Time Allowed: 3 Hours

QUESTION 1:

a) Show how matrix partitioning can be used in node elimination in power systems. What conditions should be first satisfied?

b) Show how tap changing transformers can be used to change the Y-bus of a given Power System.

QUESTION 2:

a) Show that Ybus=$\mathbf{A}^T \mathbf{Y}_{pr} \mathbf{A}$

b) Calculate the bus admittance matrix for the Power System represented in Fig.1

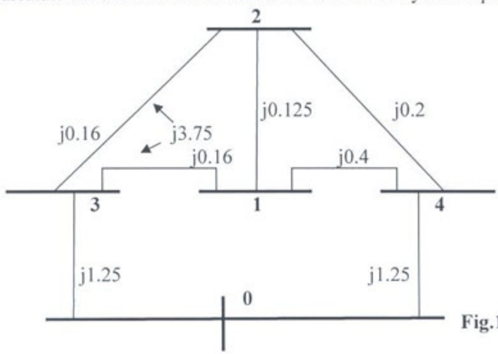


Fig.1

QUESTION 3: For the power system shown in Fig.2

a) Form the Ybus Matrix

b) Perform 2 iterations of the Gauss-Seidel load flow to calculate all missing data at each bus.

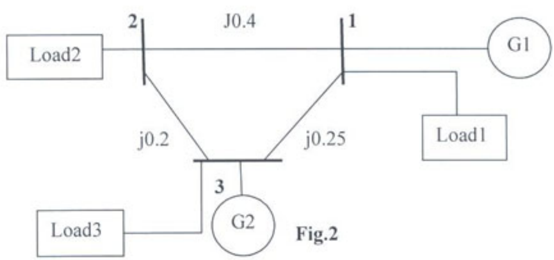


Fig.2

Given that:

$V1=1.02+j0$ $PL1=0.8$ $QL1=0.6$ (Slack Bus)

$PL2=2.4$ $QL2=1.8$ (Load Bus)

$|V3|=1$ $PG3=1.5$ $PL3=0.8$ $QL3=1.2$ (Voltage Controlled Bus)

$-5\leq Q3Gmax \leq 5$.

QUESTION 4: Consider the Power System shown in Fig.3

For a 3phase fault at the middle point of Line1-3, calculate the following:

- Current through the fault
- Voltage at each bus after the fault
- Fault current in each Transmission line.
- Fault Current drawn from each generator.

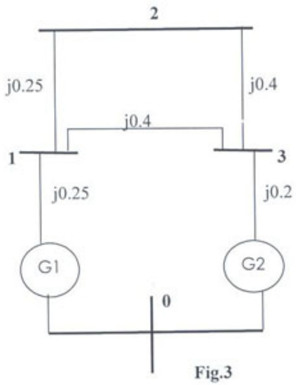


Fig.3

GOOD LUCK

Dr. Wael Ismael

Question 1 [15 Marks] 1

a) The 2 conditions are:

1. We are not interrersted in the Voltage at this bus.
2. The current is zero at this bus.

$$\begin{array}{l} \text{Non zero} \\ \text{zero} \end{array} \begin{bmatrix} IA \\ \hline IB \end{bmatrix} = \left[\begin{array}{c|c} K & L \\ \hline L^T & M \end{array}\right] \begin{bmatrix} VA \\ \hline VB \end{bmatrix}$$

$$IA = KVA + LVB$$

$$IB = L^T VA + MVB \quad ; \text{ but } IB = 0$$

$$\therefore VB = -M^{-1} L^T VA$$

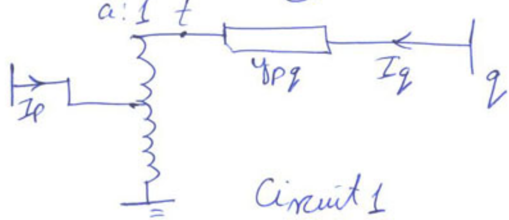
$$\therefore IA = KVA - L M^{-1} L^T VA$$

$$IA = \underbrace{[K - L M^{-1} L^T]}_{\text{New } Ybus} VA$$

5 Marks

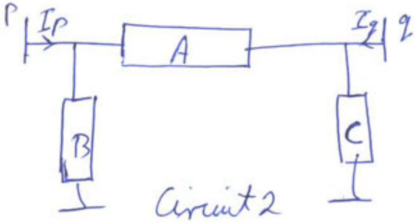
(2)

(b)



Circuit 1

It is required to represent the "Transformer T.L" system by an equivalent π section as follows.



Circuit 2

$$\frac{E_p}{E_t} = \frac{a}{1} \quad \therefore \frac{I_p}{I_t} = \frac{1}{a} \quad \therefore I_t = a I_p$$

from circuit (1) $\quad I_t = [E_t - E_q]\, y_{pq}$

$$a I_p = [\tfrac{E_p}{a} - E_q]\, y_{pq}$$

$$\boxed{\therefore I_p = E_p \frac{y_{pq}}{a^2} - \frac{y_{pq}}{a} E_q} \quad \text{I}$$

$$I_q = (E_q - E_t)\, y_{pq}$$

$$= \left(E_q - \frac{E_p}{a}\right) y_{pq}$$

$$\boxed{\therefore I_q = -\frac{y_{pq}}{a} E_p + y_{pq} E_q} \quad \text{II}$$

from circuit 2

$$I_p = (E_p - E_q)A + E_p B$$

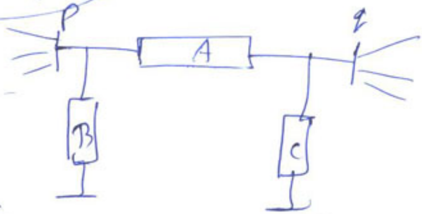
$$\boxed{\therefore I_p = (A+B)E_p - A E_q} \quad \text{III}$$

$$\boxed{I_q = -A E_p + (A+C)E_q} \quad \text{IV}$$

Equating I & III ; II & IV

$$\therefore A = \frac{y_{pq}}{a}, \quad B = \frac{y_{pq}}{a}\left(\frac{1}{a} - 1\right)$$

$$C = y_{pq}\left(1 - \frac{1}{a}\right)$$



$$Y_{pp} = y_{p1} + y_{p2} + \cdots + A + B + \cdots$$

$$Y_{pq} = Y_{qp} = -A$$

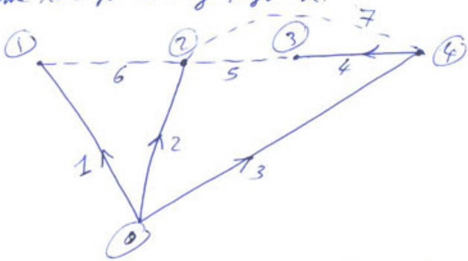
$$Y_{qq} = y_{q1} + y_{q2} + \cdots + A + C + \cdots$$

$$\therefore \left[Y_{bus\ modified}\right] = \begin{array}{c|ccccccc|} & & & & p & & q & \\ & Y_{11} & Y_{12} & \cdots & & & & Y_{1n} \\ & Y_{21} & Y_{22} & \cdots & & & & Y_{2n} \\ & \vdots & & & & & & \\ p & & & & \boxed{Y_{pp}} & & \boxed{Y_{pq}} & \\ \vdots & & & & & & & \\ q & & & & \boxed{Y_{qp}} & & \boxed{Y_{qq}} & \\ n & & & & & & & \end{array}$$

10 Marks

Question 2 15 Marks

(a) Assume the following system:



$$A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} ;\quad \begin{aligned} &v_1 = -V_1;\ v_2 = -V_2;\ v_3 = -V_4 \\ &v_4 = V_4 - V_3;\ v_5 = V_2 - V_3 \\ &v_6 = V_1 - V_2;\ v_7 = V_2 - V_4 \end{aligned}$$

$$\therefore [v_{pr}] = A[V]$$

$$I_1 = -i_1 + i_6;\ I_2 = -i_2 + i_5 - i_6 + i_7$$

$$I_3 = -i_4 - i_5 \quad I_4 = -i_3 + i_4 - i_7$$

$$\therefore A^T i_{pr} = I$$

$$v_{pr} = z_{pr}\, i_{pr}$$

$$\therefore (z_{pr})^{-1} v_{pr} = i_{pr} \quad \therefore i_{pr} = y_{pr}\, v_{pr}$$

$$\therefore A^T y_{pr}\, v_{pr} = A^T i_{pr}$$

$$\therefore A^T y_{pr}\, v_{pr} = I$$

$$\therefore \underbrace{A^T y_{pr} A}_{Y} V = I$$

7 Marks

(5)

(b)

$$A = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

$$z_{pr} = \begin{bmatrix} j1.25 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & j1.25 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & j0.16 & j3.75 & 0 & 0 & 0 \\ 0 & 0 & j3.75 & j0.16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & j0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & j0.125 \end{bmatrix}$$

$$\therefore y_{pr} = z_{pr}^{-1} = \begin{bmatrix} -j0.8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -j0.8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +j0.0114 & -j0.267 & 0 & 0 & 0 \\ 0 & 0 & -j0.267 & +j0.014 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j5 & j2.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -j8 \end{bmatrix}$$

6

$$Y_{bus} = A^T y_{pr} A$$

$$= \begin{bmatrix} -j10.4886 & j7.7328 & j0.2558 & j2.5 \\ j7.7328 & -j12.9886 & j0.2558 & j5 \\ j0.2558 & j0.2558 & -j1.3116 & 0 \\ j2.5 & j5 & 0 & -j8.3 \end{bmatrix}$$

8 Marks

Question 3 20 Marks

(a) $$Y_{bus} = \begin{bmatrix} -j6.5 & j2.5 & j4 \\ j2.5 & -j7.5 & j5 \\ j4 & j5 & -j9 \end{bmatrix}$$

5 Marks

(b) $$V_2^{(1)} = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{(0)*}} - Y_{21} V_1^{(0)} - Y_{23} V_3^{(0)} \right]$$

$$= \frac{-1}{j7.5} \left[\frac{-2.4 + j1.8}{1} - J2.5 * 1.02 - j5 * 1 \right]$$

$$= 0.7667 - j0.32$$

$$Q_3^{(1)} = -Im \left[E_3^{(0)*} \left[Y_{31} E_1 + Y_{32} E_2^{(1)} + Y_{33} E_3^{(0)} \right] \right]$$

$$= -Im \left[1 (j4 * 1.02 + j5 (0.7667 - j0.32) - j9 * 1 \right]$$

$$= -Im \left[1.6 - j1.0865 \right]$$

$$\therefore Q_3^{(1)} = 1.0865$$

$$\therefore Q_{3g} = Q_3^{(1)} + QL_3 = 2.2865$$

$$\therefore -5 \leq 2.2865 \leq 5$$

$\therefore$ in range

8

$$\therefore V_3^{(1)} = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{V_3^{(0)*}} - Y_{31} V_1^{(0)} - Y_{32} V_2^{(1)} \right]$$

$$= 1 - j0.1$$

$$\therefore \delta_3^{(1)} = -5.71$$

$$\therefore V_{3\,corrected}^{(1)} = 1 \angle -5.71$$

$$V_2^{(2)} = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{(1)*}} - Y_{21} V_1 - Y_{23} V_3^{(1)} \right]$$

$$= 0.6653 \angle -27.82$$

$$Q_3^{(2)} = -Im \left[V_3^{(1)*} \left[Y_{31} V_1^{(0)} + Y_{32} V_2^{(2)} + Y_{33} V_3^{(1)} \right] \right]$$

$$= 1.8549$$

$$\therefore Q_{3g} = 1.8549 + QL_3 = 3.0549$$

$$-5 \leqslant 3.0549 \leqslant 5$$

$$\therefore V_3^{(2)} = 0.8598 - j0.1091$$

$$\therefore \delta_3^{(2)} = -7.23$$

$$\therefore V_{3\,corrected}^{(2)} = 1 \angle -7.23 = 0.99205 - j0.126$$

$$P_1 - jQ_1 = V_1^* \left[Y_{11} V_1^{(0)} + Y_{12} V_2^{(2)} + Y_{13} V_3^{(2)} \right]$$

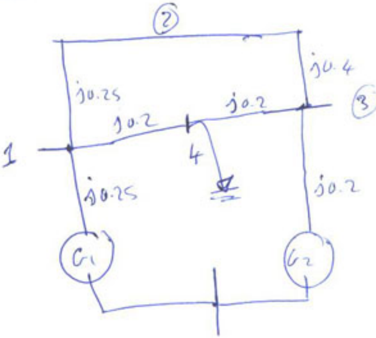
$$= 1.3308 - j1.2174$$

$$P_1 = PG_1 - P_{L1} \quad \therefore PG_1 = 2.1308\,p.u$$

$$QG_1 = 1.817\,p.u$$

15 Marks

Question 4: 20 Marks



$$Y_{bus} = \begin{bmatrix} -j13 & j4 & 0 & j5 \\ j4 & -j6.5 & j2.5 & 0 \\ 0 & j2.5 & -j12.5 & j5 \\ j5 & 0 & j5 & -j10 \end{bmatrix}$$

$$Z_{bus} = [Y_{bus}]^{-1} = \begin{array}{c} \begin{array}{cccc} & & & \text{Fault Bus} \end{array} \\ \begin{bmatrix} j0.1604 & j0.1263 & j0.0717 & j0.116 \\ j0.1263 & j0.2696 & j0.0991 & j0.1127 \\ j0.0717 & j0.0991 & j0.1426 & j0.1072 \\ j0.116 & j0.1127 & j0.1072 & j0.2116 \end{bmatrix} \end{array}$$

10 Marks

Note: Zbus may be also determined by direct Method

$$I_f = \frac{V_F}{Z_{44}} = \frac{1}{j0.2116} = -j4.726 \text{ p.u}$$

$$V_{1f} = V_f\left(1 - \frac{Z_{14}}{Z_{44}}\right) = 0.4518 \text{ p.u}$$

$$V_{2f} = V_f\left(1 - \frac{Z_{24}}{Z_{44}}\right) = \cancel{j}\ 0.4674 \text{ p.u}$$

$$V_{3f} = V_f\left(1 - \frac{Z_{34}}{Z_{44}}\right) = 0.4934 \text{ p.u}$$

$$I_{f(1-2)} = \frac{V_{1f} - V_{2f}}{z_{12}} = -j0.0624 \text{ p.u}$$

$$I_f(1-4) = \frac{V_{1f} - 0}{j0.2} = -j2.259 \text{ p.u}$$

$$I_f(2-3) = \frac{V_{2f} - V_{3f}}{j0.4} = -j0.065 \text{ p.u}$$

$$I_f(3-4) = \frac{0.4934 - 0}{j0.2} = -j2.467 \text{ p.u}$$

$$I_f G_1 = \frac{1 - 0.4518}{j0.25} = -j2.1928 \text{ p.u}$$

$$I_f G_2 = \frac{1 - 0.4934}{j0.2} = -j2.533 \text{ p.u}$$

10 Marks