الإجابة النموذجية لامتحان

ميكانيكا أولى مدنى الترم الأول

د/ أحمد [illegible]

Fayoum University
Faculty of Engineering
Dept. of Civil Engineering
First Year

Engineering Mechanics
First Term, 2009 / 2010
Final Exam
Time: 3 hours

Answer the three questions (1, 4, 5) and <u>ONLY ONE</u> of the two questions (2, 3):

[1] For the system shown in Fig. (1): the bar (AB) rotates about (A) with a constant angular velocity 5.0 rad / s , For the position shown:

(a) Use the vector solution to determine the angular velocities of the bars (BC) and (CD).

(b) Check the results obtained in (a) using the instantaneous center method.

(c) Using the graphical solution, determine the angular acceleration of the bar (CD).

(32%)

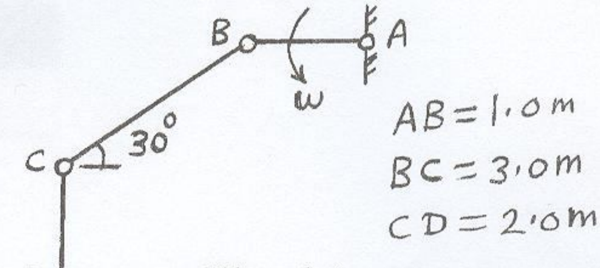


Fig. (1)

[2] The 10 kg circular bar (AB) shown in Fig. (2) moves vertically under the effect of its own weight with its ends follow the shown guides. Find the acceleration of the bar and the reactions at its ends.

(20%)

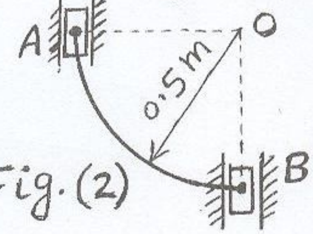


Fig. (2)

[3] The 15 kg uniform bar shown in Fig. (3) rotates in the vertical plane about a fixed axis through (A). If the bar is released from rest at $\theta = 0$, determine the angular acceleration of the bar and the reaction at (A) at $\theta = 60^0$.

(23%)

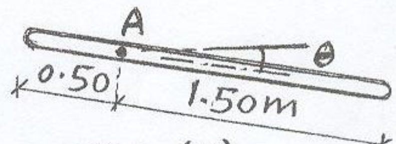


Fig. (3)

[4] The 10 kg disc shown in Fig. (4) is released from rest on the 30^0 incline, determine:

(a) the angular acceleration of the disc.

(b) the velocity of the disc after 3.0 seconds.

(c) the acceleration of the cylinder (A).

(25%)

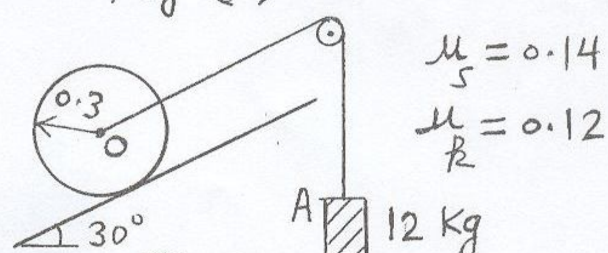


Fig. (4)

[5] The system shown in Fig. (5) is released from rest at $\theta = 70^0$. If, in this position, the spring is compressed by 0.30 m, and the disc rolls without slipping:

(a) if the spring stiffness (k) is 90 N/m, find the velocity of the collar (B) at $\theta = 30^0$.

(b) find the value of the spring stiffness (k) for which the system comes to rest at $\theta = 0$.

(25%)

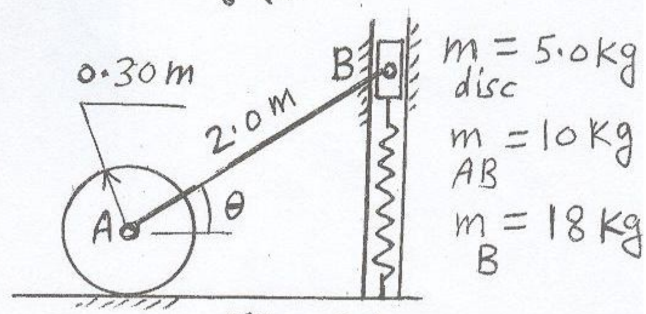


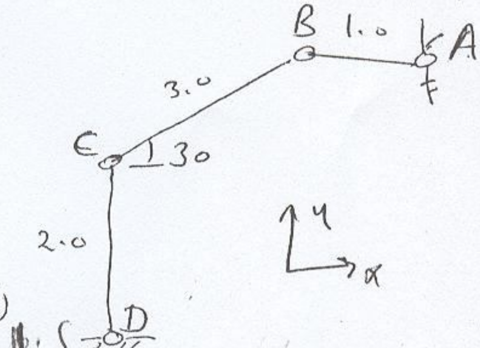
Fig. (5)

Best Wishes
Examiner:
Dr. Ahmed A. Al-Kaisy

1

(a) 9

$\underline{V}_C = \underline{V}_B + \underline{V}_{C/B}$ —①



$\underline{V}_C = \underline{w}_{CB} \times \underline{r}_{C/D}$

$\boxed{\therefore \underline{V}_C} = w_{CD}\,\underline{k} \times 2.0\,\underline{j} = \boxed{-2w_{CD}\,\underline{i}}$ —② 1.5

$\boxed{\underline{V}_B} = \underline{w}_{AB} \times \underline{r}_{B/A} = 5\underline{k} \times (-\underline{i}) = \boxed{-5\underline{j}}$ —③ 1.5

$\underline{V}_{C/B} = \underline{w}_{CB} \times \underline{r}_{C/B} = w_{CB}\,\underline{k} \times \left(-3\frac{\sqrt{3}}{2}\underline{i} - 1.5\underline{j}\right)$ 1.5

$\therefore \boxed{\underline{V}_{C/B} = +1.5\,w_{CB}\,\underline{i} - 1.5\sqrt{3}\,w_{CB}\,\underline{j}}$ —④

sub. from ②, ③, ④ in ①:

$\therefore \boxed{-2w_{CD}\,\underline{i} = 1.5\,w_{CB}\,\underline{i} - (5 + 1.5\sqrt{3}\,w_{CB})\,\underline{j}}$ 1.5

vectors 9
I.C. 5 } 14
accn 18

j-eqn;

$\therefore w_{CB} = -1.925$ rad/s

$\boxed{\therefore w_{CB} = 1.925 \text{ rad/s} \curvearrowright}$ 1.5

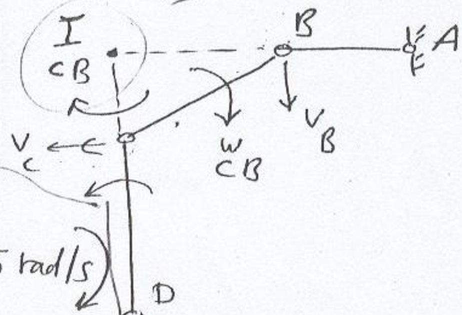
i-eqn;

$\boxed{\therefore w_{CD} = 1.443 \text{ rad/s} \circlearrowleft}$ 1.5

5 (b) I.C. Method:

1

$V_B = w_{AB} \cdot \overline{AB} = 5$ m/s



but: $V_B = w_{CB} \cdot \overline{BI_{CB}}$

2

$\therefore 5 = 3\frac{\sqrt{3}}{2}\,w_{CB} \Rightarrow \boxed{\therefore w_{CB} = 1.925 \text{ rad/s}}$

$\therefore V_C = w_{CB} \cdot \overline{CI_{CB}} = 1.925 \times 1.5 = 2.887$ m/s

but: $V_C = \overline{CD} \cdot w_{CD} \Rightarrow \therefore 2w_{CD} = 2.887$

$\boxed{\therefore w_{CD} = 1.443 \text{ rad/s} \circlearrowleft}$ 2

(c) 18

$\because w_{AB} = \text{const.} \Rightarrow \therefore \alpha_{AB} = 0$ 1

$\underline{a}_C = \underline{a}_B + \underline{a}_{C/B} \Rightarrow (\underline{a}_C)_n + (\underline{a}_C)_t = (\underline{a}_B)_n + (\underline{a}_B)_t + (\underline{a}_{C/B})_n + (\underline{a}$

$\boxed{(a_C)_t} = \alpha_{CD} \cdot \overline{CD} = 2\alpha_{CD} \leftrightarrow$ 1.0

$\boxed{(a_C)_n} = w^2_{CD} \cdot \overline{CD} = 4.164 \ m/s^2 \downarrow$ 1.0

$\boxed{(a_B)_t} = \alpha_{AB} \cdot \overline{AB} = 0$ 1.0

$\boxed{(a_B)_n} = w^2_{AB} \cdot \overline{AB} = 25 \ m/s^2 \rightarrow$ 1.0

$\boxed{(a_{C/B})_t} = \alpha_{CB} \cdot \overline{CB} = 3\alpha_{CB}$ 30 1.0

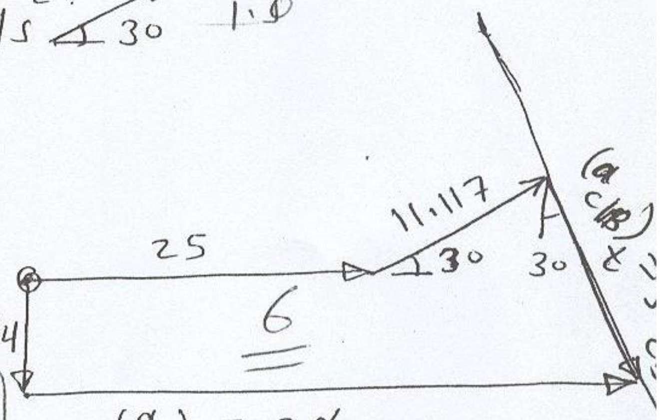
$\boxed{(a_{C/B})_n} = w^2_{CB} \cdot \overline{CB} = 11.117 \ m/s^2 \angle 30$ 1.0

~~7~~ 6

vl comp:

$11.117 \sin 30 - 3\alpha_{CB} \cos 30 = -4.164$

$\boxed{\therefore \alpha_{CB} = 3.742 \ rad/s^2 \circlearrowright}$ 2.5

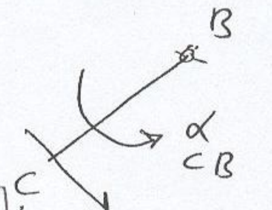


hl comp:

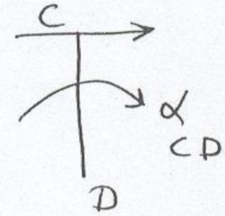
$25 + 11.117 \cos 30 + 3(3.742) \sin 30$

$= 2\alpha_{CB}$

$\boxed{\therefore \alpha_{CD} = 20.120 \ rad/s^2 \circlearrowright}$ 2.5



1.5 1



11

25

~~7~~

32

(c) 18

$\because \omega_{AB} = \text{const.} \Rightarrow \therefore \alpha_{AB} = 0$ 1

$\underline{a}_C = \underline{a}_B + \underline{a}_{C/B} \Rightarrow (\underline{a}_C)_n + (\underline{a}_C)_t = (\underline{a}_B)_n + (\underline{a}_B)_t + (\underline{a}_{C/B})_n + ($

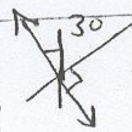
$(a_C)_t = \alpha_{CD} \cdot \overline{CD} = 2\alpha_{CD} \leftrightarrow$ 1.0

$(a_C)_n = \omega_{CD}^2 \cdot \overline{CD} = 4.164\ m/s^2 \downarrow$ 1.0

~~7~~ 6

$(a_B)_t = \alpha_{AB} \cdot \overline{AB} = 0$ 1.0

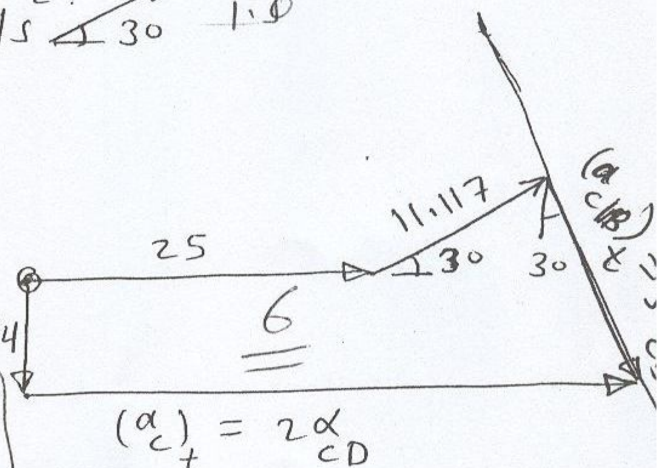
$(a_B)_n = \omega_{AB}^2 \cdot \overline{AB} = 25\ m/s^2 \rightarrow$ 1.0



$(a_{C/B})_t = \alpha_{CB} \cdot \overline{CB} = 3\alpha_{CB}$ 1.0

$(a_{C/B})_n = \omega_{CB}^2 \cdot \overline{CB} = 11.117\ m/s^2 \nearrow \angle 30$ 1.0

vl comp:



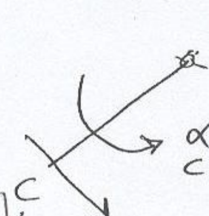
$11.117 \sin 30 - 3\alpha_{CB} \cos 30 = -4.164$

$\boxed{\therefore \alpha_{CB} = 3.742\ rad/s^2 \circlearrowleft}$ 2.5

hl comp:

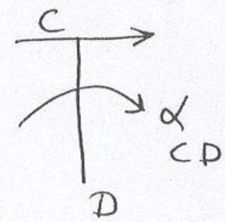
$25 + 11.117 \cos 30 + 3(3.742) \sin 30$

$= 2\alpha_{CB}$



$\boxed{\therefore \alpha_{CD} = 20.120\ rad/s^2 \circlearrowright}$ 2.5

1.5 1



~~11~~
11
25
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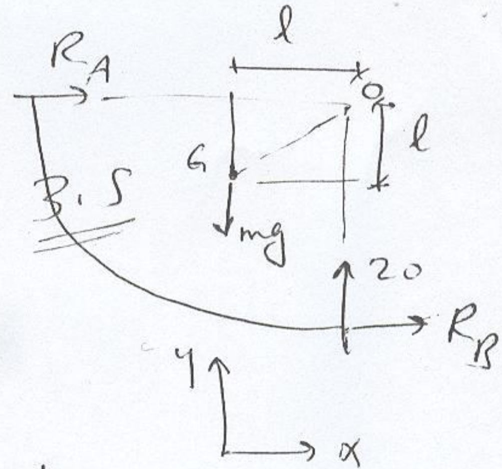
32

$\boxed{2}$ $m = 10\ kg$



$$OG = \frac{r \sin \alpha}{\alpha}$$

$$= \frac{r \sin 45^\circ}{\pi/4} = \frac{4r}{\pi\sqrt{2}}$$

$$\therefore l = \frac{OG}{\sqrt{2}} = \frac{1}{\pi} \quad 3.5$$

translational Motion in y-direction

$\Sigma F_x = m(a_G)_x$:

$$\therefore R_A + R_B = 0 \quad \text{—} (1) \quad 2$$

$\Sigma F_y = m(a_G)_y$:

$$-20 + mg = ma \rightarrow a = 7.81\ m/s^2 \downarrow \quad 3 \quad (2)$$

$\Sigma M_G = 0$:

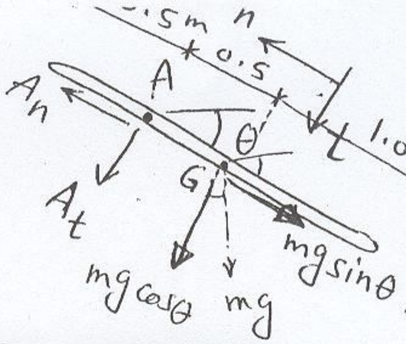
$$\therefore \frac{20}{\pi} - \frac{1}{\pi} R_A + \left(0.5 - \frac{1}{\pi}\right) R_B = 0 \quad \text{—} (3) \quad 2$$

solving (1), (3) :

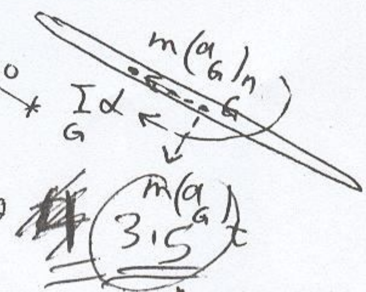
$$\therefore R_A = 12.732\ N \longrightarrow$$

$$R_B = 12.732\ N \longleftarrow \quad 6$$

$\boxed{3}$ $m = 15\,kg,\ l = 2.0\,m$



F.B.D.



3.5

K.D.

$$I_A = I_G + m\left(\frac{l}{4}\right)^2$$

$$= \frac{ml^2}{12} + m\left(\frac{l}{4}\right)^2$$

$$= \frac{15(2)^2}{12} + 15(0.5)^2$$

$$\therefore I_A = \underline{8.75\ kg.m^2}$$ 2

Rotation about a fixed axis:

$\Sigma M_A = I_A\alpha$:

$$\therefore mg\cos\theta \cdot \frac{l}{4} = 8.75\alpha$$

$$\therefore \alpha = 8.409\cos\theta \quad (1)$$ 1.5

$$\therefore \text{at } \theta = 60^\circ:\ \alpha = \underline{4.204\ rad/s^2}$$ 2

$\Sigma F_n = m(a_G)_n$:

$$\therefore A_n - mg\sin\theta = m(0.5\omega^2) \quad (2)$$ 1.5

$\Sigma F_t = m(a_G)_t$:

$$\therefore A_t + mg\cos\theta = m(0.5\alpha) \quad (3)$$ 1.5

for eqn (1):

$$\omega\frac{d\omega}{d\theta} = 8.409\cos\theta$$

$$\therefore \int \omega\, d\omega = 8.409\int\cos\theta\, d\theta$$

$$\therefore \frac{\omega^2}{2} = 8.409\sin\theta + c_1$$

$$\because \omega = 0 \text{ at } \theta = 0 \Rightarrow c_1 = 0$$

$$\therefore \omega^2 = 16.818\sin\theta$$

at $\theta = 60^\circ$: $\omega^2 = \underline{14.565} \Rightarrow \omega = \underline{3.816\ rad/s}$

3

17/8

sub. for α, w and θ in (2), (3):

$$\therefore A_t = -42.045\,N$$

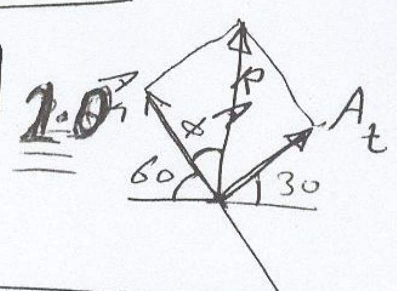
$$\therefore A_t = \underline{42.045\,N} \quad 60° \quad 2.0$$

and:

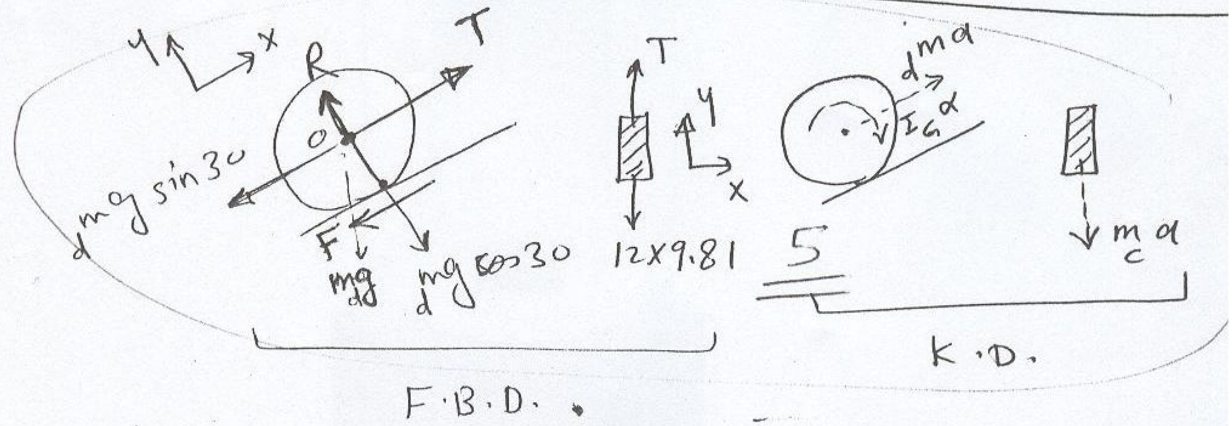
$$A_n = \underline{236.673\,N} \quad 60° \quad 2.0$$

$$\therefore R_A = \sqrt{A_n^2 + A_t^2} = \boxed{240.379\,N} \quad 2$$

$$\phi = \tan^{-1} \frac{42.045}{236.673} = \boxed{10.074°} \quad 2.0$$



4



For the disc: (G.P.M.)

The disc moves up the incline

$$a_c = a_o = a \quad 1$$

$\Sigma F_x = m a_x$:

$$\therefore T - 10 \times 9.81 \sin 30 - F = 10a$$

$$\therefore T - 49.05 - F = 10a \quad (1) \quad 1$$

$\Sigma F_y = m a_y$:

$$R - 10 \times 9.81 \cos 30 = 0$$

$$\therefore R = \underline{84.957\,N} \quad (2) \quad 1$$

$\overset{+}{\curvearrowleft}\ \sum M_o = I_o \alpha$: $\therefore -Fr = \frac{-1}{2} m_d r^2 \alpha$

$\therefore F = +1.5\alpha$ — (3) 1

For the cylinder: Translation:

$\sum F_y = m a_y$:

$\therefore 12 \times 9.81 - T = 12a$

$\therefore T = 117.72 - 12a$ — (4) 1

Assume pure rolling:

$\therefore a = \alpha r$ → (5) 1
$= 0.3\alpha$

sub. (5) in (4):

$\therefore T = 117.72 - 3.6\alpha$ — $\boxed{6}$

-3.6
$+1.5$
-2.1

sub. (3), (6), (5) in (1):

$\therefore (117.72 - 3.6\alpha) - 49.05 - 1.5\alpha = 3\alpha$

$68.67 = 8.1\alpha \Rightarrow \alpha = 8.478 \text{ rad/s}^2$

14

$\therefore F = 12.717 N$

$F_{max} = \mu_s R = 0.14R = 11.894 N$ 1

$\because F > F_{max}$.

$\therefore$ slipping will occur 2

$\therefore F = \mu_k R = 10.195 N$ → (7) 1

from (3): $\boxed{\therefore \alpha = 6.797 \text{ rad/s}^2}$ 2 sub. (7) in (1):

$$\therefore T = 59.245 + 10a \quad (8)$$

from (4), (8):

$$\therefore 117.72 - 12a = 59.245 + 10a$$

$$\therefore a = 2.658 \; m/s^2$$

$$= a_{cylinder}$$

$$\boxed{\therefore a_c = 2.658 \; m/s^2 \downarrow}$$

3

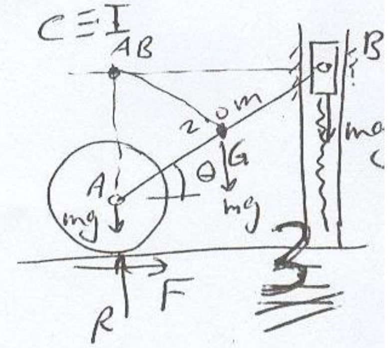
after 3 sec:

$$V = \cancel{V_0}^{\,0} + at$$

$$\therefore V = 2.658(3) = \boxed{7.974 \; m/s}$$

2

5



3

(a) $W_{1\to 2} = T_2 - T_1$ — (1) 1

$$T_1 = \frac{1}{2} m_B \cancel{v_{B_1}^2}^{\,0} + \frac{1}{2} I_C \cancel{\omega_{AB_1}^2}^{\,0} + \left(\frac{1}{2} m_d \cancel{v_{A_1}^2}^{\,0} + \frac{1}{2} I_A \cancel{\omega_{d_1}^2}^{\,0}\right) = 0 \quad 1 \quad (2)$$

$$T_2 = \frac{1}{2} m_B V_{B_2}^2 + \frac{1}{2} I_C \omega_{AB_2}^2 + \left(\frac{1}{2} m_d V_{A_2}^2 + \frac{1}{2} I_A \omega_{d_2}^2\right) \quad 1 \quad (3)$$

but:

$$V_B = \omega_{AB} \cdot \overline{BI} \Rightarrow \therefore \omega_{AB} = \frac{V_B}{\overline{BI}} = \frac{V_B}{2\cos 30} = 0.577 V_B$$

and:

$$V_A = \omega_d \cdot r = 0.3\, \omega_d$$

but:

$$V_A = \omega_{AB} \cdot \overline{AI} = \omega_{AB} * 1.0 = 0.577 V_B \quad (5) \quad 1$$

$$\therefore 0.3\omega_d = 0.577 V_B \rightarrow \therefore \omega_d = 1.925 V_B \quad 1 \quad (6)$$

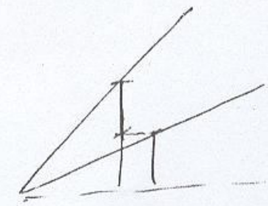
sub from (4), (5), (6) in (3):

$\overline{CG} = \frac{1}{2} AB = 1.0\,m$

$$\therefore T_2 = \frac{1}{2}(18) V_{B_2}^2 + \frac{1}{2}\left[\frac{10\times(2)^2}{12} + 10(1)^2\right](0.577 V_{B_2})^2$$

$$+ \frac{1}{2}(5)(0.577 V_{B_2})^2 + \frac{1}{2}\left[\frac{1}{2}(5)(0.3)^2\right](1.925 V_B)^2$$

$$T_2 = V_{B_2}^2 (12.469) \quad \text{--- } 3 \text{ --- (7)}$$



$$W_{1\to 2} = 10 \times 9.81(\sin 70 - \sin 30)$$

$$+ 18 \times 9.81 \times 2(\sin 70 - \sin 30) \qquad 3$$

$$- \frac{1}{2} \times 90\left[2(\sin 70 - \sin 30)\right]^2 = \cancel{238.15}\ 163.44\ J \text{ --- (8)}$$

$\sin 70 - \sin 30 = 0.440$

sub from (2), (7), (8) in (1):

$$\therefore 163.44 = 12.469 V_{B_2}^2 \Rightarrow \boxed{\therefore V_{B_2} = 4.370\,m/s \downarrow} \quad 2$$

(b) $W_{1\to 2} = T_2 - T_1$ --- (1) $\quad 1$

$T_1 = 0$ $\cancel{1}$ --- (2) & $T_2 = 0$ --- (3) $\quad 1$

$$W_{1\to 2} = 10 \times 9.81 \sin 70 + 18 \times 9.81 \times 2 \sin 70 - \frac{1}{2} k \times (2 \sin 70)^2$$

$$= 424.046 - 1.750\,k \text{ --- (4)} \quad 3$$

sub in (1):

$$\therefore 424.046 - 0.94 k = 0$$

$$\boxed{\therefore k = \cancel{451.113}\ 239.96\ N/m} \quad 2$$

25

14 (a) 8 (b)