Dr. Jehan Shazly

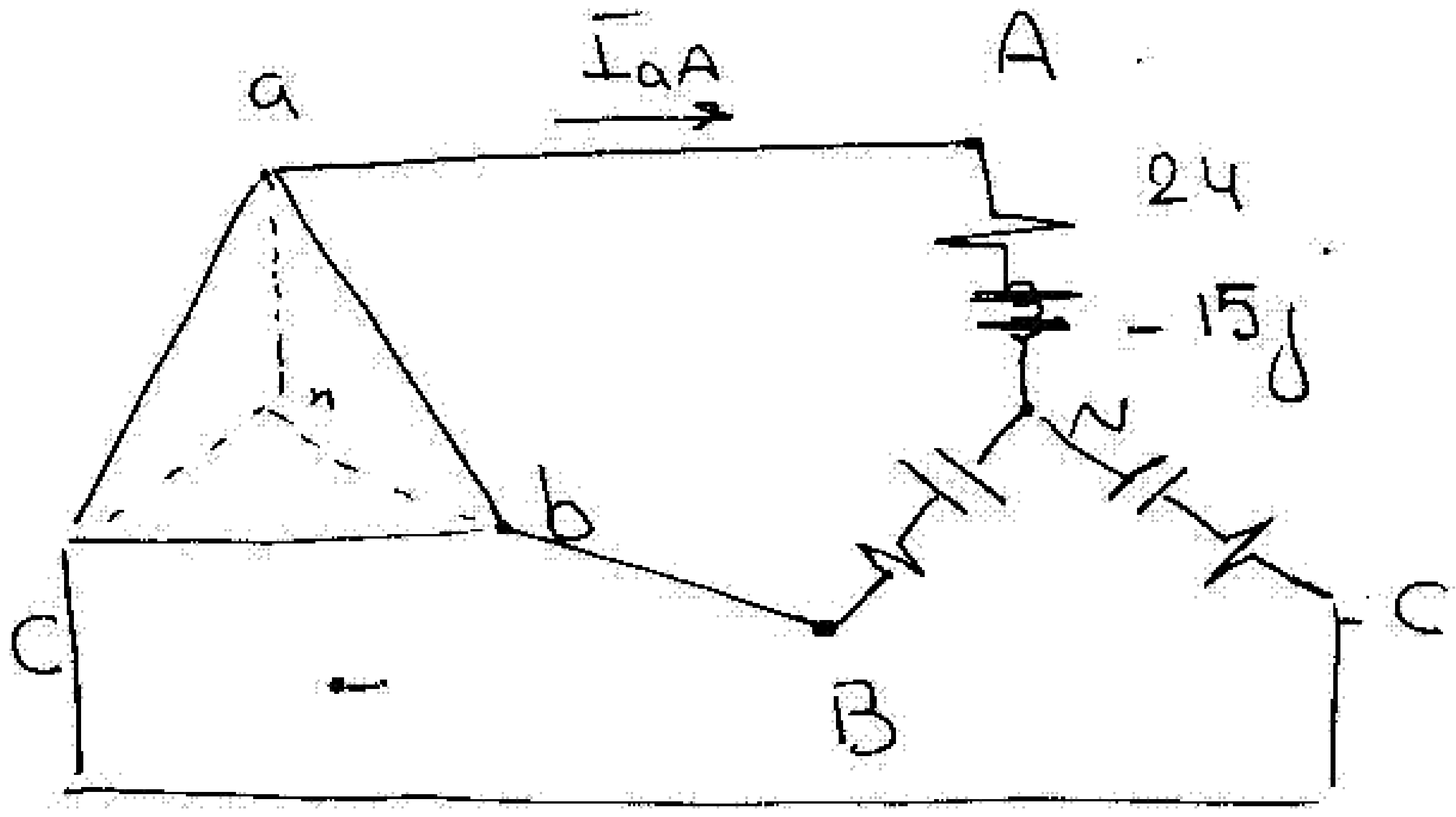
Circuits II

2nd electric power

2009 - 2010.

(1)

$V_{ab} = 125\angle 0$



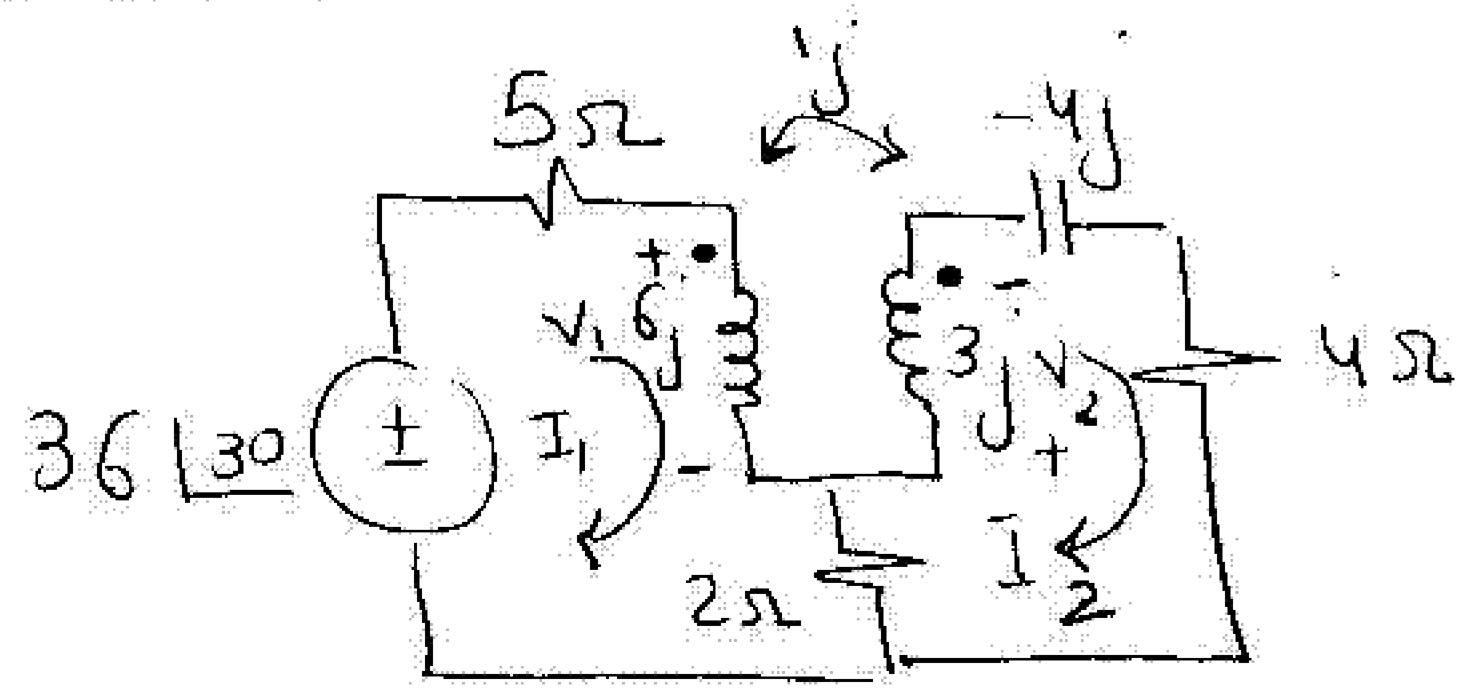
$$V_{an} = \frac{V_{ab}}{\sqrt{3}}\angle -30 = \frac{125}{\sqrt{3}}\angle -30 = 72.168\angle -30 \, V$$

$$I_{aA} = \frac{72.168\angle -30}{24-15j} = \frac{72.168\angle -30}{28.3\angle -32} = 2.55\angle 2 \, A$$

$$I_{bB} = 2.55\angle 2-120 = 2.55\angle -118 \; A$$

$$I_{cC} = 2.55\angle 2+120 = 2.55\angle 122 \; A$$

(2)



$$36\angle 30 - 5I_1 - V_1 - 2(I_1 - I_2) = 0$$

$$V_1 = 6jI_1 + -jI_2 = 6jI_1 - jI_2$$

$$2(I_2 - I_1) + 4I_2 - 4jI_2 + V_2 = 0$$

$$V_2 = 3jI_2 + jI_1 = 3jI_2 - jI_1$$

$$36\angle 30 - 7I_1 + 2I_2 - 6jI_1 + jI_2 = 0$$

$$6I_2 - 4jI_2 - 2I_1 + 3jI_2 - jI_1 = 0$$

$$36\angle 30 - I_1(7+6j) + I_2(2+j) = 0$$

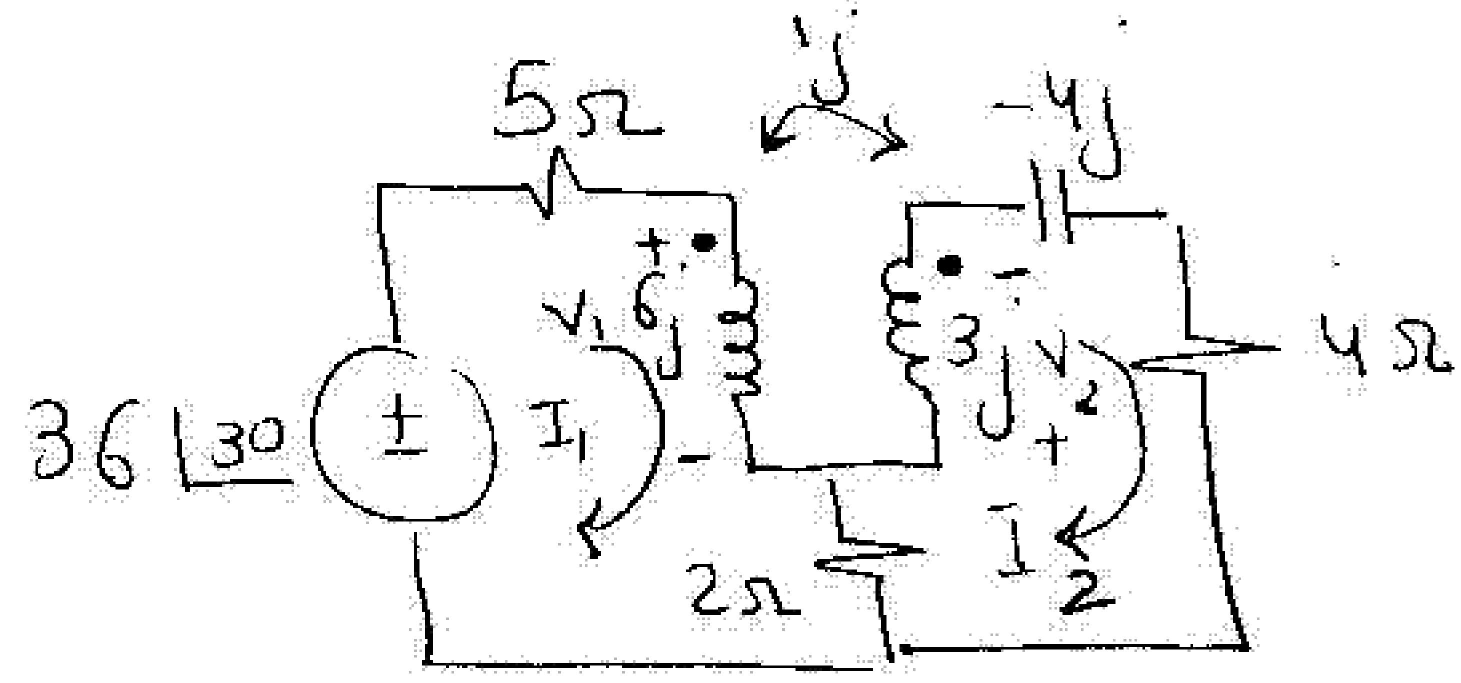
$$-I_1(2+j) + I_2(6-j) = 0$$

$$I_1 = I_2\frac{(6-j)}{(2+j)} = I_2(2.2-1.6j)$$

$$36\angle 30 - (7+6j)(2.2-1.6j)I_2 + I_2(2+j) = 0$$

$$36\angle 30 = (23+j)I_2$$

(2)



$$36\angle 30 - 5I_1 - V_1 - 2(I_1 - I_2) = 0$$

$$V_1 = 6jI_1 + -jI_2 = 6jI_1 - jI_2$$

$$2(I_2 - I_1) + 4I_2 - 4jI_2 + V_2 = 0$$

$$V_2 = 3jI_2 + jI_1 = 3jI_2 - jI_1$$

$$36\angle 30 - 7I_1 + 2I_2 - 6jI_1 + jI_2 = 0$$

$$6I_2 - 4jI_2 - 2I_1 + 3jI_2 - jI_1 = 0$$

$$36\angle 30 - I_1(7+6j) + I_2(2+j) = 0$$

$$-I_1(2+j) + I_2(6-j) = 0$$

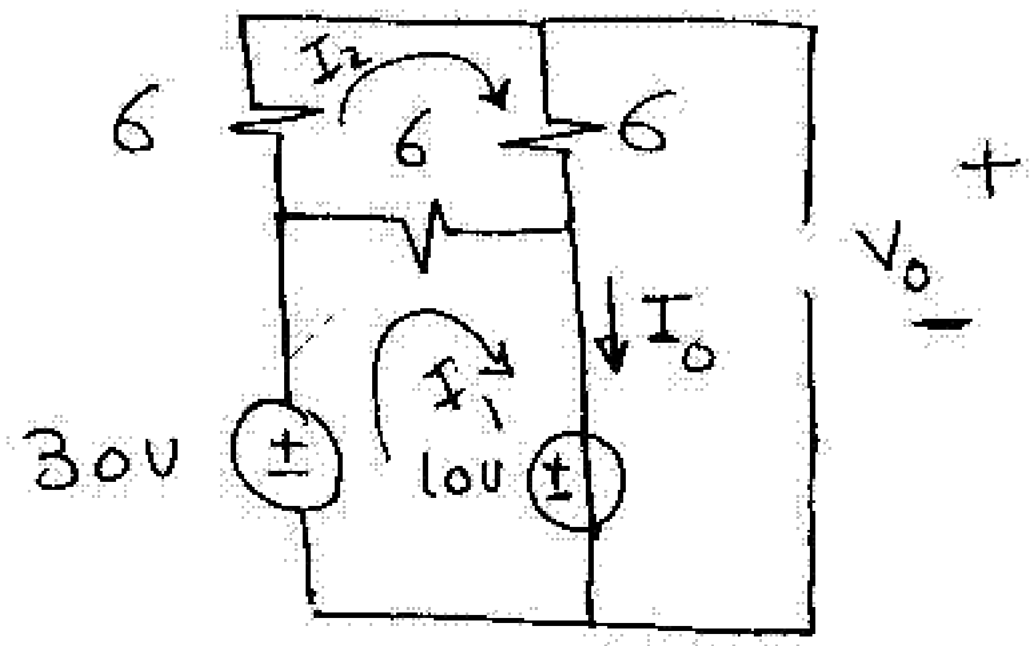
$$I_1 = I_2\frac{(6-j)}{(2+j)} = I_2(2.2-1.6j)$$

$$36\angle 30 - (7+6j)(2.2-1.6j)I_2 + I_2(2+j) = 0$$

$$36\angle 30 = (23+j)I_2$$

at $t < 0$ → switch was closed

(3)



$$30 - 10 - 6(I_1 - I_2) = 0$$

$$6I_2 + 6I_2 + 6(I_2 - I_1) = 0$$

$$18 I_2 = 6 I_1$$

$$I_2 = \frac{I_1}{3}$$

$$\cancel{3}20 - 6\left(I_1 - \frac{I_1}{3}\right) = 0$$

$$20 = 6 \times \frac{2 I_1}{3} \longrightarrow I_1 = 5\,A$$

$$I_2 = \frac{5}{3}\,A$$

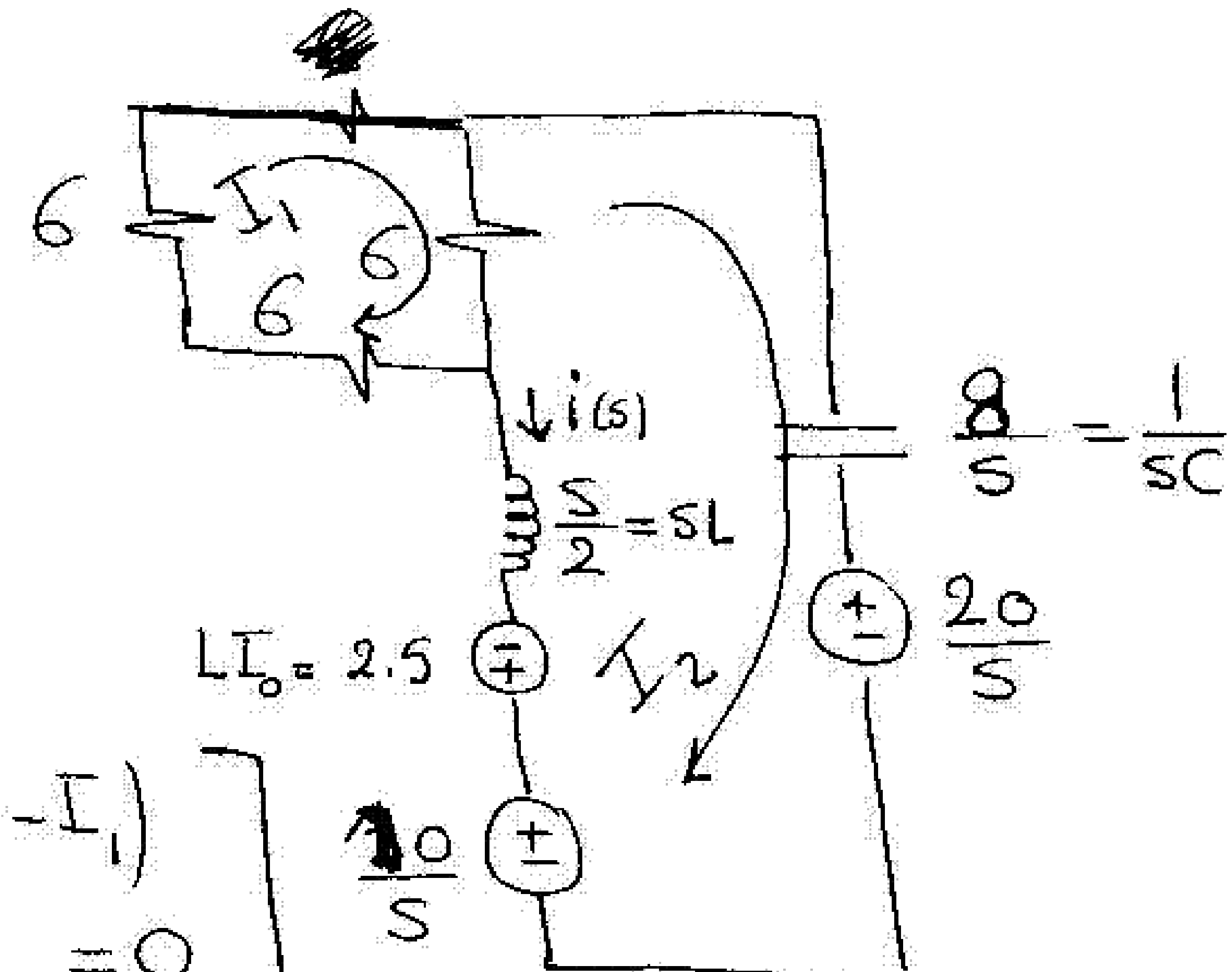
$$I_o = I_1 = 5\,A$$

$$V_o = 10 + 6 I_2 = 10 + 6 \times \frac{5}{3} = 20V$$

at $t \geq 0$ switch was opened

$L = \frac{1}{2}\,H$

$C = \frac{1}{8}\,F$



$$12 I_1 + 6(I_1 - I_2) = 0$$

$$18 I_1 = 6 I_2 \longrightarrow I_1 = \frac{1}{3} I_2$$

$$\left[\frac{10}{S} - \frac{20}{S} - 2.5 - \frac{S}{2} I_2 - 6(I_2 - I_1) - \frac{8}{S} * I_2 = 0\right]$$

$$-\frac{10}{S} - 2.5 = I_2\left(\frac{S}{2} + 6\cancel{I_2} - 2\cancel{I_2} + \frac{8}{S}\right)$$

$$I_2 = \frac{-\frac{10}{S} - 2.5}{\left(\frac{S}{2} + 4 + \frac{8}{S}\right)}$$

$$I_2 = \frac{31.17 + 18j}{23 + j} = 1.38 + j0.72 = 1.56 \angle 27.5$$

$$I_1 = 4.188 - 0.624j = 4.23 \angle -8.47$$

---

at $t = 2\,ms$

$$I_1 = 4.23 \cos\left(1000 \times \frac{2}{1000} \times \frac{180}{\pi} - 8.47\right) = -1.17\,A$$

$$I_2 = 1.56 \cos\left(1000 \times \frac{2}{1000} \times \frac{180}{\pi} + 27.5\right) = -1.23\,A$$

$$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 - M I_1 I_2$$

$$L_1 = \frac{6}{1000} \qquad L_2 = \frac{3}{1000} \qquad M = \frac{1}{1000}$$

$$W = \frac{1}{2}\, \frac{6}{1000} * (-1.17)^2 + \frac{1}{2}\, \frac{3}{1000} (-1.23)^2 - \frac{1}{1000} (-1.17)(-1.23)$$

$$= 4.93 \times 10^{-3}\ J$$

$$I_2 = \frac{-10 * 2.5\,S}{\frac{S^2}{2} + 4S + 8} = \frac{-20 - 5S}{S^2 + 8S + 16}$$

$$= \frac{-5\,(S+4)}{(S+4)^2} = \frac{-5}{S+4}$$

$$I_o(S) = -I_2(S) = \frac{5}{S+4}$$

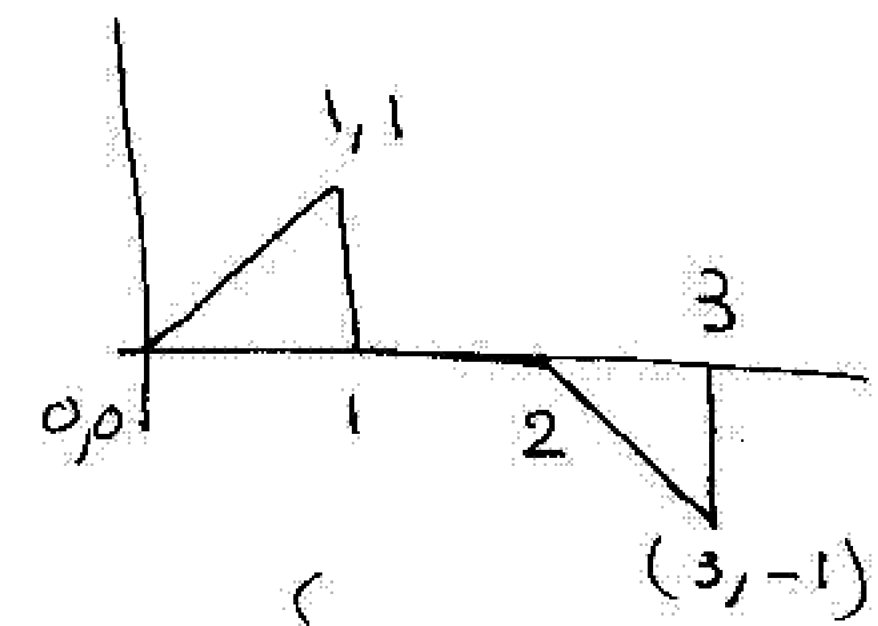
$$I_o(t) = 5\,e^{-4t}$$

---

(4)

$$\frac{y-0}{x-0} = \frac{1-0}{1-0}$$

$$y = x \longrightarrow f(t) = t$$



$$\frac{y-0}{x-2} = \frac{-1-0}{3-2} = -1$$

$$y = 2 - x \longrightarrow f(t) = 2 - t$$

$$T = 3\,sec$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$$

$$f(t) = \begin{cases} t & 0 < t < 1 \\ 0 & 1 < t < 2 \\ 2 - t & 2 < t < 3 \end{cases}$$

$$a_0 = \frac{1}{T}\left[\int_0^1 t\,dt + \int_2^3 (2-t)\,dt\right] = \frac{1}{T}\left[\left.\frac{t^2}{2}\right|_0^1 + \left.\left(2t - \frac{t^2}{2}\right)\right|_2^3\right]$$

$$= \frac{1}{T}\left[\frac{1}{2} + \left(6 - \frac{9}{2}\right) - \left(4 - \frac{4}{2}\right)\right] = \frac{1}{T}\left(\frac{1+12-9-8+4}{2}\right)$$

$$= 0$$

$$a_n = \frac{2}{T}\left[\int_0^1 t\cos n\omega_0 t\,dt + \int_2^3 (2-t)\cos n\omega_0 t\,dt\right]$$

$$= \frac{2}{T}\left\{\left[\frac{1}{(n\omega_0)^2}\cos n\omega_0 t + \frac{t}{n\omega_0}\sin n\omega_0 t\right]_0^1 + \left.\left(\frac{2}{n\omega_0}\sin n\omega_0 t\right)\right|_2^3 \right.$$

$$\left. - \left[\frac{1}{(n\omega_0)^2}\cos n\omega_0 t + \frac{t}{n\omega_0}\sin n\omega_0 t\right]_2^3\right.$$

$$b_n = \frac{2}{T}\left[\int_0^1 t\sin n\omega_0 t\,dt + \int_2^3 (2-t)\sin n\omega_0 t\,dt\right]$$

$$= \frac{2}{T}\left\{\left[\frac{1}{(n\omega_0)^2}\sin n\omega_0 t - \frac{t}{n\omega_0}\cos n\omega_0 t\right]_0^1 + \frac{-2}{n\omega_0}\cos n\omega_0 t\Big|_2^3 + \left(-\frac{1}{(n\omega_0)^2}\sin n\omega_0 t + \frac{t}{n\omega_0}\cos n\omega_0 t\right)_2^3\right\}$$

$$b_n = \frac{2}{T}(X' + y' + Z')$$

$$X' = \frac{1}{\left(n\frac{2\pi}{3}\right)^2}\sin n\frac{2\pi}{3} - \frac{1}{n\frac{2\pi}{3}}\left(\cos n\frac{2\pi}{3}\right)$$

$$= \frac{9}{n4\pi^2}\sin\frac{2\pi}{3}n - \frac{3}{2\pi n}\cos\frac{2\pi}{3}n$$

$$y' = \frac{-2}{n\frac{2\pi}{3}}\left(\cos n\frac{2\pi}{3}\times 3 - \cos n\frac{2\pi}{3}\times 2\right)$$

$$= \frac{-3}{n\pi}\left(1 - \cos\frac{4\pi}{3}n\right)$$

$$Z' = \frac{-1}{n\left(\frac{2\pi}{3}\right)^2}\left(\sin\frac{2\pi}{3}\times 3n - \sin\frac{2\pi}{3}\times 2n\right) + \frac{3}{n\frac{2\pi}{3}}\cos n\frac{2\pi}{3}\times 3 - \frac{2}{n\frac{2\pi}{3}}\cos n\frac{2\pi}{3}\times 2$$

$$= \frac{9}{n4\pi^2}\sin\frac{4\pi}{3}n + \frac{9}{n2\pi} - \frac{3}{n\pi}\cos\frac{4\pi}{3}n$$

$$b_n = \frac{2}{3}\left[\frac{9}{4n\pi^2}\sin\frac{2\pi}{3}n - \frac{3}{2\pi n}\cos\frac{2\pi}{3}n - \frac{3}{n\pi} + \cancel{\frac{3}{n\pi}\cos\frac{4\pi}{3}n} + \frac{9}{4n\pi^2}\sin\frac{4\pi}{3}n + \frac{9}{n2\pi} - \cancel{\frac{3}{n\pi}\cos\frac{4\pi}{3}}\right.$$

$$a_n = \frac{2}{T}\left\{ X + Y + Z \right\}$$

$$X = \frac{1}{\left(n\frac{2\pi}{3}\right)^2}\left[\cos\left(n\frac{2\pi}{3}\times 1\right) - 1\right] + \frac{1}{\left(n\frac{2\pi}{3}\right)}\left[\sin n\frac{2\pi}{3} \cancel{-0}\right]$$

$$= \frac{9}{4\pi^2 n}\left[\cos\left(\frac{2\pi}{3}n\right) - 1\right] + \frac{3}{2\pi n}\sin\frac{2\pi}{3}n$$

$$y = \frac{2}{n\frac{2\pi}{3}}\left(\sin n\frac{2\pi}{3}\times 3 - \sin n\frac{2\pi}{3}\times 2\right)$$

$$= \frac{2}{n\frac{2\pi}{3}}\left(\sin 2\pi n - \sin\frac{4\pi}{3}n\right) = \frac{-3}{\pi n}\sin\frac{4\pi}{3}n$$

$$Z = \frac{1}{n\left(\frac{2\pi}{3}\right)^2}\left(\cos n\frac{2\pi}{3}\times 3 - \cos n\frac{2\pi}{3}\times 2\right)$$

$$+ \frac{3}{n\frac{2\pi}{3}}\sin n\frac{2\pi}{3}\times 3 - \frac{2}{n\frac{2\pi}{3}}\sin n\frac{2\pi}{3}\times 2$$

$$= \frac{9}{n4\pi^2}\left(1 - \cos\frac{4\pi}{3}n\right) - \frac{3}{n\pi}\sin n\frac{4\pi}{3}$$

$$a_n = \frac{2}{3}\Bigg[\frac{9}{4\pi^2 n}\left(\cos\frac{2\pi}{3}n - 1\right) + \frac{3}{2\pi n}\sin\frac{2\pi}{3}n$$

$$\cancel{-\frac{3}{n\pi}\sin\frac{4\pi}{3}n} + \cancel{\frac{3}{n\pi}\sin\frac{4\pi}{3}n}$$

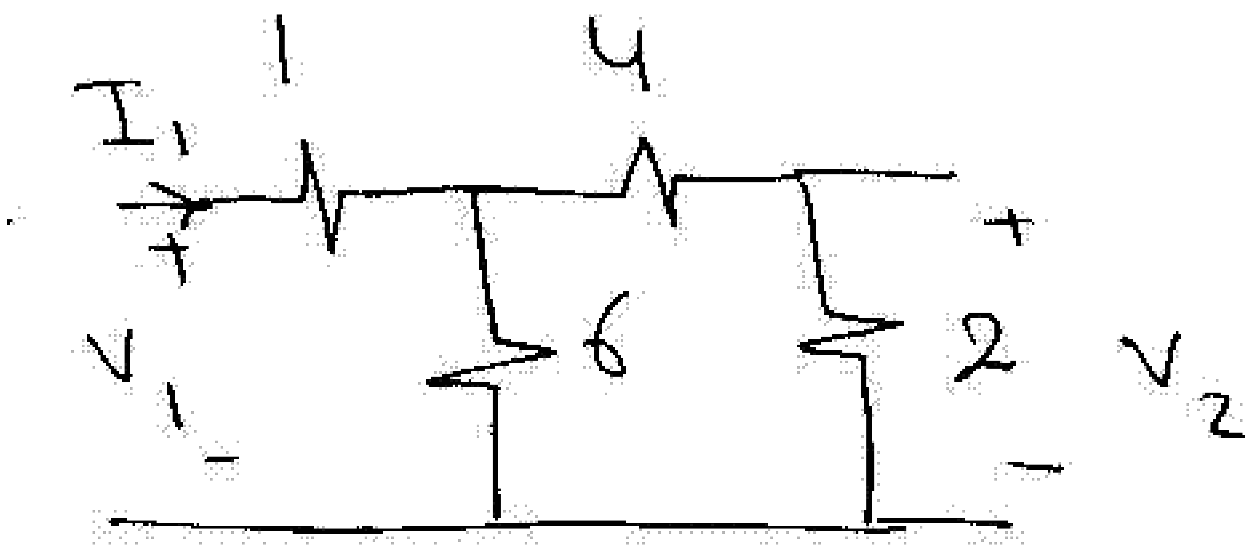
$$+ \frac{9}{4\pi^2 n}\left(\cos\frac{4\pi}{3}n - 1\right)\Bigg]$$

$$= \frac{3}{2\pi^2 n}\left(\overbrace{\cos\frac{2\pi}{3}n + \cos\frac{4\pi}{3}n}^{\text{equal}}\right) - \frac{3}{\pi^2 n} + \frac{1}{n\pi}\sin\frac{2\pi}{3}n$$

$$a_n = \frac{3}{n\pi^2}\cos\frac{2\pi}{3n} + \frac{1}{n\pi}\sin\frac{2\pi}{3}n - \frac{3}{\pi^2 n}$$

5

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

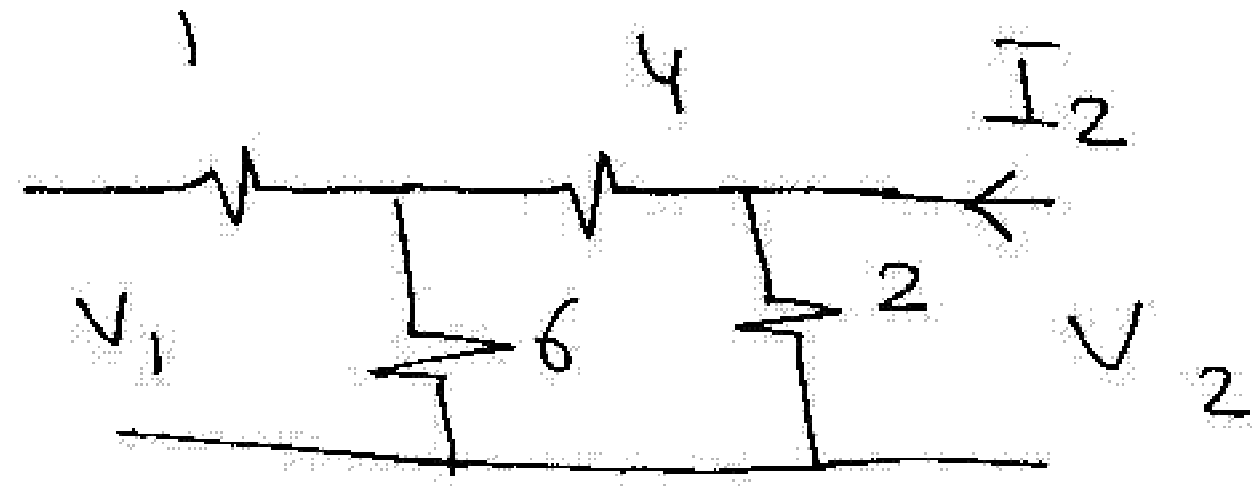


$$Z_{11} = 4\,\Omega$$

$$V_2 = \frac{1}{2} I_1 \times 2 = I_1$$

$$I_1 = \frac{V_1}{Z_{11}}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \frac{I_1}{I_1} = 1\,\Omega$$



$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$Z_{22} = \frac{10 \times 2}{12} = \frac{10}{6} = \frac{5}{3}$$

$$V_1 = \frac{I_2 \times 2}{12} \times 6 = I_2$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{I_2}{I_2} = 1\,\Omega$$

$$Z = \begin{bmatrix} 4 & 1 \\ 1 & 5/3 \end{bmatrix}$$

$$\cancel{I_2 = \frac{V_2}{Z_{22}} = \frac{V_2 \times 3}{5}}$$

$$\cancel{Z_{22} = \frac{V_2 \times 5}{3 V_2} = \frac{5}{3}\,\Omega}$$

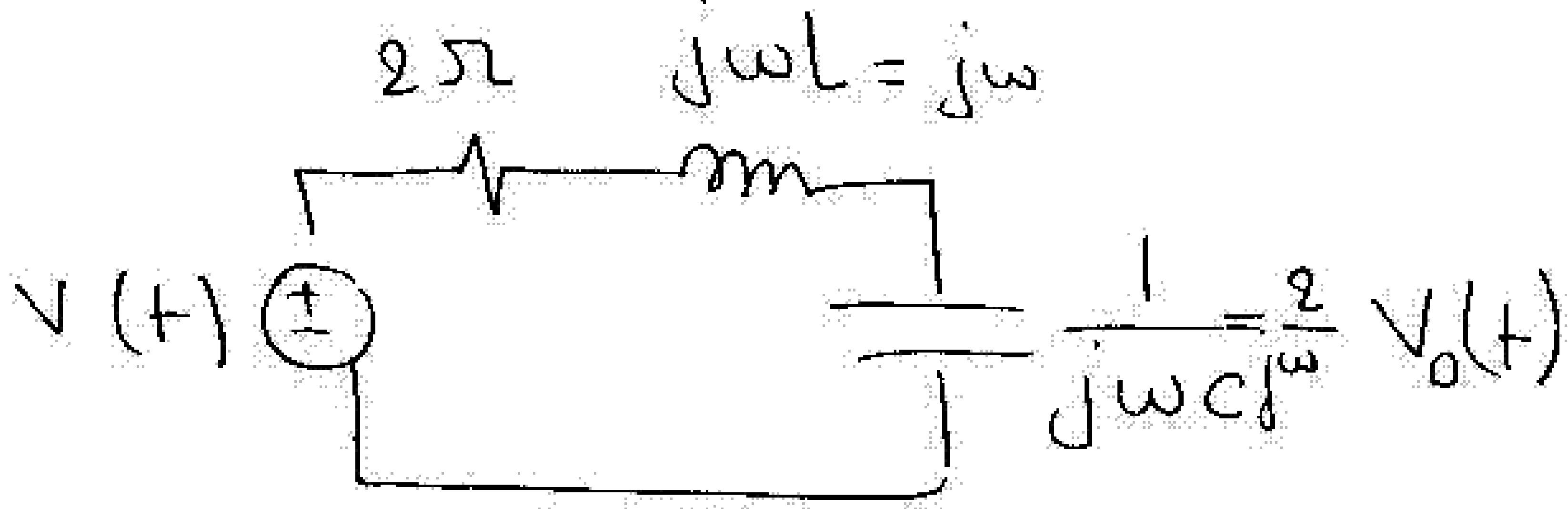
$$b_n = \frac{2}{3}\left[\frac{9}{2n\pi^2}\sin\frac{2\pi}{3}n + \frac{3}{2\pi n} - \frac{3}{2n\pi}\cos\frac{2\pi}{3}n\right]$$

$$= \frac{3}{n\pi^2}\sin\frac{2\pi}{3}n + \frac{1}{n\pi} - \frac{\cancel{3}}{\cancel{2}n\pi}\cos\frac{2\pi}{3}n$$

$$V(t) = \sum_{n=1}^{\infty}\left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)$$

$$= \sum_{n=1}^{\infty}\left(\frac{3}{n\pi^2}\cos\frac{2\pi}{3}n + \frac{1}{n\pi}\sin\frac{2\pi}{3}n - \frac{3}{\pi^2 n}\right)\cos n\omega_0 t + \left(\frac{3}{n\pi^2}\sin\frac{2\pi}{3}n - \frac{1}{n\pi}\cos\frac{2\pi}{3}n + \frac{1}{n\pi}\right)\sin n\omega_0 t$$

$$V_o(t) = \frac{V(t)}{Z}\;\frac{2}{j\omega}$$



$$= \frac{V(t) \;*\; \frac{2}{j\omega}}{2 + j\omega + \frac{2}{j\omega}} = \frac{2\,V(t)}{j^2\omega^2 + 2j\omega + 2}$$

$$= \frac{2\,V(t)}{(2-\omega^2) + 2\omega j}$$

at $\omega = \omega_0$ at $n = 1$ $\longrightarrow$ $\omega_0 = \frac{2\pi}{3}$

$$V(t) = \left(\frac{3}{\pi^2}\cos\frac{2\pi}{3} + \frac{1}{\pi}\sin\frac{2\pi}{3} - \frac{3}{\pi^2}\right)\cos\omega_0 t + \left(\frac{3}{\pi^2}\sin\frac{2\pi}{3} - \frac{1}{\pi}\cos\frac{2\pi}{3} + \frac{1}{\pi}\right)\sin\omega_0 t$$