



Answer the following questions:

Question (1) (16-points)

(a) Show that the function $f(x, y) = 2x^4 + e^{4y} - 4x^2e^y$ has exactly two critical points, both of which are local minima.

(b) Find the maximum and minimum of the function $f(x, y, z) = 4x^2 + y^2 + z^2$ subject to $x^4 + y^4 + z^4 = 1$

Question (2) (18-points)

(a) Evaluate the following integral $\int_0^1 \int_{\sqrt{y}}^1 \frac{3}{4+x^3} dx dy$

(b) Find the centre of mass of the lamina bounded by $y = x^2 - 4$, $y = 5$ if the density is the square of the distance from the y -axis.

(c) Find the area using line integral of the region bounded by $x^{2/3} + y^{2/3} = 1$
(Hint: use $x = \cos^3 t$, $y = \sin^3 t$)

Question(3) (18-points)

(a) Find the volume of the solid below $z = \sqrt{x^2 + y^2}$ and above $z = 0$ and inside $x^2 + (y-1)^2 = 1$

(b) Compute $\iiint_R \frac{1}{z} dV$, where R is the closed region bounded by $z = e^{xy}$, $z = 1$, $y = x + 1$, $y = 0$, $x = 0$.

Question(4) (18-points)

(a) Compute the line integral $\int_C (2xe^{x^2} - 2y)dx + (2y - 2x)dy$, where C runs from $(1,2)$ to $(-1,1)$

(b) Evaluate the work done by the force $\mathbf{F}(x, y) = (4xy - 2x)\mathbf{i} + (2x^2 - x)\mathbf{j}$, acting on an object as it moves along the parabola $y = x^2$ from $(-2,4)$ to $(2,4)$.

(c) Compute the surface area of the portion of the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, $a, b, c > 0$, that is in the first octant.

*With My Best Wishes
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