Fayoum University
Faculty of Engineering
First Year Civil Eng. Department

Engineering Mathematics (2A) Final Exam.
Jan., 4, 2009
Time Allowed: 3 Hours

Attempt All Questions:

Q1. Find z_{xx}, z_{yy} and z_{xy} for $z = f(x^2 y)$.

Q2. Test for extermum and saddle points the function $f(x,y) = \ln|y| + x^2 + \frac{1}{2}y^2 + 2xy - 3$

Q3. Determine the dimensions of a box of length l, breath b, and height h of fixed volume V that is open on the top and requires the minimum amount of material.

Q4. Find the integral $\int_0^{\frac{3}{2}} \int_{\sqrt{3}x}^{\sqrt{9-x^2}} e^{-(x^2+y^2)} dy dx$

Q5. Find the center of the mass of the region above the xy - plane, below the surface $z = 4 - x^2$, and between the planes $y = 0$ and $y = 4$, where the density $= 6 - z$.

Q6. Find the volume of the solid region that is inside the sphere $x^2 + y^2 + z^2 = 4$ and the above the cone $x^2 + y^2 = 2z^2$.

Q7. Find the surface area of the portion of the plane $2x + y + 3z = 1$, that is inside the cylinder $x^2 + y^2 = 4$.

Q8. Evaluate $\oint_C yx dy$; C is the semicircle $x^2 + y^2 = 4$, $y > 0$ and the line $y = 0$, $-2 \le x \le 2$.

Q9. Find the line integral $\int_C \cos x \cos y dx - \sin x \sin y dy$ where C is the octagonal path shown in Figure 1.

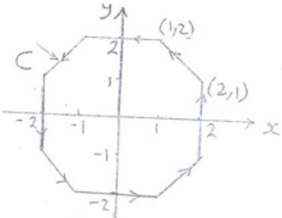

Fig. 1.

Best Wishes
Prof. Mohamed Eissa Sayed Ahmed

Q_1: $z = \quad y)$, find z_{xx}, z_{yy}, z_{xy}

Let $u = x^2 y$, then $z = f(u)$

$\therefore \frac{\partial z}{\partial x} = f'(u)\frac{\partial u}{\partial x} = f'(u)(2xy)$, $\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}[2xy f'(u)]$

$\therefore z_{xx} = 2y f'(u) + 2xy f''(u)\frac{\partial u}{\partial x} = 2y f'(u) + (2xy)^2 f''(u)$.

$z_{xy} = \frac{\partial}{\partial y}(f'(u)\, 2xy) = 2x f'(u) + 2xy f''(u)\frac{\partial u}{\partial y}$

$= 2x f'(u) + 2xy f''(u)(x^2) = 2x f'(u) + 2x^3 y f''(u)$.

$z_y = f'(u)\frac{\partial u}{\partial y} = f'(u)\, x^2$,

$z_{yy} = \frac{\partial}{\partial y}(f'(u)\, x^2) = x^2 f''(u)\frac{\partial u}{\partial y} = x^4 f''(u)$.

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Q_2: $f(x,y) = \ln|y| + x^2 + \frac{1}{2}y^2 + 2xy - 3$

$f_x = 2x + 2y = 0, \Rightarrow y = -x$

$f_y = \frac{1}{y} + y + 2x = 0$

$\frac{1}{y} + y - 2y = 0 \Rightarrow \frac{1}{y} - y = 0$

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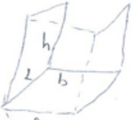
$\therefore y^2 = 1 \Rightarrow y = \pm 1$, $\therefore x = \mp 1$

x	y	$f_{xx} = 2$	$f_{yy} = \frac{-1}{y^2} + 1$	$f_{xy} = 2$	Δ	Conclusion
-1	1	2	0	2	$-4 < 0$	Saddle point
+	-1	2	0	2	$-4 < 0$	Saddle point

3:

$V = Lbh$

$\therefore Lbh - V = 0 \Rightarrow$ constraint function



$\min S = Lb + 2Lh + 2hb \Rightarrow$ min. fn.

$\therefore \min f(L,b,h) = Lb + 2Lh + 2hb,$

such that

$g(L,b,h) = Lbh - V = 0$

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$\therefore \dfrac{f_L}{g_L} = \dfrac{f_b}{g_b} = \dfrac{f_h}{g_h}, \quad \therefore \dfrac{b+2h}{bh} = \dfrac{L+2h}{Lh} = \dfrac{2L+2b}{Lb}$

for $\dfrac{b+2h}{bh} = \dfrac{L+2h}{Lh} \quad \therefore bLh + 2Lh^2 = bLh + 2bh^2$

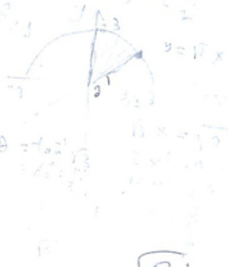
$\therefore (L = b), \quad \dfrac{L+2h}{Lh} = \dfrac{2L+2b}{Lb} \quad \therefore L^2 b + 2hLb = 2L^2 h + 2bLh$

$b = 2h$

$\therefore b(b)\dfrac{b}{2} = V \quad \therefore L = b = \sqrt[3]{2V} \Rightarrow h = \dfrac{1}{2}\sqrt[3]{2V}$

Q4: $I = \displaystyle\int_0^{\frac{3}{2}} \int_{\sqrt{3}x}^{\sqrt{9-x^2}} e^{-(x^2+y^2)}\, dy\, dx$

Using Polar Coordinates (r,θ)



$\therefore I = \displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^3 e^{-r^2}\, r\, dr\, d\theta$

$= \displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left.\dfrac{e^{-r^2}}{-2}\right]_0^3 d\theta$

$= \dfrac{1}{2}\left(1 - e^{-9}\right)\left(\dfrac{\pi}{2} - \dfrac{\pi}{3}\right)$

$I = \dfrac{(1 - e^{-9})\pi}{12}.$

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Q5:

$$M = \iiint_{R_{xy}\;0}^{\;4-x^2} \sigma \, dV$$

$$= \iint_{R_{xy}} \int_0^{4-x^2} (6-z)\, dz\, dA$$

$$= \iint_{R_{xy}} \left(6z - \frac{z^2}{2}\right)\Big|_0^{4-x^2} dA$$

$$= \iint_{R_{xy}} \left[(24-6x^2) - \frac{1}{2}(4-x^2)^2\right] dA$$

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$$= \iint_{R_{xy}} \left[24 - 6x^2 - \frac{1}{2}(16 - 8x^2 + x^4)\right] dA$$

$$= \int_{-2}^{2}\int_0^4 \left[16 - 2x^2 - \frac{1}{2}x^4\right] dy\, dx$$

$$= 4\int_{-2}^{2} \left(16 - 2x^2 - \frac{x^4}{2}\right) dx = 8\left[16x - \frac{2}{3}x^3 - \frac{x^5}{10}\right]_0^2$$

$$= 8\left[32 - \frac{16}{3} - \frac{32}{10}\right] = 8\left[\frac{960 - 160 - 96}{30}\right] = \frac{2816}{15}$$

Q6:

$$V_1 = 8 \iiint_{R^{+++}_{xyz}} dz\,dx\,dy, \quad V = \frac{4}{3}\pi(2)^3 - V_1$$

~~$= 8 \int\int\int \rho \sin\phi \, d\rho\, d\phi\, d\theta$~~

$$V_1 = 8\int_0^{\frac{\pi}{2}}\int_0^{\tan^{-1}\sqrt{2}}\int_0^2 \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= 8\left(\frac{8}{3}\right)\left(-\cos\phi\right)\Big|_0^{\tan^{-1}\sqrt{2}}\left(\frac{\pi}{2}\right)$$

$$= \frac{64}{3}\left(1 - \cos(\tan^{-1}\sqrt{2})\right)\frac{\pi}{2}$$

$$\rho^2 \sin^2\phi = 2\rho^2\cos^2\phi$$

$$\tan\phi = \sqrt{2}$$

$$\phi = \tan^{-1}\sqrt{2} =$$

$$\therefore V = \frac{4}{3}\pi(8) - \frac{32\pi}{3}\left(1 - \cos(\tan^{-1}\sqrt{2})\right)$$

$$= \frac{32}{3}\pi\left[\cos(\tan^{-1}\sqrt{2})\right]$$

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$F(x,y,z) = 2x + y + 3z - 1 = 0$

$\vec{\nabla F} = 2i + j + 3k$

$\vec{n} = \dfrac{2i + j + 3k}{\sqrt{14}}$

$S = \iint_{R_{xy}} \dfrac{\sqrt{14}}{3}\, dA$

$= \dfrac{\sqrt{14}}{3} (\pi)(2)^2 = \dfrac{4\sqrt{14}}{3}\pi$

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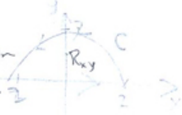
Q8: $\oint_C yx\, dy$ Green's theorem



$I = \iint_{R_{xy}} \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y}\right) dA$

$= \iint_{R_{xy}} y\, dA = \int_0^{\pi}\int_0^2 r\sin\theta\, r\,dr\,d\theta$

$= \left(\dfrac{r^3}{3}\right)_0^2 (-\cos\theta)_0^{\pi} = \dfrac{2(2)^3}{3} = \dfrac{16}{3}$

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Q9: $\int_C \cos x \cos y\, dx - \sin x \sin y\, dy$

$P(x,y) = \cos x \cos y$ ⇐ continuous

$\dfrac{\partial P}{\partial y} = -\cos x \sin y$

$Q(x,y) = -\sin x \sin y$

$\dfrac{\partial Q}{\partial x} = -\cos x \sin y$

and $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$ $\therefore I = 0$

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