A Numerical Study for Matrix Functions having Complex Eigenvalues Via Different Approximative Techniques

Thesis

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by

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Preface

The matrix functions computations have an important area of research in numerical analysis for introducing numerical solutions of some types of differential equations and linear systems of equations. Such types of prob-lems appear in linear system theory, control theory, physical applications and differential equations. Computing matrix functions of square matrices for some types of functions specially square root and logarithm functions having many applications in many fields.

The main objective of this thesis, which consists of four chapters, is to introduce an analytical and numerical treatment for computing matrix functions based on five definitions of matrix functions for some types of matrices. Namely: matrix functions for square matrices having pure complex or mixed eigenvalues using Vandermonde matrix, Lagrange-Sylvester interpolation and mixed interpolation methods. Also for square matrices having mixed or repeated real eigenvalues using extension of Sylvester's definition and Newton's divided difference. The analytical analysis and numerical treatment of the proposed methods and techniques is studied. The accuracy of these proposed methods and techniques is demonstrated by several test problems where the obtained numerical results are compared with the exact values for some types of matrix functions and in other time compared with previous methods.

The present thesis consists of four chapters as follows:

Chapter 1: In this chapter, we give a survey for approximation of matrix functions and computing matrix functions using different techniques. So this chapter includes basic definition of square matrices and matrix funct-ions. Also, this chapter contains some previous methods and definitions which are used for computing matrix functions, especially Vandermonde matrix, Lagrange-Sylvester method, Sylvester's method, Newton divided difference method and other important previous methods. In addition, some theorems, lemma and corollaries which enabled us for computing matrix functions are presented.

Chapter 2: Recently, computing the matrix functions $f(A)$ of a square matrix A using Vandermonde matrix, Lagrange-Sylvester definition as in equations (1.3.2) and (1.3.8) is considered in case of real eigenvalues. Also, computing $f(A)$ using Sylvester's definition (1.3.1) is considered in case of distinct real eigenvalues of A. In this chapter, we present a gen-eralization to Vandermonde matrix for computing matrix functions of square matrices having pure complex eigenvalues. Also, we present an extension to Lagrange-Sylvester's definition for computing matrix functions of square matrices having pure complex eigenvalues. In addition, we extend Sylvester's definition for computing $f(A)$ of square matrices having mixed eigenvalues. Moreover, we deduce a new formula of Sylvester's definition to compute $f(A)$ in case of repeated real eigenvalues of a square matrix A. Also, we can use this method for computing matrix functions of block matrices. Finally, the proposed methods are tested on several problems to illustrate their applicability and accuracy.

Chapter 3: In this chapter, the definition (1.3.6) and the theorems (1.3.2, 1.3.3) which are stated in first chapter are first modified to derive a new technique for computing matrix functions using numerical hybrid method in case of square matrices having mixed (real and pure complex) eigen-values. Finally, we give several numerical examples to illustrate the applicability of our theoretical finding and to prove the accuracy of the proposed technique.

Chapter 4: In this chapter, we propose different formulas for approxi-mating matrix functions of square matrices having (mixed or pure complex) eigenvalues. The suggested formulas are deduced from Newton's divided difference which is defined in case of real eigenvalues using definition (1.3.6) where we generalize it in our proposed cases. The theoretical analysis of these techniques is then discussed. Numerical examples are presented to illustrate the applicability and the accuracy of the obtained analytical results.