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Egypt. J. Phys., **30**, No. 1, pp. 71 - 77 (1999)

The Time Evolution of a Slightly Non-Ideal Neutral Plasma

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THE CONCEPT of a slightly non-ideal plasma was first introduced by J.B. Taylor in the context of his famous theory on plasma relaxation in the following sense : a slightly non-ideal plasma relaxes towards a state of minimum magnetic energy under the constraint that the global magnetic helicity integral and the total toroidal magnetic flux remain conserved. Based on a multiple time-scale derivative expansion scheme, we have shown in a previous work, that Taylor's conjecture is indeed true on the MHD-collision time-scale (CMHD). In this paper, the invariance properties of both the local and the global mass integrals are investigated on the ideal MHD (IMHD) and the CMHD time-scales. On the IMHD time-scale, it is shown that both the local and the global mass integral is no longer conserved while, on account of the motion of the particles across the magnetic field lines, the global mass integral is only conserved.

It is well known that in the case of an ideal, perfectly conducting plasma, the motion of the plasma particle perpendicular to the magnetic field lines is not permitted since the field lines are frozen into the plasma. Consequently, the local mass integral

$$M^{\psi} = \int \rho d\tau^{\psi},$$

is thought to be constant for each flux tube, where ρ is the particle mass density and ψ is the label of the magnetic surface. On the basis of Taylor's theory[1], the constancy of the local mass integral represents actually an infinity of topological constraints. Accordingly, because of these constraints a complete rearrangement of the plasma particles is not permitted in the ideal MHD limit. Let us now consider a slightly non-ideal plasma where the deviation from the ideal limit arises on account of resistivity effects. In the presence of a small but finite resistivity, however, the topological constraints associated with the magnetic field lines are no longer preserved, since the field lines may break and reconnect. In this case, it will be possible for the plasma particles to move across the field lines and to rearrange themselves within the whole.volume such that the global mass integral

$$M^G = \int \rho d\tau,$$

in which the integration is performed over the whole volume of plasma, is constant. A similar argument has been drawn by K. Avinash[2] to interpret the time evolution of a slightly non-ideal, *non-neutral* plasma. Contrary to this argument, Y. Kondoh[3] conjectured that, in the case of a slightly non-ideal, neutral plasma the local mass integral remains conserved.

In order to investigate the fusion plasma phenomena, such as the plasma anomalous transport and plasma relaxation, that take place on a time-scale intermediate between both of the ideal MHD (Alfven time) and the resistive diffusion, a multiple time-scale approach was developed[4]. In this approach, a multiple time-scale derivative expansion scheme has been applied simultaneously to both the dimensionless Fokker-Planck and Maxwell's equations. Within the kinetic theory, the four speciesdependent time-scales are those Larmor gyration Ω_a^{-1}, the transit time ω_a^{-1}, the collision time ν_a^{-1}, and the classical diffusion time, which can be written in the standardized form $\tau_{an} = \tau_0 \delta_a^{-n}$, where δ_a, is the ratio between the transit and the gyration frequency, *i.e.*, $\delta_a = \omega_a / \Omega_a$. Within the MHD-theory the corresponding time-scales are the ion-gyration period, the Alfven time τ_A, the MHD-collision time $\tau_c = (\nu_e^{-1} \nu_i^{-1})^{0.5}$ and the resistive diffusion time τ_{rd}, which again can be written in standardized form $t_n = t_0 \delta^{-n} = t_0(\delta_e \delta_i)^{-n/2}$. For each order of the expansion parameter δ_a, a separate set of kinetic equations has been obtained, which led, after performing the velocity moments, for each order to a separate set of the transport equations.

In a previous work[5] we have, based on a multiple time-scale derivative expansion scheme (MTS)[4], shown that during the evolution of a weakly collisional, strongly magnetized *neutral* fusion plasma on the intermediate MHD-collision (CMHD) time-scale, it behaves as a slightly non-ideal plasma. In this paper, the conjectures by K. Avinash and Y. Kondoh are analyzed and the invariance properties of the local mass integral on both the ideal MHD (IMHD) and the MHD-collision (CMHD) time-scales are investigated.

Basic Equations

Continuity Equations

Referring to the multiple time-scale approach[4] (for detailed calculations cf Ref. [6]), the dimensionless continuity equations,

$$\textit{IMHD:}\quad \frac{\partial n_0}{\partial t_1} + \frac{\partial n_1}{\partial t_0} + \nabla \cdot \left(n_0 \vec{u}_0 \right) = 0, \tag{1a}$$

$$\textit{CMHD:}\quad \frac{\partial n_0}{\partial t_2} + \frac{\partial n_1}{\partial t_1} + \frac{\partial n_2}{\partial t_0} + \nabla \cdot \left(n_0 \vec{u}_1 + n_1 \vec{u}_0 \right) = 0, \tag{1b}$$

where, n_k and u_k krefer to the k^{th} order normalized particle density and normalized average plasma velocity respectively and t_0, t_1 and t_2 are related to ion-gyration time, ideal MHD (Alfven) time (*IMHD*) and the intermediate MHD-collision time (τ_c) (CMHD), respectively.

Multiple time-scale expansion of the mass integrals

By applying the multiple time-scale derivative expansion scheme [4], one obtains for the time-evolution of the local (global) mass integral,

$$\begin{aligned}
\frac{\partial M^{\psi}}{\partial t} &= \sum_{n=0} \delta^n \sum_{s=0}^{n} \frac{\partial M^{\psi}_{n-s}}{\partial t_s} . \\
&= \left.\frac{\partial M^{\psi}}{\partial t}\right|_{ion-gyr} + \delta \left.\frac{\partial M^{\psi}}{\partial t}\right|_{IMHD} + \delta^2 \left.\frac{\partial M^{\psi}}{\partial t}\right|_{CMHD} \\
&= \sum_{n=0} \delta^n \left\{ \int \sum_{m=0}^{n} \sum_{s=0}^{n-m} \frac{\partial n_s}{\partial t_{n-s}} d\tau_m + \int \sum_{m=0}^{n} \sum_{s=0}^{n-m} n_m \frac{\partial}{\partial t_{n-s}} \left(d\tau_s \right) \right\}
\end{aligned} \tag{2a}$$

Similarly, we obtain for the time-evolution of the global mass integral

$$\frac{\partial M^G}{\partial t} = \sum_{n=0} \delta^n \sum_{s=0}^{n} \frac{\partial M^G_{n-s}}{\partial t_s},$$

$$= \left.\frac{\partial M^G}{\partial t}\right|_{ion-gyr} + \delta \left.\frac{\partial M^G}{\partial t}\right|_{IMHD} + \delta^2 \left.\frac{\partial M^G}{\partial t}\right|_{CMHD} \qquad (2b)$$

$$= \sum_{n=0} \delta^n \left\{ \int \sum_{s=0}^{n} \frac{\partial n_s}{\partial t_{n-s}} \, d\tau \right\}$$

The Time Evolution of The Volume Element

Furthermore, the dimensionless equations for the time evolution of the volume element $d\tau^{\Psi}$ (cf. Ref. [4]& [6]), read

$$\textbf{\textit{IMHD:}} \quad \frac{\partial}{\partial t_1}(d\tau_0) + \frac{\partial}{\partial t_0}(d\tau_1) = -(\Omega_i \tau_A \delta)^{-1} (\vec{u}_0 \cdot d\vec{s}_0) \;, \qquad (3a)$$

$$\textbf{\textit{CMHD:}} \quad \frac{\partial}{\partial t_2}(d\tau_0) + \frac{\partial}{\partial t_1}(d\tau_1) + \frac{\partial}{\partial t_0}(d\tau_2) = -(\Omega_i \tau_A \delta)^{-1} \{\vec{u}_0 \cdot d\vec{s}_1 + \vec{u}_1 \cdot d\vec{s}_0\}$$
$$+ (\Omega_i \tau_A \delta)^{-1} (\vec{G} + \vec{u}_0 \times \nabla \mu) \cdot d\vec{s}_0 + J_0 h(\Psi_0) \, d\theta \, d\varphi \qquad (3b)$$

$d\tau^{\Psi} = d\tau_0 + \delta d\tau_1 + \delta^2 d\tau_2$ is a volume element enclosed between two neighboring magnetic surfaces $\psi = \psi_0 + \delta\psi_1 + \delta^2\psi_2 =$ const., created by the magnetic field $\vec{B} = \vec{B}_0 + \delta\vec{B}_1 + \delta^2\vec{B}_2$. The vector function G is defined by

$$\nabla(\vec{G} \cdot \nabla\Psi_0) = \nabla\Psi_0 \times \nabla\left\{\frac{\partial \mu}{\partial t_1}\right\}, \text{ with}$$

$$\mu = \frac{\vec{u}_0 \cdot \vec{B}_0}{|\vec{B}_0|^2}, \text{ and} \qquad (4)$$

$$\vec{G} + \vec{u}_0 \times \nabla\mu = \vec{j}_1 - \left(\frac{\vec{j}_0 \cdot \vec{B}_0}{|\vec{B}_0|^2}\right) \vec{B}_1$$

Furthermore, the function "h" is yet undetermined arbitrary flux function which needs not to be zero, and ψ_0 is the zero-order poloidal magnetic flux. In Eq. (3) it is assumed that the system has been relaxed to its zero-order equilibrium state

$$\vec{j}_0 = \lambda(\Psi_0)\vec{B}_0, \text{ and } \vec{u}_0 = \mu(\Psi_0)\vec{B}_0 \qquad (5)$$

These equations are employed consistently with the boundary conditions[5].

The invariance property of the local mass integral on the imhd time-scale

The application of our multiple time-scale expansion scheme shown in Eq. (2a), yields to the order δ the following dimensionless equation for the time evolution of the local mass integral on the IMHD time-scale

$$\left.\frac{\partial M^{\psi}}{\partial t}\right|_{IMHD} = \int\left(\frac{\partial n_0}{\partial t_1} + \frac{\partial n_1}{\partial t_0}\right)d\tau_0 + \int n_0\left(\frac{\partial}{\partial t_1}d\tau_0 + \frac{\partial}{\partial t_0}d\tau_1\right) \tag{6}$$

By employing Eq. (1) and (3) together with the normalizing rules[4], we then obtain Eq. (6) in its *dimensional* form:

$$\left.\frac{\partial M^{\psi}}{\partial t}\right|_{IMHD} = -\int \nabla\cdot\left(\rho_0 \vec{u}_0\right)d\tau_0 + \oint \rho_0\left(\vec{u}_0 \cdot d\vec{s}_0\right) \equiv 0. \tag{7}$$

This means that on the IMHD time-scale the local mass integral is conserved for each flux tube. This result is consistent with Taylor's conjecture[1] concerning the nature of the IMHD invariant of motion. If we assume that the plasma is surrounded by a rigid, perfectly conducting wall so that the plasma boundary coincides with the outermost magnetic surface, then naturally we also obtain for the global mass integral

$$\left.\frac{\partial M^{G}}{\partial t}\right|_{IMHD} = \int \frac{\partial \rho_0}{\partial t_1} d\tau = -\int \nabla\cdot\left(\rho_0 \vec{u}_0\right)d\tau \equiv 0 \tag{8}$$

Thus, on the IMHD time-scale, the global mass integral is also an invariant of motion.

The invariance property of the mass integrals on the mhd-collision timescale

First, we investigate the time evolution of the local mass integral for each flux tube. Similarly, the next order in our expansion scheme yields for the time evolution on the MHD-collision time-scale the following *dimensinal form*,

$$\left.\frac{\partial M^{\psi}}{\partial t}\right|_{CMHD} = \int\left(\frac{\partial \rho_0}{\partial t_2} + \frac{\partial \rho_1}{\partial t_1}\right)d\tau_0 + \int\left(\frac{\partial d\tau_0}{\partial t_2} + \frac{\partial d\tau_1}{\partial t_1}\right)\rho_0 + \delta\int \frac{\partial \rho_0}{\partial t_1} d\tau_1 + \delta\int n_1 \frac{\partial d\tau_0}{\partial t_1} \tag{9}$$

Assuming that the plasma has been relaxed to its zero-order equilibrium state, and applying the boundary condition[5], we then end up with

$$\left.\frac{\partial M^{\psi}}{\partial t}\right|_{CMHD} = -\delta \int \rho_0 \left(\vec{G} + \vec{u}_0 \times \nabla\mu\right) \cdot d\vec{s}_0 + \iint n_0 J_0 h\left(\Psi_c\right) d\theta\, d\phi$$

$$= \delta \int \left(\vec{j}_1 - \lambda \vec{B}_1\right) \cdot d\vec{s}_0 + \iint n_0 J_0 h(\Psi_0) d\theta\, d\phi \neq 0 \qquad (10)$$

Thus, on account of the first-order magnetic flux as well as the first-order current across the magnetic flux surface the local mass integral, on the MHD-collision time-scale, is no longer conserved. On the other hand, by assuming that the plasma is surrounded by a perfectly conducting wall, it can be easily shown that the global mass integral is conserved. Thus, it can be concluded that during the evolution of a slightly non-ideal plasma on the MHD-collision time-scale, unlike to Kondoh's argument[3], plasma particles are allowed to move across the manetic field lines and to rearrange themselves such that the global mass integral remains constant. Earlier similar argument has been drawn by Avinash[2] to interpret the evolution of a slightly non-ideal non-neutral plasma. Furthermore, it is now obvious that the conjecture drawn by Kondoh[3] is no longer valid.

Summary

A multiple time-scale approach has been applied to investigate the invariance property of the local and the global mass integrals. On the IMHD time-scale, the local mass integral is conserved for each flux tube. On the MHD-collision time-scale, the invariance property of the local mass integral is violated due to the first-order magnetic flux as well as the first-order current across the magnetic flux surface. Finally, one concludes that during the evolution of a slightly non-ideal neutral plasma on the MHD-collision (CMH) time-scale, a complete rearrangement of the plasma particles within the whole plasma volume is permitted such that the global mass integral remains constant.

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(*Received* 10/3/1998)

التطور الزمنى للبلازما غير المثالية ذات المعاوقة متناهية الصغر والمتعادلة كهربيا

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جاء ذكر البلازما غير المثالية ذات المعاوقة متناهية الصغر لأول مرة فى سياق النظرية الشهيرة لتايلور ، والخاصة بدراسة استرخاء البلازما ، حيث افترض تايلور أن هذا النوع من البلازما تسترخ حتى تصل طاقة الوضع المغنطيسية إلى قيمتها الصغرى بشرط أن يظل معامل اللولبية المغنطيسية الكلى ثابتا فى عمل سابق وباستخدام الأسلوب الرياضى ذى المقاييس الزمنية المتعددة أثبتا أن فرضية تايلور صحيحة فقط فى نطاق المقياس الزمنى التصادمى الأوسط . تم اختبار ثبات كل من معامل الكتلة الموضعى ومعامل الكتلة الكلى ، حيث يشير الأول لكتلة البلازما بين سطحين فيضيين متجاورين ، بينما يشير الأخير للكتلة الكلية للبلازما ، وذلك ى نطاق كل من المقياس الزمنى المثالى "زمن الفن" والمقياس الزمنى التصادمى الأوسط . فى نطاق المقياس الزمنى التصادمى الأوسط ، حيث تتصرف البلازما كما لو كانت بلازما غير مثالية ذات معاوقة متناهية الصغر ، لم يعد معامل الكتلة الموضعى ثابتا حيث وجد أن الجسيمات المشحونة تتحرك عبر الأسطح الفيضية حتى تحتفظ فقط بمعامل الكتلة الكلى ثابتا مع الزمن .