FAYOUM UNIVERSITY
FACULTY OF ENGINEERING
January, 17th, 2010

Fluid Mechanics
2nd Year Civil
Time: 3 hours

- **Start every question in a new page.**

(1) (a) A vertical concentric annulus, with outer radius $(\mathbf{r_o})$ and inner radius $(\mathbf{r_i})$, is lowered into fluid of surface tension $(\boldsymbol{\sigma})$ and contact angle $(\boldsymbol{\theta < 90^\circ})$. Derive an expression for the capillary rise (**h**) in the annular gap, if the gap is very narrow.

$\Sigma F_y = 0$

$\sigma \cos\theta\,(2\pi r_o + 2\pi r_i) = \gamma \pi (r_o^2 - r_i^2) h$

$\therefore \; h = \dfrac{2\sigma\cos\theta}{\gamma(r_o - r_i)}$

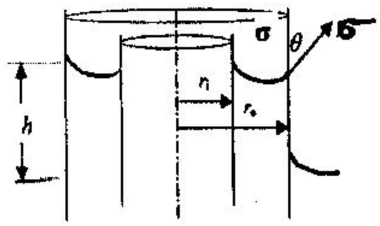

Figure 1

(b) A layer of water flows down an inclined fixed surface with the velocity profile shown in **figure 2**. Determine the magnitude and direction of the shearing stress that the water exerts on the fixed surface for **U = 2 m/s** and **h = 0.1 m**. $(\mu = 1.12 \times 10^{-3})$

$\tau = \mu \dfrac{du}{dy}$

$\dfrac{du}{dy} = U\left(\dfrac{2}{h} - \dfrac{2y}{h^2}\right)$

at $y = 0$ $\left.\dfrac{du}{dy}\right)_{y=0} = \dfrac{2U}{h}$

$\therefore \; \tau = \mu\left(\dfrac{2U}{h}\right) = 1.12 \times 10^{-3} \times 2 \times \dfrac{2}{0.1}$

$= 4.48 \times 10^{-2}\ N/m^2$ in the flow direction

4 μ

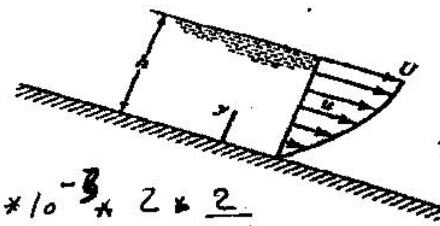

Figure 2

$\dfrac{u}{U} = 2\dfrac{y}{h} - \dfrac{y^2}{h^2}$

$u = U\left(\dfrac{2y}{h}\right.$

(2) (a) A **Circular** gate **ABC (figure 3)** is hinged at **B**. Compute the force **P** just sufficient to keep the gate from opening when h = 8.0 m. Neglect atmospheric pressure.

$F = \gamma A \bar{h} = 9810 \times \pi r^2 \times 8 = 8\pi\ t = 25.13\ t$

$= 246552.2\ N$

$z = \dfrac{I_x}{A\bar{y}} = -\dfrac{\frac{\pi}{4}(1)^4}{8\pi} = 0.03125\ m$

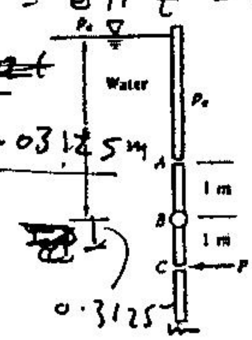

Figure 3

$\Sigma M)_B = 0$ $\quad P \times 1.0 = 246552.2 \times 0.03125$

$\therefore \; P = 7704.76\ N = 0.7856\ t$

$7704.76\ N$

(b) Analyze problem **(2 – a)** for arbitrary depth (**h**) and gate radius (**R**) and derive a formula for the opening force (**P**). Prove that the force **P** is not dependent on the depth **h**.

$F = \gamma A \bar{h} = \gamma \pi R^2 h$

$z = -\dfrac{I}{A\bar{h}} = -R^2/4h$

$\Sigma M)_B = 0.0 = \underbrace{(\gamma \pi R^2 h)}_{F} \times \dfrac{R^2}{4h} = PR$

$\therefore \; \boxed{P = \dfrac{\gamma \pi R^3}{4}}$ independent on

(b) The tank in **Figure 7** is being filled with water by two inlets. The area of the tank is A_t. The water height is **(h)**. Compute **(dh/dt)** for $D_1 = 5.0$ **cm**, $D_2 = 7.0$ **cm**, $V_1 = 1.0$ **m/s and** $V_2 = 0.7$ **m/s and** $A_t = 0.2\ m^2$.

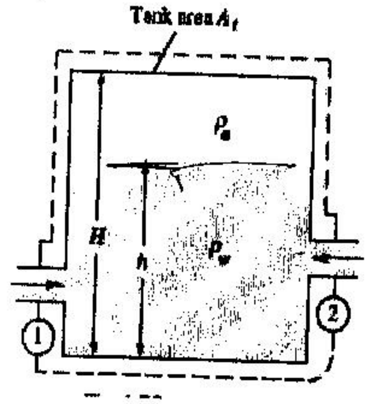

Figure 7

14.95
118.45
22.50

(c) A closed rectangular tank **1.2 m high, 2.5 m long, and 1.5 m wide** is filled with water. The pressure at the top is raised to **60 KN/m²**. Calculate the pressure at the corners of this tank when it is decelerated horizontally along the direction of its length at **5.0 m/s²**. Calculate the force on the ends of the tank, and check their difference by **Newton's law.**

Best Wishes

Examiners:

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$\tan\theta = \frac{a_x}{g} = \frac{h}{2.5}$

$\therefore \frac{5}{9.81} = \frac{h}{2.5} \rightarrow h = 1.275\ m$

$P_A = \gamma h + 60 = 72.495\ kN/m^2$

$P_B = \gamma(h+1.2) + 60 = 84.255\ t/m^2$

~~$P_C = 60.0\ kN/m^2$~~

$P_D = \gamma(1.2) + 60 = 71.76\ kN/m^2$

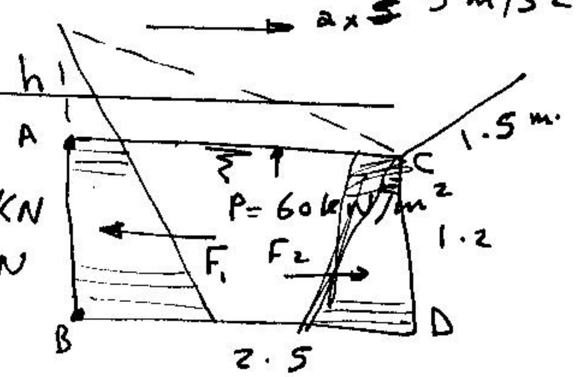
$P_A = 72.495\ kN$

$P_B = 84.255\ kN$

$P_C = 60\ kN/m^2$

$P_D = 71.76\ kN/m^2$

$\longrightarrow a_x = 5\ m/s^2$



$F_1 = F_{AB} = \left(\frac{P_A + P_B}{2}\right) * 1.2 * 1.5 = 140.951\ kN$

$F_2 = \left(\frac{P_C + P_D}{2}\right) * 1.2 * 1.5 = 118.4517\ kN$

$\Delta F = F_1 - F_2 = 22.5\ kN \rightarrow (1)$

$\Sigma F_x = m * acc. = \frac{\gamma}{9.81} * 2.5 * 1.2 * 1.5 * 5 = 22.5\ kN \rightarrow (2)$

from (1) & (2) $\therefore \Sigma F_x = m * acc. = \Delta F$ ok

ا.د. محمد حسن
[illegible]

Model Answer

① (a) $\sum F_y = 0.0 \qquad F_\sigma \uparrow = w \downarrow$

$$\sigma \cos\theta \, (2\pi r_o + 2\pi r_1) = \gamma (\pi r_o^2 - \pi r_1^2) h$$

$$\therefore \boxed{h = \frac{2\sigma \cos\theta}{\gamma (r_o - r_1)}}$$

(b) $\tau = \mu \frac{du}{dy}$ & $u = U\left(\frac{2y}{h} - \frac{y^2}{h^2}\right) \Rightarrow \frac{du}{dy} = U\left(\frac{2}{h} - \frac{2y}{h^2}\right)$

$$\left.\frac{du}{dy}\right|_{y=0} = \frac{2U}{h} = \frac{2 * 2}{0.1} = 40 \ \text{1/sec.}$$

$\therefore \tau = \underline{40\mu} = \underline{40 * 1.12 * 10^{-3}} = \underline{4.48 * 10^{-2}} \ N/m^2$ in flow direc.

② (a) $F = \gamma A \bar{h} = 9810 * \pi (1)^2 * 8 = \underline{246552.2 \ N} = \underline{25.13} \ t = 8\pi \ t$

$$y_p = \frac{I_{c.g}}{A\bar{y}} + \bar{y} = \frac{\pi (1)^4/4}{\pi (1)^2 * 8} + 8 = \underline{\underline{8.03125 \ m.}}$$

$$z = 0.03125 \ m$$

$\sum M_B = 0.0 \qquad F * z = P * r$

$$F * 0.03125 = P * 1 \longrightarrow P = \underline{7704.76 \ N} = 0.785 \ t = \frac{\pi}{4}$$

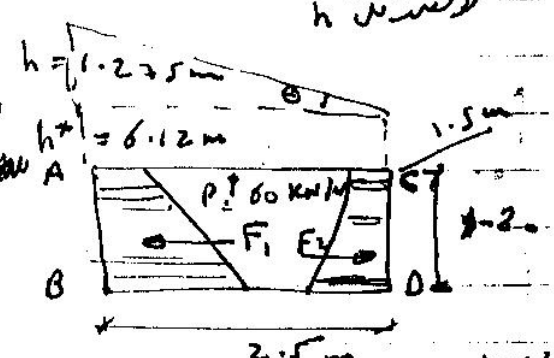
(b) $F = \gamma A \bar{h} = \gamma * \pi R^2 * h = \gamma \pi R^2 h$

$$z = \frac{I_{c.g}}{A\bar{y}} = \frac{\pi R^4/4}{\pi R^2 * h} = \frac{R^2}{4h}$$

$$\sum M_B = 0.0 \quad \therefore P * R = \gamma \pi R^2 h * \frac{R^2}{4h} \Longrightarrow \boxed{P = \frac{\gamma \pi R^3}{4}}$$

h [illegible]

⑥ $\tan\theta = \frac{a_x}{g} = \frac{h}{2.5} \longrightarrow \underline{h = 1.275 \ m}$



$F_1 = F_{AB} = \frac{(P_A + P_B)}{2} * 1.2 * 1.5 = \underline{140.951 \ KN}$

$F_2 = F_{CD} = \frac{(P_C + P_D)}{2} * 1.2 * 1.5 = \underline{118.4517 \ KN}$

$\Delta F = F_1 - F_2 = \underline{22.5} \ KN \longrightarrow ①$

$\sum F_x = \text{mass} * \text{acc.} = \frac{\gamma}{g} * 2.5 * 1.2 * 1.5 * 5$

$$= \frac{9.81}{9.81} * 2.5 * 1.2 * 1.5 * 5 = \underline{22.5 \ KN} \longrightarrow ②$$

$P_A = \gamma h + 60 = \underline{72.495} \ KN/m^2$

$P_B = \gamma (h + 1.2) + 60 = \underline{84.255}$

$P_C = \underline{60} \ KN/m^2$

$P_D = \gamma (1.2) + 60 = \underline{71.76} \ KN/m^2$

from ① & ②

$\sum F_x = \text{mass} * \text{acc.} = \Delta F$ ※.

Question 3 (b)

$F_{h_1} = (1.0)\ (2)\ (4 * 8) = 64t$

$F_{h_2} = (1)\ (2 * 8) = 16t$

$F_{V_1} = \frac{\pi}{2}\ (4)\ (8) = 16\pi = 50.27t$

$F_{V_2} = \frac{\pi}{4}\ (4)\ (8) = 8\pi = 25.13t$

$R_H = 64 - 16 = 48t$

$R_V = (0.8)\ 4\pi\ (8) - 8\pi - 16\pi = 1.6\pi = 5.03t$

Question 4 (b)

$MB = \frac{I}{\forall} = \frac{(1)(4.6)^3/12}{(1.2)(4.6)} = 1.47\ m$

$BG = 0.45 + 1.2 = 1.65\ m$

Since $BG < MB \longrightarrow$ unstable

Question 5 (a)

$0.6\ \gamma_w\ \forall_{ol} = \gamma_{Fluid}\ (0.75\ \forall_{ol}) \Rightarrow \gamma_{fluid} = 7832\ N/m^3$

Air Pressure $\Rightarrow \quad 0 - (7832)\ (0.4) = P_{air}$

$\Rightarrow -3130$ (Pa, gauge) $[N/m^2]$

Question 5 (b)

$$\frac{dh}{dt} = \frac{A_1 V_1 + A_2 V_2}{A_t} = \frac{Q_1 + Q_2}{A_t}$$

$Q_1 = 1.963\ l/s \quad , \quad Q_2 = 2.694\ l/s$

$$\frac{dh}{dt} = \frac{4.657}{(0.2)(1000)} = 0.0233\ m/s = 2.33\ cm/s$$