Fayoum University
Faculty of Engineering
Industrial Engineering Dept.
Third Year 2008-2009

Fayoum University

Operations Research
1st Term Final Exam
Time allowed: 3 Hrs.
Jan. 2009

Attempt All the Questions

Question 1 (15 pts.)

A company produces two products, A and B. The sales volume for A is at least 80% of the total sales of both A and B. However, the company cannot sell more than 100 units of A per day. Both products use one raw material, of which the maximum daily availability is 240 lb. The usage rates of the raw material are 2 lb per unit of A and 4 lb per unit of B. The profit units for A and B are \$20 and \$50, respectively. Determine the optimal product mix for the company.

Question 2 (25 pts.)

Maximize: $Z = 5X_1 + 2X_2$ interpret the solution results

Subject to:

$$6X_1 + X_2 \geq 6$$
$$4X_1 + 3X_2 \geq 12$$
$$X_1 + 2X_2 \geq 4 \qquad x_1, x_2 \geq 0$$

If there is no clear solution, suggest a modification in the problem to get a clear solution, and solve the problem again graphically.

Question 3 (20 pts.)

A private carrier must deliver wheat in sacks from the warehouses having the weekly supplies as listed in table 1. Delivery must be made to mills which require the quantities in sacks per week as given table 2. The transportation and handling costs (cents per sack) from each warehouse to each mill are given in table 3. Use Vogel's approximation method to determine how many sacks should be delivered from each warehouse to each mill in order to achieve the least cost.

Table 1

Warehouse	Supply/Week
1	10,000
2	12,000
3	15,000
Total	37,000

Table 2

Mill	Demand/Week
A	8,000
B	9,000
C	10,000
D	8,000

Table 3

	Mill			
Warehouse	A	B	C	D
1	13	14	13	20
2	16	13	20	12
3	19	12	17	15

Question 4 (20 pts.)

A fast-food restaurant has one drive-in window. Cars arrive according to a Poisson distribution at the rate of 2 cars every 5 minutes. The space in front of the window can accommodate at most 10 cars, including the one being served. Other cars can wait outside this space if necessary. The service time per customer is exponential, with a mean of 1.5 minutes. Determine the following:

(a) The probability that the facility is idle. (5 pts.)
(b) The expected number of customers waiting to be served. (5 pts.)
(c) The expected waiting time until a customer reaches the window to place an order. (5 pts.)
(d) The probability that the waiting line will exceed the to-space capacity. (5 pts.)

Question 5 (20 pts.)

A. Find the minimum of the following function by using the dichotomous search method. Assume that Δ=0.05, the convergence criterion is based on $\left|X_R - X_L\right| \leq 0.2$. (10 pts.)

$$F = X^3 - 4X^2 + 2$$

B. Carry out at least five iterations using the method of steepest descent to minimize the following objective function. Assume that $X_O = (1,1)$. (10 pts.)

$$F = (X_1 - 3)^2 + 9(X_2 - 5)^2$$

Hint: compute the step size in the search direction in the first iteration only and then use it for all the following iterations

Good luck
Dr. Mohammad Abdel-karim

Operations Research Final Exam Solution
January 2009

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1- Mathematical model formulation

The decision variables are

X1= number of units of product A
X2=number of units of product B

The problem constraints are

1- Market constraints

$X1 \geq 0.8(X1+X2)(1)$

$X1 \leq 100(2)$

2- Material constraint

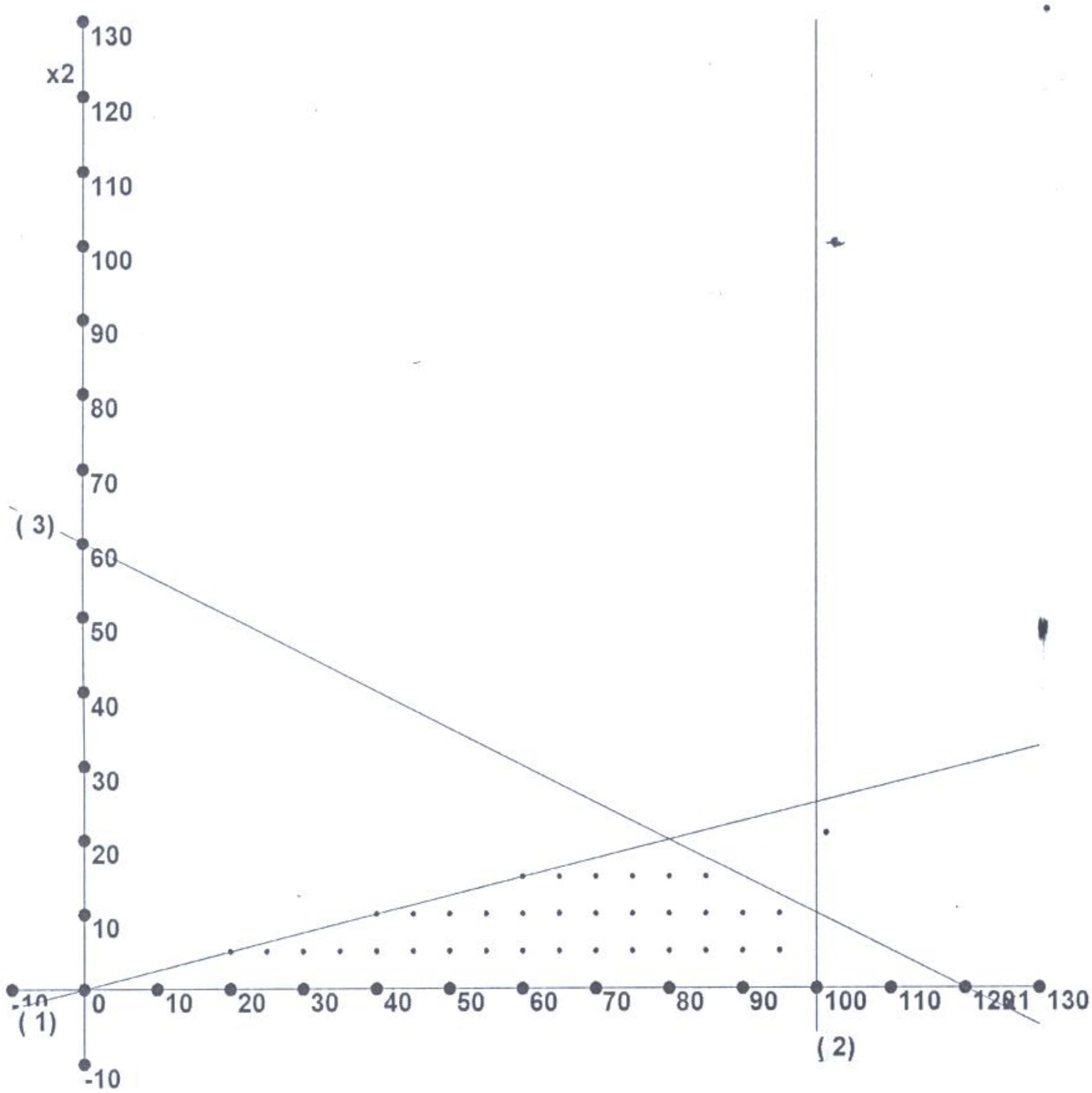
$2X1+4X2 \leq 240$ (3)

Objective function

Maximize $Profit(Z)=20X1+50X2$

Solution obtained by the graphical solution method(see figure 1)

X1 = 80.0 units
X2 = 20.0 units
Profit(Z)=2600 $



Figure(1)The problem feasible region

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Table 2	
Mill	Demand/Week
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D	8,000

Table 3	Mill			
Warehouse	A	B	C	D
1	13	14	13	20
2	16	13	20	12
3	19	12	17	15

Solution:

Table 3	Mill				Supply
Warehouse	A	B	C	D	
1	13	14	13	20	10,000
2	16	13	20	12	12,000
3	19	12	17	15	15,000
Demand	8,000	9,000	10,000	8,000	

Σ supply = 37,000 Σ demand=35,000 Σ supply $\neq$ Σ demand

To balance the problem a dummy demand column must be added to the above transportation table

Required amount = 37,000-35,000=2,000

The balanced transportation table is as following

	Mill					
	A	B	C	D	Dummy	supply / 1000
Warehouse 1	13	14	13	20	0	10
Warehouse 2	16	13	20	12	0	12
Warehouse 3	19	12	17	15	0	15
Demand/1000	8	9	10	8	2	37

Vogel's Approximation Method

	A	B	C	D	Dummy	supply / 1000	Penalties 1	Penalties 2
Warehouse 1	13	14	**8** [13]	20	**2** [0]	10	13	
Warehouse 2	**8** [16]	13	**2** [20]	**2** [12]	0	12	12	1
Warehouse 3	19	**9** [12]	17	**6** [15]	0	15	12	3
Demand/1000	8	9	10	8	2	37		
1	8	9	2	8	0			
2	8	0	2	2	0			
3	0	0	0	0	0			
1	3	1	4	3	0			
2	3	1	3	3	-			

Total transportation cost = 494,000

Final Transportation table

	A	B	C	D
Warehouse 1	13	14	**8** [13]	20
Warehouse 2	**8** [16]	13	**2** [20]	**2** [12]
Warehouse 3	19	**9** [12]	17	**6** [15]

Question 4 (20 pts.)

A fast-food restaurant has one drive-in window. Cars arrive according to a Poisson distribution at the rate of 2 cars every 5 minutes. The space in front of the window can accommodate at most 10 cars, including the one being served. Other cars can wait outside this space if necessary. The service time per customer is exponential, with a mean of 1.5 minutes. Determine the following:

(a) The probability that the facility is idle. (5 pts.)
(b) The expected number of customers waiting to be served. (5 pts.)
(c) The expected waiting time until a customer reaches the window to place an order. (5 pts.)
(d) The probability that the waiting line will exceed the to-space capacity. (5 pts.)

1- Solve the problem by using simplex method

Q1)

Maximize: $Z = 5x_1 + 2x_2$

Subject to:

$6x_1 + x_2 \geq 6$

$4x_1 + 3x_2 \geq 12$

$x_1 + 2x_2 \geq 4$ $\qquad x_1, x_2 \geq 0$

Standard Form

O.F $\quad$ Z - 5x1 - 2x2 = 0

S.T.

6x1 + x2 - s1 + r1 = 6

4x1 + 3x2 - s2 + r2 = 12

x1 + 2 x2 - s3 +r3 =4

min $\quad$ New O.F $\quad$ r0 = r1 + r2 + r3 $\qquad$ r0 - r1 - r2 - r3 = 0

Initial Phase

BV	r0	x1	x2	s1	s2	s3	r1	r2	r3	R.H.S	Ratio
r0(min)	1	0	0	0	0	0	-1	-1	-1	0	
r1	0	6	1	-1	0	0	1	0	0	6	
r2	0	4	3	0	-1	0	0	1	0	12	
r2	0	1	2	0	0	-1	0	0	1	4	

1st iter

BV	r0	x1	x2	s1	s2	s3	r1	r2	r3	R.H.S	Ratio
r0(min)	1	11	6	-1	-1	-1	0	0	0	22	
r1	0	6	1	-1	0	0	1	0	0	6	1
r2	0	4	3	0	-1	0	0	1	0	12	3
r2	0	1	2	0	0	-1	0	0	1	4	4

2nd iter

BV	r0	x1	x2	s1	s2	s3	r1	r2	r3	R.H.S	Ratio
r0(min)	1	0	4 1/6	5/6	-1	-1	-1 5/6	0	0	11	
x1	0	1	1/6	- 1/6	0	0	1/6	0	0	1	6
r2	0	0	2 1/3	2/3	-1	0	- 2/3	1	0	8	3 3/7
r2	0	0	1 5/6	1/6	0	-1	- 1/6	0	1	3	1 2/3

3rd iter

BV	r0	x1	x2	s1	s2	s3	r1	r2	r3	R.H.S	Ratio
r0(min)	1	0	0	4/9	-1	1 2/7	-1 1/2	0	-2 2/7	4 1/6	
x1	0	1	0	- 1/5	0	0	1/5	0	-0	5/7	8
r2	0	0	0	4/9	-1	1 2/7	- 4/9	1	-1 2/7	4 1/6	3 2/7
x2	0	0	1	0	0	- 5/9	-0	0	5/9	1 2/3	-3

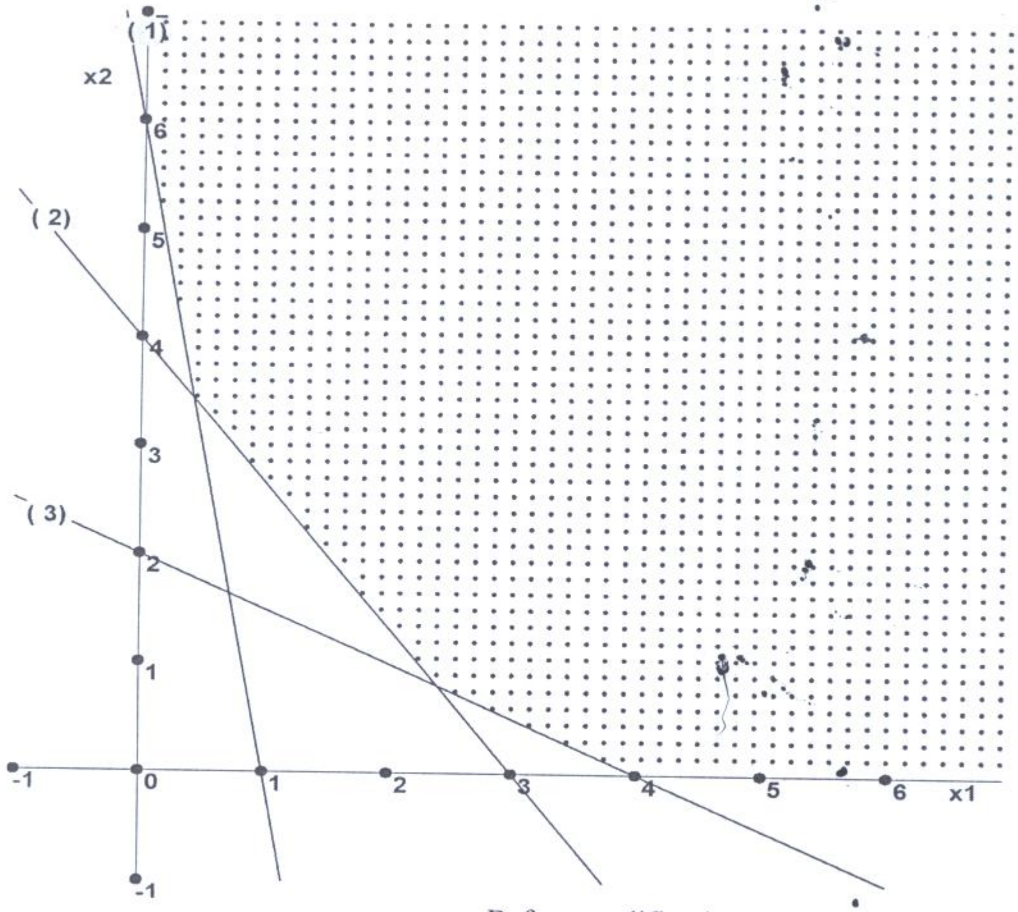
4th iter

BV	r0	x1	x2	s1	s2	s3	r1	r2	r3	R.H.S	Ratio
r0(min)	1	0	0	0	0	0	-1	-1	-1	0	
x1	0	1	0	- 1/5	0	0	1/5	-0	0	3/7	
s3	0	0	0	1/3	- 7/9	1	- 1/3	7/9	-1	3 2/7	
x2	0	0	1	2/7	- 3/7	0	- 2/7	3/7	0	3 3/7	

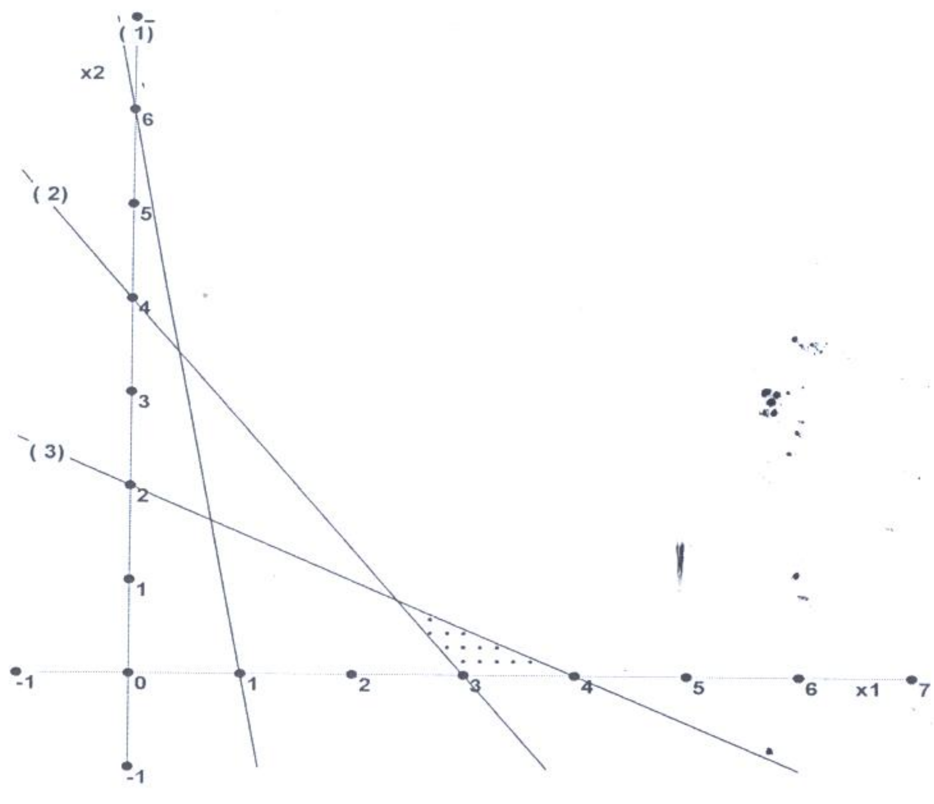
r0=0 this mean that the problem has an optimal solution

second phase

BV	Z	x1	x2	s1	s2	s3	r1	r2	r3	R.H.S	Ratio
Z(max)	1	-5	-2	0	0	0				0	
x1	0	1	0	- 1/5	0	0				3/7	
s3	0	0	0	1/3	- 7/9	1				3 2/7	
x2	0	0	1	2/7	- 3/7	0				3 3/7	



Before modification



After modification

1st iter

BV	Z	x1	x2	s1	s2	s3	r1	r2	r3	R.H.S	Ratio
Z(max)	1	0	0	- 1/2	- 1/2	0				9	
x1	0	1	0	- 1/5	0	0				3/7	-2
s3	0	0	0	1/3	- 7/9	1				3 2/7	9 1/5
x2	0	0	1	2/7	- 3/7	0				3 3/7	12

2nd iter

BV	Z	x1	x2	s1	s2	s3	r1	r2	r3	R.H.S	Ratio
Z(max)	1	0	0	0	-1 3/5	1 2/5				13 3/5	
x1	0	1	0	0	- 2/5	3/5				2 2/5	-6
s1	0	0	0	1	-2 1/5	2 4/5				9 1/5	-4 1/6
x2	0	0	1	0	1/5	- 4/5				4/5	4

3rd iter

BV	Z	x1	x2	s1	s2	s3	r1	r2	r3	R.H.S	Ratio
Z(max)	1	0	8	0	0	-5				20	
x1	0	1	2	0	0	-1				4	-4
s1	0	0	11	1	0	-6				18	-3
s2	0	0	5	0	1	-4				4	-1

Solution Results interpretation

All the ratios are negative in S3 Column, this mean that the solution region is un bounded

2- <u>Problem modification</u>

The problem has no feasible region because the constraints does not make a closed bounded area, so if we changed the constraints signs to make the constraints intersects and form a close area, a clear solution can be obtained.

The new modified problem is

Maximize: $Z = 5x_1 + 2x_2$

Subject to: $6x_1 + x_2 \geq 6$

$4x_1 + 3x_2 \geq 12$

$x_1 + 2x_2 \leq 4$ $x_1, x_2 \geq 0$

- Solution by graphical method

See the bounded region after modifying the problem by changing the third constraint greater than or equal (>=) to smaller than or equal (<=).

The solution is

Z = 20.0
X1 = 4.0
X2 = 0.0

Solution

$l = (\frac{2}{5}{'}\ 60) = 24 cars / hour$

$m = (\frac{60}{1.5}) = 40 customer / hour$

(e) The probability that the facility is idle. (5 pts.)

$r = \frac{l}{m}$

$r = \frac{24}{40} = 60\%$

The probability that the facility is idle=100-60=40%

(f) The expected number of customers waiting to be served(Lq)

$$L_q = \frac{r^2}{1- r}$$

L_q=0.9 car

Nearly one car is expected to wait in the waiting line for the service

(g) The expected waiting time until a customer reaches the window to place an order. (5 pts.)

$$\text{Average waiting time}(W_q) = \frac{l}{m(m-l)}$$

Wq=0.0375 hr or Wq=2.25 min

(h) The probability that the waiting line will exceed the to-space capacity. (5 pts.)

$$P_n = (1- r)r^n$$

The waiting line will exceed the space capcity when the there are fsix cars in the system one in the server and the other five cars waiting outside in the space

$$P_6 = (1- 0.6)0.6^6$$

The probability that the waiting line will exceed the to-space capacity=0.0186624

Question 5 (20 pts.)

A. Find the minimum of the following function by using the dichotomous search method. Assume that Δ=0.05, the convergence criterion is based on $|X_R - X_L| \le 0.2$. (10 pts.)

$$F = X^3 - 4X^2 + 2$$

min f(x) = x^3-4*x^2+2

XL=0
XR=10
delta= 0.05

X1=XL+(XR-XL)/2-(delta/2)
X2=XL+(XR-XL)/2+(delta/2)

iter #	XL	XR	X1	X2	F(X1)	F(X2)	Diff
1	0	10	4.975	5.025	26.131859	27.881891	
2	0	5.025	2.4875	2.5375	-7.358830	-7.416900	5.025
3	2.4875	5.025	3.73125	3.78125	-1.741598	-1.127655	2.5375
4	2.4875	3.78125	3.109375	3.159375	-6.610752	-6.390825	1.29375
5	2.4875	3.159375	2.798438	2.848438	-7.409739	-7.343313	0.671875
6	2.4875	2.848438	2.642969	2.692969	-7.479248	-7.478696	0.360938
7	2.4875	2.692969	2.565234	2.615234	-7.441371	-7.471036	0.205469
8	2.565234	2.692969	2.604102	2.654102	-7.466069	-7.480852	0.127734

X= 2.629102

X= 2.629102 Optimum value

B. Carry out at least five iterations using the method of steepest descent to minimize the following objective function. Assume that X_O = (1,1). (10 pts.)

$$F = (X_1 - 3)^2 + 9(X_2 - 5)^2$$

b)

F = (x1-3)^2+9*(x2-5)^2

Fx1= 2*x1-6
Fx2=18*x2-90

gradient =[2*x1-6;18*x2-90]   target   gradient = [0;0]

X0=(1,1)

X(k+1)=X(k)-r*gradient(X(k))

x1 = 1
x2 = 1
Fx1 = -4
Fx2 = -72

$$X(1) = \begin{vmatrix} 1+4*r \\ 1+72*r \end{vmatrix}$$

x1 = 1+4*r

x2 = 1+72*r

h(r) = (-2+4*r)^2+9*(-4+72*r)^2

hr = -5200+93344*r

r = 325/5834

F = (x1-3)^2+9*(x2-5)^2

r = 0.0557

iter #1	X(k)		X(k+1)		FUN Value	Gradient Value	
1	x1	x2	x1	x2			
2	1	1	1.2228	5.0104	3.15941	-3.55	0.2
3	1.2228	5.0104	1.42078	4.999973	2.49394	-3.16	-0
4	1.42078	4.999973	1.596705	5	1.96924	-2.81	0
5	1.596705	5	1.753032	5	1.55493	-2.49	-0
6	1.753032	5	1.891944	5	1.22779	-2.22	0
7	1.891944	5	2.015382	5	0.96947	-1.97	0

x1= 2.0154

x2= 5.0