

المادة : تحليل مركب	الفرقة الرابعة	جامعة الفيوم
امتحان دور يناير ٢٠١١	عام " قديم "	كلية التربية
الزمن : ثلاث ساعة	شعبة الرياضيات	قسم الرياضيات

(١)(a) Find the image of (with drawing):

(i)  $x^2 - y^2 = a$  and  $xy = b$  under  $w = z^2$

(ii)  $y > 1$  under  $w = (1-i)z$

(b) Find  $\int_c \frac{z^3}{z+i} dz$  where  $c: |z|=2$

(٢)(a) Does the limit:  $\lim_{z \rightarrow 0} [z^2 / |z|^2]$  exists?

(b) Find  $\int_c z^2 dz$  where  $c: a$  Segment from 0 to  $(2+i\frac{\pi}{2})$

(٣)(a) Prove that: If the limit of a function  $f$  at a point exists then it is unique.

(b) Find  $\int_c \frac{z^2 \sin \pi z + \cos^2 \pi z}{(z-1)(z-2)} dz$  where

(i)  $c: |z-1| = \frac{1}{2}$       (ii)  $c: |z| > 2$ .

(٤)(a) Prove that: If  $f(z) = u(x,y) + i v(x,y)$  and  $f'(z)$  exists at a point

$z_0 = x_0 + i y_0$  then  $u_x, u_y, v_x, v_y$  exists at  $(x_0, y_0)$  and satisfy

The equations  $u_x = v_y$  and  $u_y = -v_x$

(b) Find the values of  $z$  where  $e^z = -2$  (by Detail)

(٥)(a) Prove that  $\lim_{z \rightarrow z_0} z^2 = z_0^2$

(b) Prove that  $f(z) = \frac{\bar{z}}{z}, z \neq 0$

$= 1 \quad z = 0$

Is not continuous at  $z = 0$

( مع تمنياتي بالنجاح )