Statistical Inferences for Weibull-Exponential Model: Bayesian and non-Bayesian Approaches

A Thesis

Submitted to Department of Mathematics
Faculty of Science (Girls), Al-Azhar University
For the degree of Ph.D in Mathematics (Mathematical Statistics)

By

Heba Nagaty Mohammed Mahmoud
Assistant Lecturer - Basic Science Department- Faculty of Computers and Information- Fayoum University

Supervised By

Prof. Dr. Mohamed Abdel Wahab Mahmoud
Professor of Mathematical Statistics
Department of Mathematics
Faculty of Science (Men)
Al-Azhar University

Prof. Dr. Lamiaa Sabry Gad Diab
Professor of Mathematical Statistics
and Head of Math Department Faculty of Science (Girls)
Al-Azhar University

Prof. Dr. Nabila Mohamed Hassan Mostafa
Professor of Basic Science Department –
Faculty of Computers and Information
Fayoum University

2021

Summary

Reliability theory played an important role in industrial and system engineering. Customers and users are always looking for high quality and long life products. In measuring reliability, we are observing lifetimes of products of interest. During the life-test, successive times to failure are noted and lifetime data are collected from the tested sample can be divided into two basic parts. First, the data collected from the complete samples, in which the time of life failure is recorded for each unit of the tested sample. Second, the data collected from the censored samples, in which it some of the units will be removed or withdrawn from the test and it is difficult to record the time of life failure of all tested sample units.

The researcher can make statistical inferences more accurately for different samples. Statistical inference is divided into two major areas parameter estimation and tests of hypotheses. The objective of statistical inference is to make an inference about a population based on information contained in a sample taken from that population. Any function of the sample is called a statistic. Statistics is used to estimate the unknown parameter of the population by point estimation or interval estimation. Estimation can be classified into two categories non-Bayesian (Classical) estimation and Bayesian estimation. While tests of hypotheses are used to make a correct decision about specific hypotheses for population parameters.

Censoring is an important feature and plays a vital role in lifetime data analysis. There are a number of censoring schemes available to the experimenter. The conventional Type-I censoring has the advantage of fixing the experiment time but it may result in a very few or even no observation. Another disadvantage of this type of censoring is the randomness of number of failures, which may result in a complicated likelihood function of parameters. While the conventional Type-II censored sample contains a specified number of failures and the failure times of remaining units are unobserved. Fixing the number of failures would make the duration of the completion of the life test random and so the test time would also be unknown, which is a disadvantage of the conventional Type-II censoring.

One of the drawbacks of the conventional Type-I or Type-II censoring

schemes is that they do not allow for removal of units at points other than the terminal point of the experiment. To avoid the disadvantage of the conventional Type-I or Type-II censoring schemes, another censoring scheme has been introduced which allows the removal of the unit at any point during the experiment and is called the progressive Type-II censoring scheme. The reason behind its popularity is that it provides the facility of removals of the units during the experiments and reduces the cost and experiment times both.

The main aim of this thesis is to obtain statistical inference about the parameters of WED using different types of censoring schemes namely PRO-II-C, JPRO-II-C, PRO-I-F-C and A-II-PRO-C.

The whole thesis is divided into six Chapters in addition to the list of references and two summaries, one in Arabic and the other in English. Among them, the first Chapter, basic concepts, some important definitions, different types of censoring and notations are given. The Bayesian and non-Bayesian approaches with some related topics are described in details.

In Chapter 2 point and interval estimations for the parameters of k WE populations based on JPRO-II-C samples are obtained. The ML, Bayes, and parametric bootstrap methods are used to obtain point estimations for the k distribution parameters. Furthermore, the ACIs, Bayes estimates and there credible intervals are derived under the SEL function and LINEX loss function using MCMC technique. The formula of the expected value of the number of failures for the two WE populations is given. Finally, numerical examples using simulated and real data sets are applied to illustrate all different methods. In addition, the comparison between the A.E and S.E of M_{r} are obtained.

Chapter 3 is focused on the estimation of the parameters for WED under an A-II-PRO-C. The ML, parametric bootstrap and Bayes estimators for the parameters of a WED are derived. Moreover, the approximate ACIs and Fisher information matrix are obtained. MCMC technique is applied to estimate the unknown parameters of WED. The Metropolis--Hastings algorithm is used to generate samples from the posterior density functions. Furthermore, the ACIs, Bayes estimates using MCMC technique and there credible intervals are derived. A real data set and simulated example are presented to illustrate the different methods of inference.

Finally, a Monte Carlo simulation study is carried out to compare the performance of the different methods.

Chapter 4 is concentrated on obtaining the point and interval estimation for the parameters of WED and their survival (reliability) and hazard rate functions under PRO-I-F-C scheme. The ML, parametric percentile bootstrap and the Bayes estimates under symmetric and asymmetric loss functions are studied. Based on the asymptotic normality of MLEs, we construct the ACIs for the parameters. Furthermore, depending on the delta and parametric bootstrap methods, we calculate the ACIs for survival and hazard rate functions. Furthermore, MCMC approach is used to obtain the Bayes estimators and their corresponding CRIs. Examples using simulated and real data sets are presented for illustrative purposes. Finally, Some comparisons between classical and Bayesian estimation methods are studied by using Monte Carlo simulation study.

Chapter 5 is focused on the estimation problem of the parameters for WED using PRO-II-C under CSPALT. The ML, Bayes and parametric bootstrap methods are used for estimating the unknown parameters and acceleration factor. Moreover, the approximate ACIs and Fisher information matrix are obtained. MCMC and Lindley's approximation are used to get Bayes estimates of various parameters under symmetric and asymmetric loss functions. A numerical example presented to illustrate the results. Finally, a Monte Carlo simulation study is conducted to examine and compare the performance of the different methods.

In Chapter 6, the purpose of this chapter is to obtain the point and interval estimation for the parameters of WED using PRO-II-C under SSPALT. The ML and Bayes methods are used to estimate the unknown parameters and the acceleration factor. The MCMC is used to compute the approximate Bayes estimates and the corresponding CRIs using Metropolis-Hastings method. An example is presented to illustrate the results. Finally, a Monte Carlo simulation study is conducted to examine and compare the performance of the different methods.

Lastly, a collection of the references are provided at the end. Therefore, we have included only those references which are cited in the thesis and are directly related to our considered problems.