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RINGS WITH SOME KINDS OF MAPPINGS

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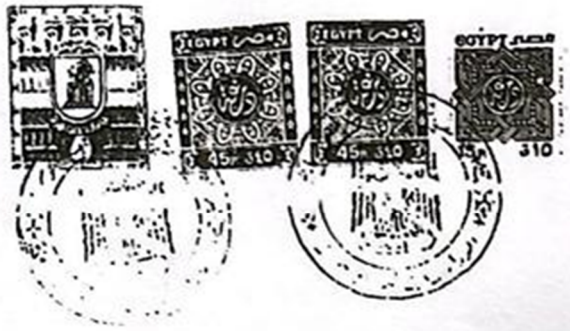

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ABSTRACT

In this thesis we introduce a new notion of a mapping on a given ring. This notion that we call ring homoderivation is analogous to the well-known notions of a ring endomorphism and a ring derivation. A series of results are derived and chapter three is completely devoted for these results. Chapters one and two survey –in a fairly intensive manner- the previously and relatively recent results akin to our study.

INTRODUCTION

INTRODUCTION

This thesis continues a line of investigation in the literature concerning specific known kinds of mappings of rings. Such mappings preserve certain combinations of elements in rings, for example, homomorphisms, derivations and the like.

The notion of derivations and homomorphisms of rings has been extensively investigated. Chapter II is completely allocated for this study carried out by several authors. We introduce here a new parallel to it notion; namely homoderivations of a rings, with the aim of obtaining similar, though different results. To be more precise we say that a mapping d: $R \to R$ on a given ring R is a homoderivation if:

(1) $d(x + y) = d(x) + d(y)$, and

(2) $d(xy) = d(x)d(y) + xd(y) + d(x)y$, for all $x, y \in R$.

The thesis comprises three chapters. The first chapter includes several types of mappings in the area of rings and the terminology that are needed in either of the last two chapters. The second chapter covers, as mentioned above, all these results related to derivations of rings that will be needed, so to speak, in the sequel. The purpose of the last chapter is completely devoted to our study of rings homoderivations. Besides some problems of rings with derivations are

treated using homoderivations. Also conditions used by Bell and Daif [4],[5],[6],[13] and [14] studied under derivations are here treated under homoderivations. Some results necessary for commutativity of rings are obtained via homoderivations.

Prime rings with involution that earn homoderivations as well are proved to be satisfying S_4 in the sense it is a commutative integral domain or an order in a 4-dimensional central simple algebra.

Throughout this thesis R stands for an associative ring not necessarily having 1; whereas C designates the center of R.