \cos(\theta_r+\delta) \end{bmatrix} \begin{bmatrix} X \end{bmatrix}$$

$$\therefore \boxed{Z_{sr} = P L_o \begin{bmatrix} \cos 0 & -\sin 0 \\ \sin 0 & \cos 0 \end{bmatrix} + \dot{\theta}_r L_o \begin{bmatrix} 0 & - \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} P L_o & -\dot{\theta}_r L_o \\ \dot{\theta}_r L_o & P L_o \end{bmatrix}}$$

3 iii) Cont'd

$$Z_{rs} = C_r^{*T} \; Z_{rs\alpha\beta} \; C_s$$

$$= \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} pL_o\cos\theta_r & pL_o\sin\theta_r \\ -pL_o\sin\theta_r & pL_o\cos\theta_r \end{bmatrix} *$$

$$\begin{bmatrix} \cos(\theta_r+\delta) & -\sin(\theta_r+\delta) \\ \sin(\theta_r+\delta) & \cos(\theta_r+\delta \end{bmatrix}$$

$$= C_r^{*T} \; p \begin{bmatrix} L_o\cos\delta & -L_o\sin\delta \\ L_o\sin\delta & L_o\cos\delta \end{bmatrix}$$

$$\therefore Z_{rs} = C_r^{*} \; pL_o \; C_r = \begin{bmatrix} pL_o & \\ 0 & pL \end{bmatrix}$$

$$\text{for} \quad Z_{ss} = C_s^{*T} \; Z_{ss\alpha\beta} \; C_s$$

$$= C_s^{*T} \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} C_s + C_s^{*T} \begin{bmatrix} pL_s & 0 \\ 0 & pL_s \end{bmatrix} C$$

$$= \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} + C_s^{*T} * L_s \begin{bmatrix} \cos(\theta_r+\delta) & -\sin \\ \sin(\theta_r+\delta) & \cos(\end{bmatrix}$$

$$* P \quad + C_s^{*T} (\dot{\theta}_r) \begin{bmatrix} -\sin(\theta_r+\delta) & -\cos(\\ \cos(\theta_r+\delta) & -\sin \end{bmatrix}$$

$$= \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} + \begin{bmatrix} pL_s & 0 \\ 0 & pL_s \end{bmatrix} + \dot{\theta}_r \begin{bmatrix} 0 & -L_s \\ L_s & 0 \end{bmatrix}$$

for $Z_{rr} = C_r^{*T} \, Z_{rr\alpha\beta} \, C_r$

Since C_r is not fn of time (δ is constant)

$$\therefore Z_{rr}' = C_r^{*} (R_r + PL_r) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [C_r]$$

$$= (R_r + PL_r) \left[C_r^{*} \; C_r \right]$$

$$= \begin{bmatrix} R_r + PL_r & 0 \\ 0 & R_r + PL_r \end{bmatrix}$$

$$\therefore \begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \left[\begin{array}{cc|cc} R_s + PL_s & -\dot{\theta}_r L_s & PL_o & -\dot{\theta}_r L_o \\ \dot{\theta}_r L_s & R_s + PL_s & \dot{\theta}_r L_o & PL_o \\ \hline PL_o & 0 & R_r + PL_r & 0 \\ 0 & PL_o & 0 & R_r + PL_r \end{array}\right] \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

— o — o — o —

The resultant differential equation are similar to that obtained for a frame conciding with the rotor refrence since the frame used here is also attached to rotor and is rotating with rotor speed.

Rotational voltage terms appears only in the stator equations.

EPM405A; Final exam 2009/2010 soln.

11

4a.)

$$L_r = L_{\ell r} + L_o$$

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_f \\ v_{kd} \\ v_{kq} \end{bmatrix} = \begin{bmatrix} R_s + p(L_s + L_z\cos 2\theta_r) & pL_z \sin 2\theta_r & pL_d\cos\theta_r & pL_d\cos\theta_r & -pL_q\sin\theta_r \\ pL_z \sin 2\theta_r & R_s + p(L_s - L_z\cos 2\theta_r) & pL_d\sin\theta_r & pL_d\sin\theta_r & pL_q\cos\theta_r \\ pL_d\cos\theta_r & pL_d\sin\theta_r & R_r + p(L_{\ell r}+L_d) & pL_d & 0 \\ pL_d\cos\theta_r & pL_d\sin\theta_r & pL_d & R_{kd} + p(L_{\ell kd}+L_d) & 0 \\ -pL_q\sin\theta_r & pL_q\cos\theta_r & 0 & 0 & R_{kq} + p(L_{\ell kq}+L_q) \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix}$$

$$L_d = L_o + L_z$$
$$L_q = L_o - L_z$$

4 b).

Transformation matrix for transforming stator variables to rotor is

$$C_s = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}$$

4c) $Z_{ss}' = C_s^{*T} Z_{ss} \, C_s$

$$= \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} + C_s^{*T} \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix} P\, C_s + C_s^{*T} P L_2 \begin{bmatrix} \cos 2\theta_r & \sin 2\theta_r \\ \sin 2\theta_r & -\cos 2\theta_r \end{bmatrix} + C_s$$

$$\therefore Z_{ss} = \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} + C_s^{*T} \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix} C_s \cdot P + C_s^{*T} \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix} \dot{\theta}_r \begin{bmatrix} -\sin\theta_r & -\cos\theta_r \\ \cos\theta_r & -\sin\theta_r \end{bmatrix} + \left(C_s^{*T} P L_2 \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix} \right)$$

$$= \begin{bmatrix} R_s + PL_s & -\dot{\theta}_r L_s \\ \dot{\theta}_r L_s & R_s + PL_s \end{bmatrix} + y$$

$$y = C_s^{*T} L_2 \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix} [p]$$

$$+ C_s^{*T} L_2 \, \dot{\theta}_r \begin{bmatrix} -\sin\theta_r & \cos\theta_r \\ \cos\theta_r & \sin\theta_r \end{bmatrix}$$

$$\therefore y = L_2 \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix} [p]$$

$$+ \dot{\theta}_r L_2 \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} -\sin\theta_r & \cos\theta_r \\ \cos\theta_r & \sin\theta_r \end{bmatrix}$$

$$= \begin{bmatrix} pL_2 & 0 \\ 0 & -pL_2 \end{bmatrix} + \begin{bmatrix} 0 & \dot{\theta}_r L_2 \\ \dot{\theta}_r L_2 & 0 \end{bmatrix}$$

$$\therefore Z_{ss} = \begin{bmatrix} R_s + p(\overbrace{L_{ls} + L_0 + L_2}^{L_d}) & -\dot{\theta}_r (\overbrace{L_{ls} + L_0 - L_2}^{L_q}) \\ \dot{\theta}_r (\underbrace{L_{ls} + L_0 + L_2}_{L_d}) & R_s + p(\underbrace{L_{ls} + L_0 - L_2}_{L_q}) \end{bmatrix}$$

4c Cont'd

$$Z_{sr} = C_s^{*T} \; Z_{s\alpha\beta}$$

$$= \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix}_{2\times 2} \begin{bmatrix} pL_d\cos\theta_r & pL_d\cos\theta_r & -pL_q\sin\ldots \\ pL_d\sin\theta_r & pL_d\sin\theta_r & pL_q\cos\ldots \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \left[\begin{bmatrix} L_d\cos\theta_r & L_d\cos\theta_r & -L_q\sin\theta_r \\ L_d\sin\theta_r & L_d\sin\theta_r & L_q\cos\theta_r \end{bmatrix} p \right.$$

$$\left. + \; \dot{\theta}_r \begin{bmatrix} -L_d\sin\theta_r & -L_d\sin\theta_r & -L_q\cos\theta_r \\ L_d\cos\theta_r & L_d\cos\theta_r & -L_q\sin\theta_r \end{bmatrix} \right]$$

$$= \begin{bmatrix} pL_d & pL_d & 0 \\ 0 & 0 & pL_q \end{bmatrix} + \dot{\theta}_r \begin{bmatrix} 0 & 0 & -L_q \\ L_d & L_d & 0 \end{bmatrix}$$

$$= \begin{bmatrix} pL_d & pL_d & -\dot{\theta}_r L_q \\ \dot{\theta}_r L_d & \dot{\theta}_r L_d & pL_q \end{bmatrix}$$

4c Cont'd

For $Z_{rs} = C_r^{*T} \, Z_{rs\alpha\beta} \, C_s$

$$= Z_{rs\alpha\beta} \, C_s$$

$$\therefore \; Z_{rs} = p \underset{3\times 2}{\begin{bmatrix} L_d \cos\theta_r & L_d \sin\theta_r \\ L_d \cos\theta_r & L_d \sin\theta_r \\ -L_q \sin\theta_r & L_q \cos\theta_r \end{bmatrix}} \underset{\cancel{2}\times 2}{\begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}}$$

$$= p \begin{bmatrix} L_d & 0 \\ L_d & 0 \\ 0 & L_q \end{bmatrix}$$

$$Z_{rr} = C_r^{*T} \, Z_{rr\alpha\beta} \, C_r$$

$$= Z_{rr\alpha\beta}$$

therefore the voltage diff equations are given as.

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_f \\ v_{kd} \\ v_{kq} \end{bmatrix} = \begin{bmatrix} R_s + p(\overbrace{L_{ls}+L_0+L_2}^{L_{ls}+L_d}) & -\dot\theta_r(\overbrace{L_{ls}+L_0-L_2}^{L_{ls}+L_q}) & pL_d & pL_d & -\dot\theta_r L_q \\ \dot\theta_r(\underbrace{L_{ls}+L_0+L_2}_{L_{ls}+L_d}) & R_s + p(\underbrace{L_{ls}+L_0-L_2}_{L_{ls}+L_q}) & \dot\theta_r L_d & \dot\theta_r L_d & pL_q \\ pL_d & 0 & R_f + p(L_{lf}+L_d) & pL_d & 0 \\ pL_d & 0 & pL_d & R_{kd} + p(L_{lkd}+L_d) & 0 \\ 0 & pL_q & 0 & 0 & R_{kq} + p(L_{lkq}+L_q) \end{bmatrix}$$

4d

$$T_e = P\,[I]^T\,[G]\,[I]$$

P = no. of pair poles

$$I = \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} , \quad G = \left[\begin{array}{cc|cc} 0 & -L_{qs} & 0 & 0 \\ L_{ds} & 0 & L_d & L_d \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right.$$

$$L_{ds} = L_{ls} + L_d$$
$$L_{qs} = L_{ls} + L_q$$

$$I^T G = \Big[L_{ds} i_{qs} \;\Big|\; -L_{qs} i_{ds} \;\Big|\; L_d i_{qs} \;\Big|\; L_d i_{qs} \;\Big|\; L_q$$

$$\therefore T_e = P \Big[i_{ds} i_{qs} L_{ds} - i_{ds} i_{qs} L_{qs} + i_f i_{qs} L_d + i_{kd} i_{q}$$
$$- i_{kq} i_{ds} L_q \Big]$$

$$= P \Big[-i_{ds} \big(\underbrace{i_{qs} L_{qs} + i_{kq} L_q}_{\lambda_{qs}} \big) + i_{qs} \big(\overbrace{L_{ds} i_{ds} + i_f L_d + L_d i_{kd}}^{\lambda_{ds}} \big) \Big]$$

$$\therefore \boxed{T_e = P \left[i_{qs} \lambda_{ds} - i_{ds} \lambda_{qs} \right]}$$