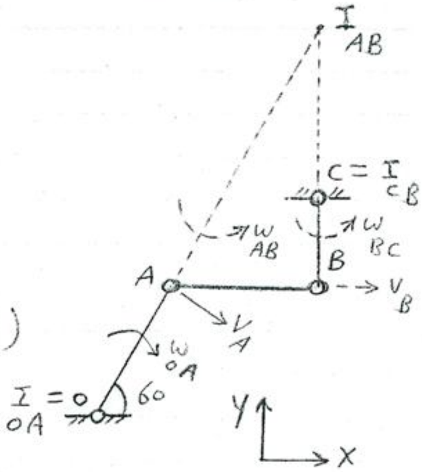
* Eng. Mechanics – Jan. 2011 – Model Answer *

1 . **26**

(a): (8)



$$\underline{V}_B = \underline{V}_A + \underline{V}_{B/A} \quad (1)$$

$$\underline{V}_A = \underline{\omega}_{OA} \times \underline{r}_{A/O}$$

$$= -4\underline{k} \times (1.3\cos 60\, \underline{i} + 1.3 \sin 30\, \underline{j})$$

$$\therefore \underline{V}_A = 4.503\,\underline{i} - 2.6\,\underline{j} \quad (2)$$

$$\underline{V}_B = \underline{\omega}_{BC} \times \underline{r}_{B/C} = \omega_{BC}\, \underline{k} \times (-1.0\,\underline{j})$$

$$\therefore \underline{V}_B = +\omega_{BC}\, \underline{i} \quad (3)$$

$$\underline{V}_{B/A} = \underline{\omega}_{AB} \times \underline{r}_{B/A} = \omega_{AB}\, \underline{k} \times 1.5\,\underline{i} = 1.5\,\omega_{AB}\, \underline{j} \quad (4)$$

sub. from (2), (3), (4) in (1):

$$\therefore \omega_{BC}\, \underline{i} = 4.503\, \underline{i} + (1.5\,\omega_{AB} - 2.6)\, \underline{j}$$

$$\therefore \boxed{\omega_{BC} = 4.503 \text{ rad/s} \circlearrowleft} \quad \& \quad \boxed{\omega_{AB} = 1.733 \text{ rad/s} \circlearrowleft}$$

(b): (5)

$$V_A = \omega_{OA} \cdot \overline{OA} = 4 \times 1.3 = 5.2 \text{ m/s}$$

but; $V_A = \omega_{AB} \cdot \overline{AI_{AB}} = 3\,\omega_{AB}$

$$\therefore 3\,\omega_{AB} = 5.2 \implies \boxed{\omega_{AB} = 1.733 \text{ rad/s} \circlearrowright}$$

$$\therefore V_B = \omega_{AB} \cdot \overline{BI_{AB}} = 1.733 * 3 \sin 60 = 4.503 \text{ m/s}$$

but: $V_B = \omega_{BC} \cdot \overline{BC} = 1.0 * \omega_{BC}$

$$\therefore \boxed{\omega_{BC} = 4.503 \text{ rad/s} \circlearrowleft}$$

(c): (13)

$\because \underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$\therefore (\underline{a}_B)_n + (\underline{a}_B)_t = (\underline{a}_A)_n + (\underline{a}_A)_t + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$(a_B)_n = \omega_{BC}^2 \cdot \overline{BC} = 20.28 \text{ m/s}^2 \uparrow$

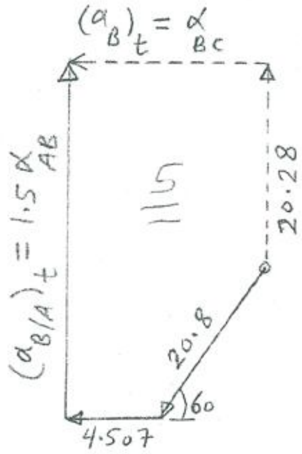
$(a_B)_t = \alpha_{BC} \cdot \overline{BC} = \alpha_{BC} \longleftrightarrow$ 6

$(a_A)_n = \omega_{OA}^2 \cdot \overline{OA} = 20.8 \text{ m/s}^2$ $\measuredangle 60°$

$(a_A)_t = \alpha_{OA} \cdot \overline{OA} = 0.0$

$(a_{B/A})_n = \omega_{AB}^2 \cdot \overline{AB} = 4.507 \text{ m/s}^2 \leftarrow$

$(a_{B/A})_t = \alpha_{AB} \cdot \overline{AB} = 1.5\, \alpha_{AB} \updownarrow$

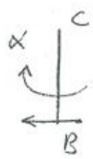


The Acceleration Diagram is as shown

i-eqn:

$$-20.8 \cos 60 - 4.507 = -\alpha_{BC}$$

$$\therefore \boxed{\alpha_{BC} = 14.907 \text{ rad/s}^2 \circlearrowright}$$ 2

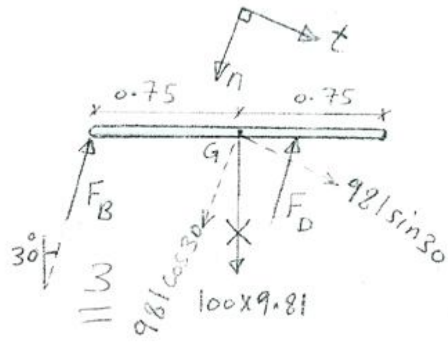


2 **16** Curvilinear translation.

$\Sigma F_t = m(a_G)_t$:

$\therefore 100 \times 9.81 \sin 30 = 100 \times \overline{AB} \times \alpha_{AB}$

$\therefore \boxed{\alpha = 6.131 \ rad/s^2}$ 3

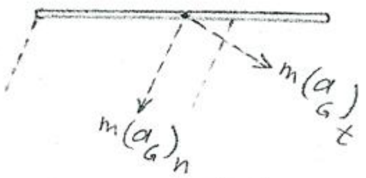


F.B.D.

$\Sigma F_n = m(a_G)_n$:

$\therefore 981 \cos 30 - F_B - F_D = \omega^2_{AB} \cdot \overline{AB} = 0.0$

$\therefore F_B + F_D = 849.571$ —— ① 3



K.D.

$\Sigma M_G = 0.0$:

$\therefore (F_B \cos 30)(0.75) - (F_D \cos 30)(0.25) = 0.0$

$\therefore F_D = 3 F_B$ —— ② 3

from ①, ② :

$\therefore 4 F_B = 849.571$

2 $\therefore \boxed{F_B = 212.393 \ N \ \angle 60}$ (compression)

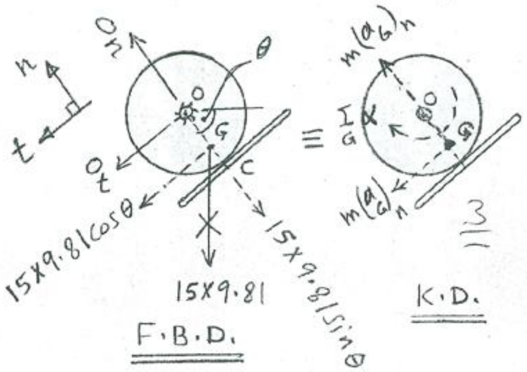
2 $\therefore \boxed{F_D = 637.178 \ N \ \angle 60}$ (compression)

3 | 18

$OG = 0.40\,m$

$I_o = \frac{1}{2}(5)(0.6)^2 + \left[\frac{10(1.5)^2}{12} + 10(0.6)^2\right]$

$\therefore I_o = 6.375\ kg.m^2$ 3



$\Sigma M_o = I_o \alpha$:

$\therefore (15 \times 9.81 \cos\theta)(0.4) = 6.375\alpha$

$\therefore \alpha = 9.233 \cos\theta$ —— (1)

sub. $\alpha = \omega \frac{d\omega}{d\theta}$ in (1) and integrate, we get:

$\frac{\omega^2}{2} = 9.233 \sin\theta + c_1$

$\because \omega = 0$ at $\theta = 0 \Rightarrow c_1 = 0 \Rightarrow \omega^2 = 18.466 \sin\theta$ —— (I)

at $\theta = 90°$: $\alpha = 0.0\ ,\ \omega^2 = 18.466$ —— (II)

$\Sigma F_t = m(a_G)_t$:

$\therefore O_t + 15 \times 9.81 \cos\theta = 15\alpha(0.4) = 6\alpha$ —— (2)

sub. $\theta = 90°$ & $\alpha = 0.0 \Rightarrow O_t = 0.0$

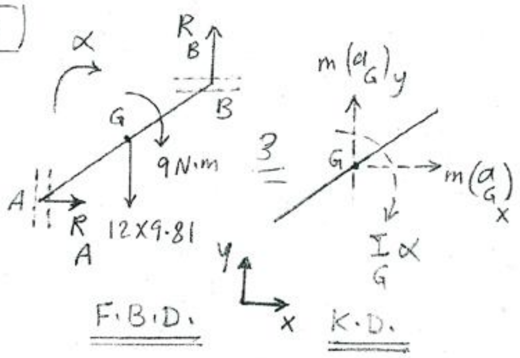
$\Sigma F_n = m(a_G)_n$:

$\therefore -15 \times 9.81 \sin\theta + O_n = 15 \times 0.4\omega^2 = 6\omega^2$ —— (3)

sub. $\theta = 90°$, $\omega^2 = 18.466 \Rightarrow O_n = 257.95N \uparrow$

$\therefore R_o = \sqrt{O_n^2 + O_t^2} = 257.95N \uparrow$

4 General plane Motion [20]



$\Sigma F_x = m(a_G)_x$:

$$\therefore R_A = 12\, a_x \quad —(1) \qquad 2$$

$\Sigma F_y = m(a_G)_y$:

$$\therefore R_B - 12 \times 9.81 = 12 a_y$$

$$\therefore R_B - 117.72 = 12 a_y \quad —(2) \qquad 2$$

$\overset{+}{\curvearrowright}$ $\Sigma M_G = I_G \alpha$:

$$\therefore -0.5 R_A - \frac{\sqrt{3}}{2} R_B + 9 = \frac{12(2)^2}{12}\alpha = 4\alpha$$

$$\therefore R_A + \sqrt{3}\, R_B = 18 - 8\alpha \quad —(3) \qquad 2$$

To get a_x, a_y:

$$\underline{a}_A = \underline{a}_B + \cancel{(\underline{a}_{A/B})_n}^{\to 0} + (\underline{a}_{A/B})_t$$

$$\therefore a_A \underline{j} = a_B \underline{i} + (-\alpha \underline{k}) \times (-\sqrt{3}\, \underline{i} - 1.0 \underline{j})$$

$$= (a_B - \alpha)\, \underline{i} + \sqrt{3}\alpha\, \underline{j}$$

$$\therefore a_A = \sqrt{3}\,\alpha \quad \& \quad a_B = \alpha$$

$$\underline{a}_G = \underline{a}_B + \cancel{(\underline{a}_{G/B})_n}^{\to 0} + (\underline{a}_{G/B})_t$$

$$= \alpha \underline{i} + (-\alpha \underline{k}) \times \left(-\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}\right)$$

$$\therefore \underline{a}_G = 0.5\alpha\, \underline{i} + 0.5\sqrt{3}\,\alpha\, \underline{j}$$

$$\therefore \boxed{a_x = 0.5\alpha} \quad \& \quad \boxed{a_y = 0.5\sqrt{3}\,\alpha} \qquad 4$$

Sub. a_x, a_y in ①, ②:

$\therefore R_A = 6\alpha$ — ①' & $R_B = 117.72 + 6\sqrt{3}\,\alpha$ — ②'

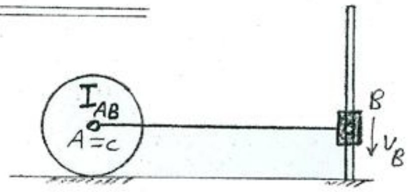
sub. from ①', ②' in ③:

$\therefore 185.897 = -32\alpha \Rightarrow \therefore \boxed{\alpha = 5.809\ rad/s^2}$ ↺

$\therefore \boxed{R_A = 34.854\,N \leftarrow}$ & $\boxed{R_B = 57.351 \downarrow}$

5 [20]

<u>(a):</u> (12) $W_{1\to 2} = T_2 - T_1$ — ①



$T_1 = 0.0$ — ②

at position(2), the disc will come To rest instantaneously

$\therefore T_2 = \frac{1}{2} m_B V_{B_2}^2 + \frac{1}{2} I_c\, \omega_{AB_2}^2$

$= \frac{1}{2}(10) V_{B_2}^2 + \frac{1}{2} \cdot \frac{12(1.5)^2}{3}\, \omega_{AB_2}^2$

but: $V_B = 1.5\,\omega_{AB}$

$\therefore T_2 = (11.25 + 4.5)\,\omega_{AB_2}^2 = 15.75\,\omega_{AB_2}^2$ — ③

For The spring:

$\ell_0 = 1.5 \sin 20 + 0.05 = 0.563\ m$

$\therefore \Delta x_2 = 1.5 - 0.563 = 0.937\ m$

$\Delta x_3 = 1.5 \sin 75^\circ - 0.563 = 0.886\ m$

$$\therefore W_{1\to 2} = 10\times 9.81\times 1.5\cos 20 + 12\times 9.81\times 0.75\cos 20 - 9\times \frac{70\pi}{180} + \frac{1}{2}\times 20\left[(0.05)^2-(0.937)^2\right]$$

$$\therefore W_{1\to 2} = 138.276 + 82.965 - 10.996 - 8.755 = \underline{\underline{201.49\,N.m}}$$ — (4)

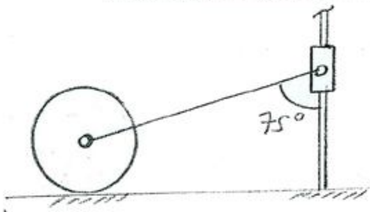
sub. from (2), (3), (4) in (1):

$$\therefore 201.49 = 15.75\, w_{AB_2}^2 \Rightarrow \boxed{w_{AB_2} = 3.577\,rad/s \circlearrowright}$$

(b): (8)

$$W_{1\to 2} = T_2 - T_1 \quad \text{— (1)}$$

$$T_1 = 0 \quad \text{— (2)}, \quad T_2 = 0 \quad \text{— (3)}$$



$$W_{1\to 2} = 10\times 9.81\times 1.5(\cos 20 - \cos 75) + 12\times 9.81\times 0.75(\cos 20 - \cos 75) - 9\times\frac{55\pi}{180} + \frac{1}{2}k\left[(0.05)^2-(0.886)^2\right]$$

$$= 100.191 + 60.114 - 8.639 - 0.391k$$

$$\therefore W_{1\to 2} = 151.666 - 0.391k \quad \text{— (4)}$$

sub. from (2), (3), (4) in (1):

$$\therefore 151.666 - 0.391k = 0$$

$$\therefore \boxed{k = 387.893\,N/m}$$