Mathematical Modelling based on Filippov Systems for Pest Control

By

Doaa Mahmoud Fawzy Abdel Hamid

A Thesis Submitted to the
Graduate School of Basic and Applied Science,

Egypt-Japan University of Science and Technology **(E-JUST)**

In Partial Fulfillment of the Requirements for the Degree of

Master of Science

in

Applied and Computational Mathematics

May 2024

Mathematical Modelling based on Filippov Systems for Pest Control

By
Doaa Mahmoud Fawzy Abdel Hamid

For The Degree
of

Master of Science

in
Applied and Computational Mathematics

Supervision Committee

Title, Name	**Affiliation**	**Signature, Date**
Prof. Waheed Zahra	Applied and Computational Mathematics Program, Basic and Applied Sciences Inst., E-JUST, Egypt.	…………………
Prof. Ahmed Elsaid	Applied and Computational Mathematics Program, Basic and Applied Sciences Inst., E-JUST, Egypt.	…………………
Asst. Prof. Ayman Arafa	Applied and Computational Mathematics Program, Basic and Applied Sciences Inst., E-JUST, Egypt.	…………………

Examination Committee

Title, Name	**Affiliation**	**Approved, Date**
Prof. Gamal Mokhtar Mahmoud	Mathematics department, Faculty of Science, Assiut University	…………………
Prof. Mohamed Soror Abdel Latif Mahmoud	Mathematics department, Faculty of Science, New Mansoura University	…………………
Prof. Waheed Zahra	Applied and Computational Mathematics Program, Basic and Applied Sciences Inst., E-JUST, Egypt.	…………………
Prof. Ahmed Elsaid	Applied and Computational Mathematics Program, Basic and Applied Sciences Inst., E-JUST, Egypt.	…………………

SUMMARY

Mosquito-borne diseases persist globally despite control efforts, urging exploration of innovative strategies such as sterile insect technique (SIT) and incompatible insect technique (IIT). Previous studies used ordinary differential equations to simulate the application of these strategies, aiming to reduce mosquito vectors. However, these studies assumed continuous control implementation, resulting in increased costs.

Inspired by this, we introduce a non-smooth Filippov SIT model with threshold policy control (TPC), examining the interaction between wild and sterile mosquitoes. In this model, sterile mosquitoes are released when wild mosquito density exceeds an economic threshold. We incorporate a density-dependent birth rate for wild mosquitoes and account for immigration effects. Our study reveals complex dynamics, encompassing local sliding bifurcation and bistability, as key parameters, including the threshold value are varied. Additionally, the system demonstrates hysteresis phenomena with varying sterile mosquito release rates. We exclude the existence of three types of limit cycles in the Filippov system. Our results suggest that reducing the threshold value can enhance wild mosquito control effectiveness, emphasizing the economic advantages of employing SIT with TPC to combat disease vectors while improving economic outcomes.

However, SIT faces limitations since sterile insects cannot establish themselves in the environment, requiring continuous laboratory production. To address this, we propose a Filippov IIT model with TPC, which implements biological and chemical control strategies only when the wild mosquito density surpasses an economic threshold. The biological strategy involves releasing Wolbachia-infected male mosquitoes, offering advantages such as shortened lifespan and disease blocking. Concurrently, moderate insecticide and larvicide spraying serve as a chemical approach to expedite control. This model provides economic benefits since the control strategies are not continuously applied, given the costly production and release of Wolbachia-infected mosquitoes. Additionally, it provides environmental benefits by reducing reliance on chemical strategies, combining them with biological ones. Theoretical analysis reveals the model's long-term dynamics, encompassing sliding bifurcations, pseudo-equilibria, and stability. Utilizing Filippov convex method, we identify necessary conditions for the sliding segment existence and determine switching line dynamics. The model exhibits boundary equilibrium and

transcritical bifurcations in both free and controlled systems. Our main findings emphasize the Filippov IIT model's effectiveness and sustainability in managing wild mosquito populations. Integrated strategies efficiently reduce both larvae and adult wild mosquito densities, with varying threshold values significantly impacting control outcomes.

This study highlights the potential of the proposed Filippov SIT and IIT models for controlling mosquito-borne diseases, emphasizing the importance of integrated strategies for economic and sustainable results.