ON POWER GRAPHS AND LABELINGS

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Abstract

This work is a contribution to the study of two problems of graph theory, namely: power graphs and graph labeling. Graph products are viewed as a convenient method to describe the structure of a graph in terms of its factors. There are three products - Cartesian, strong and lexicographic product which have many applications and theoretical interpretations. These products have the property that projection into at least one factor is a weak homomorphism. For this reason, the three standard products are most extensively studied and have the widest range of applications. When dealing with product graphs, one of the main source of reference is the book by R. Hammack et al. [15]. An interconnection network may be modelled by a simple graph whose vertices represent components of the network and the edges represent physical communication links. A basic feature of a network is that its components are connected by physical communication links to transmit information according to some pattern. Many graph theoretic techniques can be used to study the efficiency and reliability of a network. v For designing large-scale interconnection networks, the product graph operation is an important method to obtain large graphs from smaller ones, with a number of parameters that can be calculated from the corresponding parameters of the factor graph. The r th power of a graph G, denoted by Gr , is the graph with vertex set V (G) where two vertices are adjacent if they are within distance r in G, i.e., the length of the shortest path joining them is at most r. Here, we determine the r th power of the graph G∗H in terms of Gr , Hr and Gr ∗ Hr , when ∗ is the join, Cartesian, symmetric difference, disjunctive, composition, skew and corona product. Then we solve the equation (G ∗ H) r = Gr ∗ Hr . Graph labeling is one major research area in graph theory. New results are being discovered and published at a rapidly increasing rate. Further we have an enormous number of open problems and conjectures on graph labelings. For an excellent and up to date dynamic survey on graph labeling we refer to Gallian [17]. Most of the graph labeling methods trace their origin to the concept of the ones presented in 1967 by Rosa [37]. One of these labelings, β−labeling, originated as a means of attacking the conjecture of Ringel [36], which states that the complete graph K2m+1 can be decomposed into 2m+1 copies of a given tree of size m. The same concept was introduced by Golomb who called it a graceful labeling [13]. Various vi types of graph labelings such as graceful labeling, harmonious labeling, cordial labeling, arithmetic labeling, Skolem graceful labeling, magic labeling, antimagic labeling, α−labeling, multiplicative and strongly multiplicative labeling, have been investigated by several authors. The concept of graph labeling has a wide range of applications to other branches of science such as X-ray crystallography, coding theory, cryptography, astronomy, circuit design and communication networks design. Here, we study labelings related to graceful labelings that give some approaches to Ringel's conjecture [36], as well as to another conjecture by Graham and H¨aggkvist that, in a weak form, asks for the decomposition of a complete bipartite graph by a given bipartite graph of appropriate size