ayoum University	**Civil Engineering**	**Second Year**
Faculty of Engineering	**Final Term Exam Answer**	**Numerical Analysis**
Time: 3 hours		**19-1-2009**

1-Fit the curve $y=1/(a+bx^2)$ for the following readings: (10points)

x	1	2	4	6
y	0.5	0.42	0.26	0.18

Hence compute dy/dx at x =0.5

a=2.0021

b=0.1013

dy/dx(0.5)=0.02464449

2-Define sin(A), ln(I-A)and e^A. (3points)

$$\sin(A) = TD_{\sin\lambda}T^{-1}$$

$$\ln(I-A) = TD_{\ln(1-\lambda)}T^{-1}$$

$$e^A = TD_{e^\lambda}T^{-1}$$

3-If

$$A = \begin{pmatrix} 5 & -2 & 4 \\ -5 & 0 & 0 \\ 4 & -2 & 5 \end{pmatrix}$$

i) Find the inverse of A (2points)

$$A^{-1} = \begin{pmatrix} 0 & -0.2 & 0 \\ -2.5 & -0.9 & 2 \\ -1 & -0.2 & 1 \end{pmatrix}$$

ii) Evaluate the eigenvalues and the eigenvectors (6points)

$$\lambda_1 = 1.0, \begin{pmatrix} -2 \\ 10 \\ 7 \end{pmatrix}, \lambda_2 = 10.0, \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}, \lambda_3 = -1.0, \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix}$$

iii) Compute the determinant of $A^{10}+I$ (3points)

$$\left|A^{10}+I\right| = 2*2*(10^{10}+1)$$

4-Are the following set of vectors linearly independent or dependent? If the answer is independent, transform them to orthogonal ones.

$(0 \;\; 0 \;\; 1)^T \; (-2 \;\; 1 \;\; 0)^T \; (1 \;\; -2 \;\; 1)^T$ (10points)

It is linear independent

$$Y_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, Y_2 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, Y_3 = \begin{pmatrix} -0.6 \\ -1.2 \\ 0 \end{pmatrix}$$

5- Find the conditions for the matrix A to have an eigenvalue equal 2, hence compute the other eigenvalues (6points)

$$\text{If } A = \begin{pmatrix} 2 & 0 & h \\ 1 & 4 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

$h \in R$

$\lambda = 2,4,2$

6- Use Runge Kutta 2 's method to find y at x=0.1 if $y' = (1+y^2)\tan x$ and $y(0)=\sqrt{3}$. (10point)

$$k_1 = 0, k_2 = .0401339, k_{avr} = .02006695$$
$$y = 1.752117742$$

7-Explain how to fit the curves

i)y=sin(a+bx)

$$a + bx = \sin^{-1} y$$

ii)y=ln10-ln(c+dx) (show how to find the coefficient a, b, c and d without calculation) (6 points)

$$e^{-y} = (c + dx)/10$$

8-Show how to solve the differential equation using Runge Kutta 4's method

$$2(\frac{dy}{dx})^2 + 5y\frac{dy}{dx} + 3y^2 = 0$$ (6 points)

$$\frac{dy}{dx} = 1.5y$$
$$\frac{dy}{dx} = y$$

9- Use Modified Euler's method to find y at x=-1.8 6 if $y' = 2x + 3y + 1$ and y(-2)=2 (four steps are enough) (8points)

$\Delta y_1 = 0.42$
$\Delta y_1 = 0.5278$
$\Delta y_1 = 0.5278$
$\Delta y_1 = 0.5504$
$y = 2.5504$