Fayoum University
Faculty of Engineering

First Term Final Exam. (2016-2017) January, 2, 2017.

CALCULS I (MTHC002)

Prof. Dr. Samy El Badawy Yehia Time: 180 Min.

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1) Find : 5 points

$$\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+t}}-\frac{1}{t}\right)$$

By substitution, we get $\infty - \infty. So$:

$$\lim_{t\to 0}\frac{1-\sqrt{1+t}}{t\sqrt{1+t}}\ X\ \frac{1+\sqrt{1+t}}{1+\sqrt{1+t}} = \lim_{t\to 0}\frac{1-(1+t)}{t\sqrt{1+t}+t\,(1+t)} =$$

$$= \lim_{t\to 0}\frac{-t}{t\,(\sqrt{1+t}+(1+t)} = \lim_{t\to 0}\frac{-1}{\sqrt{1+t}+(1+t)} = \frac{-1}{2}$$

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2) Find the points P on the graph of the curve $y = x^3$ such that the tangent at this point P has x intercept equals 4. 10 points

Let the point be (a, a^3) . The slope $of\ the\ tangent\ is$ $3\,a^2$. It passes through the point (4 , 0).

The equation of the tangent is:

$$3\,a^2 = \frac{y-a^3}{x-a}$$

Then $3\,a^2 = \frac{0-a^3}{4-a}$. Then $12\,a^2 - 3\,a^3 = -a^3$.

$Then\ 12\,a^2 - 2\,a^3 = 0$

Then a = 6 (a = 0 is refused because the tangent will be the x axis) and the point is (6, 216).

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3) Find $\frac{dy}{dx}$: 15 points

a) y =$(\sin 3x)^{(\cos 5x)} + (\cos 7\,x)^{(\sin 9x)}$.

A = $(\sin 3x)^{(\cos 5x)}$, $B = (\cos 7\,x\,)^{(\sin 9x)}$

ln A = (cos 5 x) ln (sin 3 x) , ln B = $(\sin 9\,x\,)\ln(\cos 7\,x)$

$$\frac{A'}{A} = \cos 5\,x\,\left(\frac{3\cos 3\,x}{\sin 3\,x}\right) + \left(\ln(\sin 3x)\,(-\,5\sin\;5\,x)\right)$$

$$\frac{B'}{B} = \sin 9\,x\,\left(\frac{-\,7\sin 7\,x}{\cos 7\,x}\right) + \;(\ln(\cos 7\,x))\;(9\cos 9\,x)$$

$$\frac{dy}{dx} = \;A' + B' =$$

$$\left((\sin 3x)^{(\cos 5x)}\right)\left(\cos 5\,x\,\left(\frac{3\cos 3\,x}{\sin 3\,x}\right) + \;\left(\ln(\sin 3x)\,(-\,5\sin\;5\,x)\right)\right)+$$

$$+\;\left((\cos 7\,x\,)^{(\sin 9x)}\right)\left(\sin 9\,x\;\left(\frac{-\,7\sin 7\,x}{\cos 7\,x}\right)\;+\right.$$

$$\left.\left(\ln\left(\cos 7\,x\right)\right)\;\left(9\cos 9\,x\right)\right)$$

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b) $\tan\left(\frac{x}{y}\right) = x + y.$ Put it in the form $(\frac{dy}{dx} = \cdots)$

$$\left(sec^2\;\left(\frac{x}{y}\right)\right)\;\left(\frac{y - x\,y'}{y^2}\right) = 1 + y'.$$

Then y $\left(sec^2\;\left(\frac{x}{y}\right)\right) - x\,y'\left(sec^2\;\left(\frac{x}{y}\right)\right) = \;y^2 + y^2y'$

Then y' $\left(x\;\left(sec^2\;\left(\frac{x}{y}\right)\right) + y^2\right)$ = y $\left(sec^2\;\left(\frac{x}{y}\right)\right) - \;\;y^2$

Then y' = $\dfrac{\text{y}\left(sec^2\left(\frac{x}{y}\right)\right) - \;y^2}{x\left(sec^2\;\left(\frac{x}{y}\right)\right) + y^2}$

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c) y = $tanh^{-1}(\sqrt{1+\ln(x^2+5x}\,)$

$$y' = \frac{\frac{\frac{2x+5}{x^2+5x}}{2\sqrt{1+\ln(x^2+5x)}}}{1-(1+\ln(x^2+5x))}$$

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4) Find the points on the curve : 10 points

$$y = \text{cosec } x + \sec x \qquad 0 \le x \le \pi/2 .$$

Such that the tangent at these points are horizontal.

y' = - (cosec x)(cot x)+(sec x)(tan x) = 0.

Then $\frac{-1}{\sin x} \cdot \frac{\cos x}{\sin x} + \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = 0.$

Then $\frac{-\cos x}{sin^2\,x} + \frac{\sin x}{cos^2\,x} = 0.$

Then $\frac{sin^3\,x - cos^3\,x}{sin^2\,x\ cos^2\,x} = 0.$

Then $(\sin x - \cos x)\,(sin^2\,x + \sin x\cos x + cos^2\,x) = 0.$

Which means:

Either sin x = cos x then tan x = 1 and x = $\pi/4$

And the point is $(\pi/4\ , 2\sqrt{2}\ )$

OR sin x cos x = -1 which is refused since x is in the first quadrant.

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5)

$$y = f(x) = \frac{x^2+7\,x+3}{x^2} \;.$$

Showing:

a) The domain of this function, intersection with the axes, and all asymptotes and the points of intersections with them (if any)
b) Critical points and local extreme.

Intersection with y axis none. Intersection with the x axis :

$$x^2 + 7\,x + 3 = 0.$$

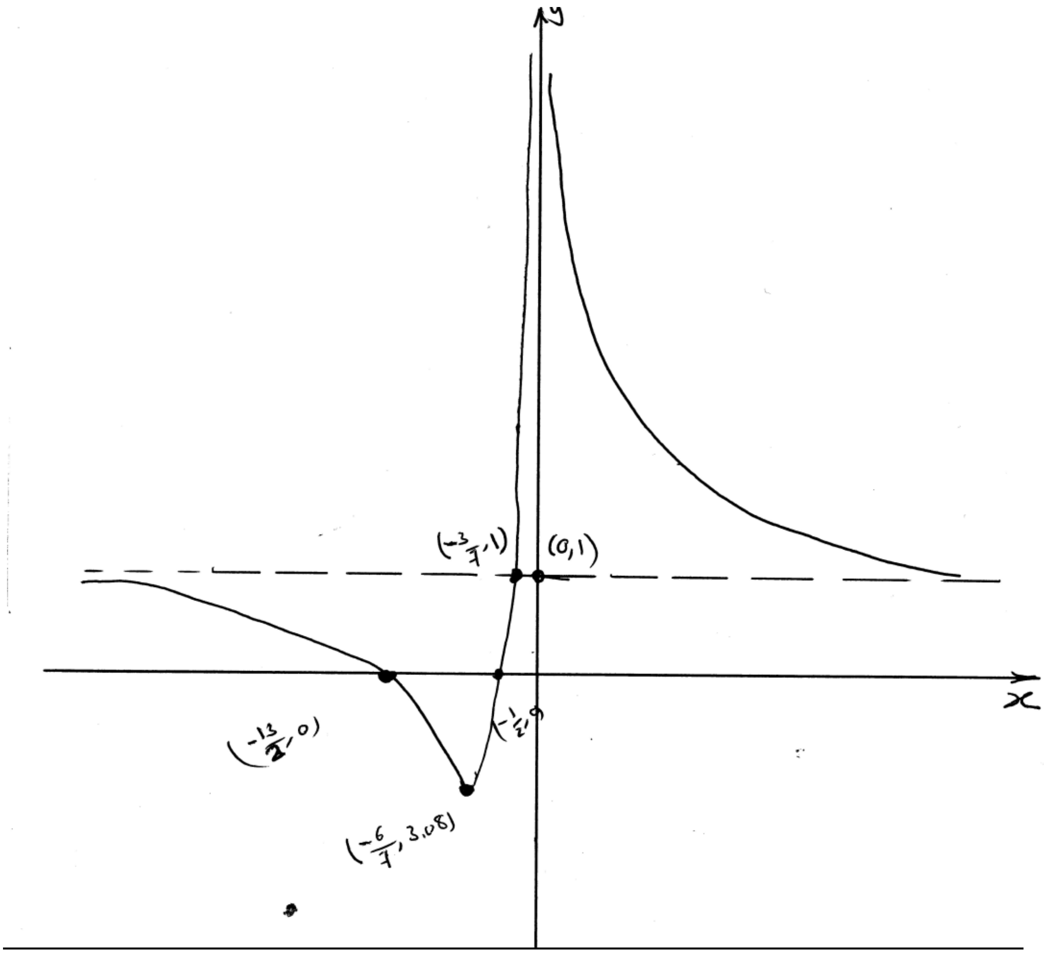
$$x = \frac{-\,7 \pm \sqrt{49-12}}{2} = \frac{-\,7\;\pm\sqrt{37}}{2} \cong \frac{-7 \pm 6}{2}$$

Then x $\cong$ -1/2 Or -13/2.

Domain R – {0}.

$$\lim_{x\to\infty} \frac{x^2 + \;\; 7\,x + 3}{x^2} = \;\; 1. Then$$

Horizontal asymptote y = 1 and Vertical asymptote x = 0
Intersection with the horizontal asymptote:
1 = $\frac{x^2+7\,x+3}{x^2}$. then x = -3/7. Point of intersection (-3/7 , 1).
y' = f'(x) = $\frac{-\,7\,x-6}{x^3}$. Critical point ($\frac{-\,6}{7}$, -3.08).Since f' changes its sign from negative to positive before then after x = -6/7 this makes this point a local minimum.



6) Find the following limit: 10 points

$$\lim_{x\to(0)^+} (1+\sin 4x)^{(\cot x)}$$

By substitution, we get 1^{∞} then we take (ln) :

ln y = (cot x) ln (1 + sin 4 x) = $\frac{\ln(1+\sin 4x)}{\tan x}$.

by substitution , get $\frac{0}{0}$ Applying L'Hopital Theorem.

$$\lim_{x\to(0)^+} \ln y = \lim_{x\to(0)^+} \frac{\frac{4\cos 4x}{1+\sin 4x}}{sec^2 x} = \lim_{x\to(0)^+} \frac{4cos^3 4x}{1+\sin 4x}$$

$$= 4.$$

$$\lim_{x\to(0)^+} (1+\sin 4x)^{(\cot x)} = e^4.$$