جامعة الفيـــــــوم	الفرقة الرابعة	المادة : تحليل دالــى
كليـــــة التربيـــــة	عـــــام	امتحان دور مايو 2011
قســـم الرياضيـــات	شعبة رياضيات"قديم"	الزمن : ثلاث ساعة

(1)(a) Defind a normed space and Prove that ℓ_2^K is a normed space and also prove that ℓ_2^K is a Banach space.

(b) Prove that , If X is a Banach space and Y is a closed subspace in X then Y is a Banach space.

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(2)(a) Prove that: If $T:X \to Y$;X,Y are normed space and T is continuous then T is continuous at the zero element in X.

(b) Let $T \in L(X)$ where X is an inner product space , prove that if $\|Tx\| = \|x\| \ \forall x \in X \ then \ \langle Tx, Ty \rangle = \langle x, y \rangle \ \forall x, y \in X$.

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(3)(a) state the Hahn-Banach theorem and prove that If Y be a subspace of a normed space X and suppose

$x_0 \in X$ satisfying $d = d(x_0, Y) = \inf_{x \in Y} \|x_0 - x\| > 0$

then there is a bounded linear functional F on X such that

$\|F\| = 1 \ , F(x_0) = d \ and \ F(x) = 0 \ for \ x \in Y$.

(b) Prove that, $\sigma_p(T) \subset \sigma(T)$.

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(4)(a) Prove that, the inner product space X can be considered as a normed space with the norm $\|x\| = \sqrt{\langle x, x \rangle}$; $x \in X$.

(b) Prove that, if $T \in B(H)$ and $T_1 = \frac{1}{2}(T + T^*)$, $T_2 = \frac{-i}{2}(T - T^*)$ then

T_1 and T_2 are self – adjoint operators and $T = T_1 + iT_2$.

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(5)(a) Prove that, if X be an inner product space and $x, y \in X$ then

$|\langle x, y \rangle|^2 \le \langle x, x \rangle \ \langle y, y \rangle$.

(b) Prove that : If X be a real inner product space and $x, y \in X$ then

$\langle x, y \rangle = \frac{1}{4}\left[\|x + y\|^2 - \|x - y\|^2\right]$.

(مع تمنيـــــاتي بالنجـــاح)