Fayoum University
Faculty of Engineering

Preparatory Year | Time : 90 Minutes.
Mathematics | May, 4, 2010.
Prof. Dr. Samy El Badawy Yehia | Second term Midterm Test.

الإسم :                    الفصل :                    رقم الجلوس :

1) $\int \frac{4x+5}{x^2+2x+2}dx = 2\int \frac{2x+2}{x^2+2x+2}dx + \int \frac{1}{x^2+2x+2}dx = 2\ln\left|x^2+2x+2\right| + \int \frac{1}{(x+1)^2+1}dx =$

$= 2\ln\left|x^2+2x+2\right| + \tan^{-1}(x+1) + C.$

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2) $\int \frac{\sqrt{x^3+1}}{x}dx =$

Let u = $\sqrt{x^3+1}$ then $u^2 = x^3+1$ , $u^2-1 = x^3$.

and $2udu = 3x^2dx$. then $\frac{2}{3}udu = x^2dx$.

$= \int \frac{\sqrt{x^3+1}}{x^3}(x^2dx) = \int \frac{u}{u^2-1}(\frac{2}{3}udu) = \frac{2}{3}\int \frac{u^2}{u^2-1}du = \frac{2}{3}\left[\int 1+\frac{1}{u^2-1}\right]du = \frac{2}{3}\left[u - \tan sh^{-1}u\right] + C =$

$= \frac{2}{3}\left[\sqrt{x^3+1} - \tan sh^{-1}\sqrt{x^3+1}\right] + C.$

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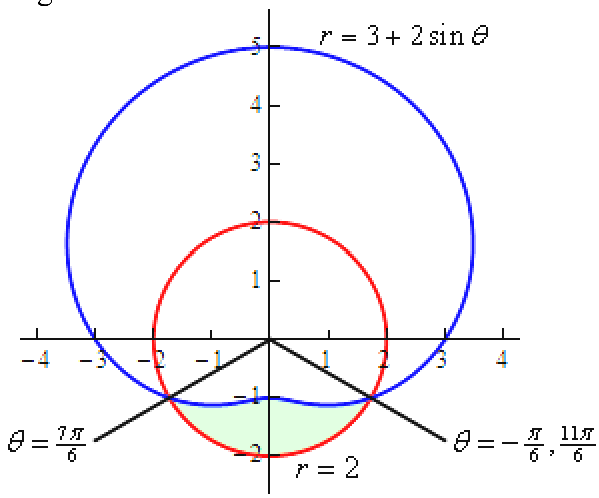
3)Determine the area of the region outside r = 3 + 2 sin Θ and inside r = 2 ( shaded area) .




$$A = \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \frac{1}{2}\left[(2)^2 - (3+2\sin\theta)^2\right]d\theta = \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \frac{1}{2}\left[-5-12\sin\theta - 4\sin^2\theta\right]d\theta ===$$

$$= \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \left[-7-12\sin\theta + 2(\cos(2\theta)\right]d\theta = \frac{1}{2}\left[-7\theta + 12\cos\theta + \sin(2\theta)\right]_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} = \frac{11\sqrt{3}}{2} - \frac{7\pi}{3} = 2.196.$$

4) Show that the angle between the pair of lines :

$$3x^2 - 7xy + 4y^2 = 0$$

is equal to the angle between the pair

$$6x^2 - 5xy + y^2 = 0$$

The First Pair is (3x – 4y)(x - y) =0. Which gives the two lines 3x-4y =0 and x-y =0 with slopes ¾ and 1 then the angle Θ between them is :

$$\tan\theta = \pm\frac{m_1 - m_1}{1 + m_1 m_2} = \pm\frac{3/4 - 1}{1 + (3/4)(1)} = \pm 1/7 .$$

The Second Pair is (3x – y)(2x-y)=0 . Which gives the two lines 3x-y = 0 and 2x –y = 0 with slopes 3 and 2.then the angle ϕ between them is :

$$\tan\phi = \pm\frac{m_3 - m_4}{1 + m_3 m_4} = \pm\frac{3 - 2}{1 + 6} = \pm 1/7$$

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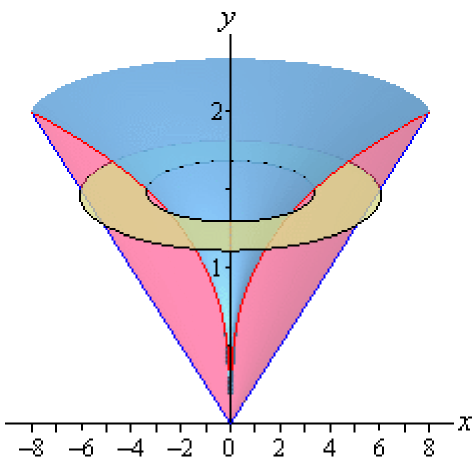
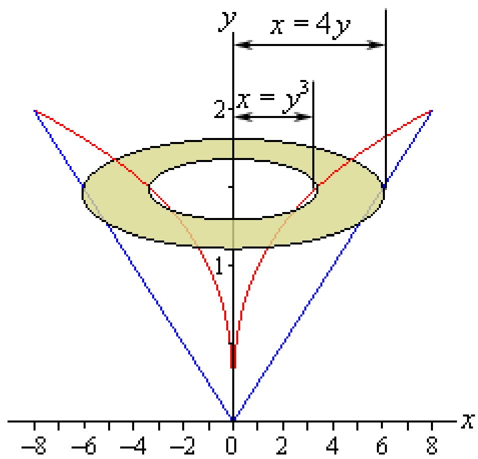
5) The area bounded between the curves $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ is revolved about the y axis. Find the volume of the resulting solid.




Thickness dy

Inner Radius $y^3$

Outer Radius $4y$

$$V = \pi\int_0^2 \left[16y^2 - y^6\right]dy = \pi\left[\frac{16}{3}y^3 - \frac{1}{7}y^7\right]_0^2 = \frac{512\pi}{21}.$$

6) The space ship Apollo 11 orbited the moon before landing on it. The orbit was an elliptical shape with the centre of the moon as one of its focii. If the radius of the moon is 1728 km. The nearest

point of the orbit to the surface of the moon was 110 km. The farthest point was 314 km. Find the equation of this ellipse.

2a = 2 times the radius of the moon + min. distances + max. Distance = 2 x 1728 + 110 + 314 = 3880.
Then a = 1940. c = 1940 – (1728+110) = 102.
$b^2 = a^2 - c^2 = 3753196$. The equation of the ellipse is :

$$\frac{x^2}{3763600} + \frac{y^2}{3753196} = 1 or \frac{x^2}{(1940)^2} + \frac{y^2}{(1937.32)^2}$$

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7) Given the two circles :

$$S_1 : x^2 + y^2 + 2g_1 x + 2f_1 y + c_1 = 0,$$
$$S_2 : x^2 + y^2 + 2g_2 x + 2f_2 y + c_2 = 0$$

Show that for each $\lambda \in$ R ( R is the set of real numbers) , the circle $S_1 + \lambda S_2 = 0$ is coaxial with both $S_1$ and $S_2$.

Let λ and μ be any two real numbers. Then the radical axis of the two circles : $S_1 + \lambda S_2 = 0$ and $S_1 + \mu S_2 = 0$ is :
$\frac{S_1 + \lambda S_2}{1+\lambda} - \frac{S_1 + \mu S_2}{1+\mu} = 0$ which gives $\frac{(1+\mu)(S_1 + \lambda S_2) - (1+\lambda)(S_1 + \mu S_2)}{(1+\lambda)(1+\mu)} = 0$. Then

$S_1 + \lambda S_2 + \mu S_1 + \lambda\mu S_2 - S_1 - \mu S_2 - \lambda S_1 - \lambda\mu S_2 = 0$. Then :

$\mu S_1 - \lambda S_1 + \lambda S_2 - \mu S_2 = 0.$ Then $S_1(\lambda - \mu) - S_2(\lambda - \mu) = 0.$ Which gives the radical axis :

$$S_1 - S_2 = 0$$

which is independent of the particular choice of λ and μ .

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