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L-fuzzy syntopogenous structures

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ABSTRACT

In this paper we introduce the concept of L-fuzzy syntopogenous structures in the framework of U. Hőhle, S. E.Rodabaugh L-fuzzy topology. We investigate some of their properties. The relationship amonge L-fuzzy syntopogenous structures, L-fuzzy topology, L-fuzzy proximity and L-fuzzy uniformity is studied.

Keywords: Quantales; L-fuzzy topology; L-fuzzy proximity;

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1 Introduction:

Ramadan et al introduced a notion of a fuzzifying syntopogenous structures as a framework of Mingsheng Ying fuzzifying topological spaces and a notion of a smooth syntopogenous structures as a framework of Ŝostak fuzzy topology. For a fixed basis L, algebraic structures in L (quantales, MV-algebras) are extended for a completely distributive lattice L. In this paper we establish the concept of L-fuzzy syntopogenous structures as a unified approach to theories of (Hőhle and Rodabaugh) L-fuzzy topology , (Kim and Min) L-fuzzy proximity spaces and (Ramadan and Kim) L-fuzzy uniformity spaces. Some fundamental properties of them are established. Finally, the relationship among L-fuzzy syntopogenous structures, L-fuzzy topology, L-fuzzy proximity and L-fuzzy uniformity is studied. In this article let X be a nonempty set, $L = (L,\ \leq,\ \oplus,\ \odot,\ 0, 1)$ be a completely distributive lattice with the least element 0 and the greatest element 1 in L . $L_0 = L - \{0\}$ and $L_1 = L - \{1\}$. For each $\alpha \in L$ let $\overline{\alpha}$ and $\widetilde{\alpha}$ be the constant fuzzy subsets of X and $X \times X$ with value α, respectively. We denote the characteristic function of a subset A of X by 1_A .

2 Preliminaries:

Definition 2.1.(Hőhle and Rodabaugh)
A triple $(L\ , \leq, \odot)$ is called a strictly two-sided, commutative quantal (stsc-quantale, for short) iff it satisfies the following properties:
(L1) $(L, \odot)$ is a commutative semigroup.
(L2) $a = a \odot 1$, for each $a \in L$.
(L3) $\odot$ is distributive over arbitrary joins, i.e., $(\bigvee_{i\in\Gamma} a_i) \odot b = \bigvee_{i\in\Gamma} (a_i \odot b)$.

Remark 2.2. (Hőhle and Rodabaugh)
(1) Each frame is a stsc-quantale. In particular, the unit interval ($[0,1], \leq, \wedge, 0, 1)$ is a stsc-quantales.
(2) Every continuous t-norm T on ($[0,1], \leq, t)$ with $\odot = t$ is a stsc-quantales.
(3) Every GL-monoid is a stsc-quantale.
(4) Let $(L, \leq, \odot)$ be a stsc-quantale. For each $x, y \in L$, we define
$x \rightarrow y = \vee \{z \in L \mid x \odot z \leq y\}.$
Then it satisfies Galois correspondence, that is $(x \odot y) \leq z \Leftrightarrow x \leq (y \rightarrow z)$.
In this paper, we always assume that $(L, \leq, \odot, \oplus, ^*)$ is a stsc-quantale with an order-reversing involution * defined by $x \oplus y = (x^* \odot y^*)^*$ unless otherwise specified.

Definition 2.3. (Hőhle and Rodabaugh)
A stsc-quantale $(L, \leq, \odot, ^*)$ is called a complete MV-algebra iff it satisfies the following property:
(MV) $(x \rightarrow y) \rightarrow y = x \vee y, \ \forall x, y \in L$ which is defined as
$x \rightarrow y = \vee\{z \in L \mid x \odot z \leq y\}, \quad x^* = x \rightarrow 0.$

Lemma 2.4. (Hőhle and Rodabaugh)

Let $(L, \leq, \odot, \oplus, ^*)$ be a stsc-quantale with an order-reversing involution *. For each $x, y, z \in L, \{y_i \mid i \in \Gamma\} \subset L$, we have the following properties:
(1) If $y \leq z$ then $(x \odot y) \leq (x \odot z)$ and $(x \oplus y) \leq (x \oplus z)$.
(2) $x \odot y \leq x \wedge y \leq x \vee y \leq x \oplus y$.
(3) $\wedge_{i \in \Gamma} y_i^* = (\vee_{i \in \Gamma} y_i)^*$ and $\vee_{i \in \Gamma} y_i^* = (\wedge_{i \in \Gamma} y_i)^*$.
(4) $x \oplus (\wedge_{i \in \Gamma} y_i) = \wedge_{i \in \Gamma} (x \oplus y_i)$.
(5) $(x \vee y) \odot (z \vee w) \leq (x \vee z) \vee (y \odot w) \leq (x \oplus z) \vee (y \odot w)$.
(6) $x \odot (x \rightarrow y) \leq y$ and $x \rightarrow y \leq (y \rightarrow z) \rightarrow (x \rightarrow z)$.
(7) If $x^* = x \rightarrow 0$, then $x \rightarrow y = y^* \rightarrow x^*$.
(8) If $x^* = x \rightarrow 0$, then $x \odot (x^* \oplus y^*) \leq y^*$.
(9) If L is a complete MV-algebra, then
$x \odot y = (x \rightarrow y^*)^*, \ (x \oplus y) = x^* \rightarrow y,$
$(x \oplus z) \odot y \leq x \oplus (y \odot z),$
$(x \odot y) \odot (z \oplus w) \leq (x \odot z) \oplus (y \odot w),$
$x \oplus (\vee_{i \in \Gamma} y_i) = \vee_{i \in \Gamma} (x \oplus y_i)$ and $x \odot (\wedge_{i \in \Gamma} y_i) = \wedge_{i \in \Gamma} (x \odot y_i)$.
All algebraic operations on L can be extended pointwise to the set L^X as follows:
(1) $\lambda \leq \mu$ iff $\lambda(x) \leq \mu(x), \forall x \in X$.
(2) $(\lambda \odot \mu)(x) = \lambda(x) \odot \mu(x), \forall x \in X$.
(3) $(\lambda \rightarrow \mu)(x) = \lambda(x) \rightarrow \mu(x), \forall x \in X$.

Definition 2.5.(Hőhle and Rodabaugh)
A map $\tau : L^X \rightarrow L$ is called an L-fuzzy topology if it satisfies the following conditions:
(o1) $\tau(\underline{0}) = \tau(\underline{1}) = 1$,
(o2) $\tau(\mu_1 \odot \mu_2) \geq \tau(\mu_1) \odot \tau(\mu_2), \ \forall \mu_1, \mu_2 \in L^X$.
(o3) $\tau(\vee_{i \in \Gamma} \mu_i) \geq \wedge_{i \in \Gamma} \tau(\mu_i)$ for any $\{\mu_i\}_{i \in \Gamma} \subset L^X$.
The pair (X, τ) is called an L-fuzzy topological spaces.

Let τ_1 and τ_2 be L-fuzzy topologies on X. We say that τ_1 is finer than τ_2 (τ_2 is coarser than τ_1), denoted by $\tau_2 \leq \tau_1$, if $\tau_2(\lambda) \leq \tau_1(\lambda)$, for all $\lambda \in L^X$.
Let (X, τ_1) and (Y, τ_2) be L-fuzzy topological spaces.
A function $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is called L-fuzzy continuous map if $\tau_2(\lambda) \leq \tau_1(f^{-1}(\lambda))$ for all $\lambda \in L^Y$.

Definition 2.6.(Hőhle and Rodabaugh)

A map $I : L^X \times L_1 \to L^X$ is called L-fuzzy interior operator on X iff I satisfies the following condations:

$(I1)$ $I(\underline{1}\,, r) = \underline{1}$ for all $r \in L_1$.

$(I2)$ $I(\lambda, r) \leq \lambda$ for all $r \in L_1$.

$(I3)$ If $\lambda \leq \mu$ and $r \leq s$, then $I(\lambda, s) \leq I(\mu, r)$.

$(I4)$ $I(\lambda \odot \mu,\ r \odot s) \geq\ I(\lambda, r) \odot I(\mu, s)$.

The pair (X, I) is called L-fuzzy interior space.

The L- fuzzy interior operator I is called topological if

$$I\ (I\ (\lambda, r)) \geq I(\lambda, r),\ \forall \lambda \in L^X,\ r \in L_1.$$

Let I_1 and I_2 be two L-fuzzy iterior operators on X. We say that I_1 is finer than I_2 ($I2$ is coarser than $I1$), denoted by $I_2 \leq I_1$, if $I_2(\lambda, r) \leq I_1(\lambda, r)$ for all $\lambda \in L^X, r \in L_1$.

Definition 2.7.(Y.C.Kim and K.C.Min)

A function $\delta : L^X \times L^X \to L$ is called an L-fuzzy preproximity on X if it satisfies the followig axioms:

(P1) $\delta(\underline{1}, \underline{0}) = 0$ and $\delta(\underline{0}, \underline{1}) = 0$.

(P2) If $\lambda \leq \rho$, then $\delta(\lambda, \mu) \leq \delta(\rho, \mu)$ and $\delta(\mu, \lambda) \leq \delta(\mu, \rho)$.

(P3) If $\delta(\lambda, \rho) \neq 1$, then $\lambda \leq \rho^*$.

(P4) $\delta(\lambda_1 \odot \lambda_2, \rho_1 \oplus \rho_2) \leq \delta(\lambda_1, \rho_1) \oplus \delta(\lambda_2, \rho_2)$.

An L-fuzzy preproximity space is called L-fuzzy quasi-proximity space if

(P5) $\delta(\lambda, \rho) \geq \wedge_{\gamma \in L^X} \{\delta(\lambda, \gamma) \oplus \delta(\gamma^*, \rho)\}$.

An L-fuzzy preproximity space is called principal if

(P6) $\delta\ (\vee_{j \in J} \lambda_j, \rho) \leq \vee_{j \in J}\ \delta(\lambda_j, \rho)$.

An L-fuzzy quasi-proximity space is called L-fuzzy proximity space if

(P) $\delta(\lambda, \rho) =\ \delta(\rho, \lambda)$.

Definition 2.8. (B.Hutton)

Let X be a set and Ω_X be the set of all mappings $\alpha : L^X \to L^X$ such that

(1) $\alpha(\underline{0}) = \underline{0}$,

(2) $\alpha(\mu) \geq \mu$,

(3) $\alpha(\vee_{i \in \Gamma}\ \mu_i) = \vee_{i \in \Gamma}\ \alpha(\mu_i)$.

Definition 2.9. (B.Hutton)

(1) If $\alpha_1, \alpha_2 \in \Omega_X$, then $\alpha_1 \odot \alpha_2 \in \Omega_X$ where $(\alpha_1 \odot \alpha_2)(\mu) = \wedge\{\ \alpha_1(\mu_1) \oplus \alpha_2(\mu_2) \mid \mu = \mu_1 \oplus \mu_2\}$.

(2) If $\alpha \in \Omega_X$, then $\alpha^{-1} \in \Omega_X$ where $\alpha^{-1}(\mu) = \wedge\{\lambda \in L^X \mid \alpha(\lambda^*) \leq \mu^*\}$.

Definition 2.10.

A function $U : \Omega_X \to I$ is called L-fuzzy quasi-uniformity on X if it satisfies for $\alpha, \beta \in \Omega_X$, the following conditions:

(U1) $U\ (\alpha \odot \beta) = U\ (\alpha) \odot U\ (\beta)$.

(U2) There exists $\alpha \in \Omega_X$ such that $U\ (\alpha) = 1$.

(U3) $U\ (\alpha) \leq \vee\{U(\beta) \mid \beta \circ \beta \leq \alpha\}$

The pair (X, U) is said to be L-fuzzy quasi-uniform space.

The L-fuzzy quasi-uniform space (X, U) is said to be L-fuzzy uniform space if it satisfies

(U) $U\ (\alpha) = U\ (\alpha^{-1})$.

Definition 2.11.

A function $B : \Omega_X \to I$ is called L-fuzzy quasi-uniform base on X if it satisfies for $\alpha, \beta \in \Omega_X$, the following conditions:

(UB1) $B(\alpha_1) \odot B(\alpha_2) \geq \vee\{B(\beta) \mid \beta \leq \alpha_1 \odot \alpha_2\}$.
(UB2) There exists $\alpha \in \Omega_X$ such that $B\ (\alpha) = 1$.
(UB3) $B\ (\alpha) \leq \vee\{B(\beta) \mid \beta \circ \beta \leq \alpha\}$.
The L-fuzzy quasi-uniform base B on X is said to be L-fuzzy uniform base if it satisfies
(UB) $B\ (\alpha) \leq \vee\{B(\beta) \mid \beta \leq \alpha^{-1}\}$.

Theorem 2.12.
Let $B : \Omega_X \to I$ be the L-fuzzy uniform base on X. Define $U_B : \Omega_X \to I$ as $U_B(\alpha) = \vee\{B(\beta) \mid \beta \leq \alpha\}$,then U_B is an L-fuzzy uniformity on X.

3 L-fuzzy topogenous order and L-fuzzy topologies.

Definition 3.1.
A function $\eta : L^X \times L^X \to L$ is called an L-fuzzy semi-topogenous order on X,if it satisfies the following axioms:
(T1) $\eta(\underline{1}, \underline{1}) = \eta(\underline{0}, \underline{0}) = 1$.
(T2) If $\eta(\mu, \lambda) \neq 0$, then $\mu \leq \lambda$.
(T3) If $\mu \leq \mu_1,\ \lambda_1 \leq \lambda$ then $\eta(\mu_1, \lambda_1) \leq \eta(\mu, \lambda)$.

Definition 3.2.
Let η be an L-fuzzy semi-topogenous order on X and let the mapping $\eta^* : L^X \times L^X \to L$ defined by $\eta^*(\lambda, \mu) = \eta(\mu^*, \lambda^*),\ \forall \lambda, \mu \in L^X$. Then η^* is an L-fuzzy semi-topogenous order on X.

Definition 3.3
An L-fuzzy semi-topogenous order η is called symmetric if $\eta = \eta^*$, that is
(T4) $\eta(\lambda, \mu) = \eta(\mu^*, \lambda^*), \forall \lambda, \mu \in L^X$.

Definition 3.4.

An L-fuzzy semi-topogenous order η is called L-fuzzy topogenous if for any $\lambda, \lambda_1, \lambda_2, \mu, \mu_1, \mu_2 \in L^X$, we have:
(T5) $\eta(\lambda_1 \oplus \lambda_2, \mu) = \eta(\lambda_1, \mu) \odot \eta(\lambda_2, \mu)$.

(T6) $\eta(\lambda, \mu_1 \odot \mu_2) = \eta(\lambda, \mu_1) \odot \eta(\lambda, \mu_2)$.

Definition 3.5. An L-fuzzy semi-topogenous order η is called perfect if
(T7) $\eta(\vee_{i\in\Gamma}\lambda_i, \mu) = \wedge_{i\in\Gamma}\ \eta(\lambda_i, \mu)$, for any $\{\mu, \lambda_i \mid i \in \Gamma\} \subset L^X$.
A perfect L-fuzzy semi-topogenous order η is called biperfect if
(T8) $\eta(\lambda, \wedge_{i\in\Gamma}\mu_i) = \wedge_{i\in\Gamma}\ \eta(\lambda, \mu_i)$,for any $\{\lambda, \mu_i \mid i \in \Gamma\} \subset L^X$.

Theorem 3.6.
Let $\eta_1, \eta_2 : L^X \times L^X \to L$ be a perfect L-fuzzy semi-topogenous orders on X (resp. biperfect L-fuzzy topogenous). Define the composition $\eta_1 \circ \eta_2$ of $\eta_1$and η_2 on X by

$$(\eta_1 \circ \eta_2)\ (\lambda, \mu) = \bigvee_{\nu \in L^X} (\eta_1(\lambda, \nu) \odot \eta_2(\nu, \mu)).$$

Then $(\eta_1 \circ \eta_2)$ is a perfect (resp.L-fuzzy topogenous, biperfect) L-fuzzy semi-topogenous order on X.

Proof:

We prove (T2) and (T7) . Let $\eta_1, \eta_2 : L^X \times L^X \to L$ be a perfect L-fuzzy semi-topogenous orders on X.
(T2) If $(\eta_1 \circ \eta_2)(\lambda, \mu) \neq 0$, the there exists $\nu \in L^X$ such that $(\eta_1 \circ \eta_2)(\lambda, \mu) \geq \eta_1(\lambda, \nu) \odot \eta_2(\nu, \mu) \neq o$. It implies $\lambda \leq \nu \leq \mu$.
(T7) It is proved from

$$\begin{aligned}(\eta_1 \circ \eta_2)(\bigvee_{i\in\Gamma} \lambda_i, \mu) &= \bigvee_{\nu \in L^X} (\eta_1(\bigvee_{i\in\Gamma} \lambda_i, \nu) \odot \eta_2(\nu, \mu)) \\ &= \bigwedge_{i\in\Gamma} (\bigvee_{\nu\in L^X} \eta_1(\lambda_i, \nu) \odot \eta_2(\nu, \mu)) \\ &= \bigwedge_{i\in\Gamma} (\eta_1 \circ \eta_2)(\lambda_i, \mu).\end{aligned}$$

Others are easily proved.

Definition 3.7.
A fuzzy syntopogenous structures on X is a non-empty family Ψ of L-fuzzy topogenous orders on X satisfying the following two conditions:
(S1) Ψ is directed, i.e., given two L-fuzzy topogenous orderes $\eta_1, \eta_2 \in \Psi$,there exists an L-fuzzy topogenous order $\eta \in \Psi$ such that $\eta \geq \eta_1, \eta_2$.
(S2) For every $\eta \in \Psi$, there exists $\eta_1 \in \Psi$ such that $\eta \leq \eta_1 \circ \eta_1$.
The pair (X, Ψ) is called L-fuzzy syntopogenous space.

Definition 3.8.

An L-fuzzy syntopogenous structures Ψ is called L-fuzzy topogenous if Ψ consists of a single element. In this case, $\Psi = \{\eta\}$ is called L-fuzzy topogenous structure, dentoted by $\Psi = \{\eta\} = \eta$ and (X, Ψ) is called L-fuzzy topogenous space.

An L-fuzzy syntopogenous structures Ψ is called perfect (resp. biperfect, symetric etc.) if each L-fuzzy topogenous order $\eta \in \Psi$ is perfect (resp. biperfect, symetric etc.)

Proposition 3.9.
Let η be L-fuzzy topogenous order on X. Define a mapping $I_\eta : L^X \times L_1 \to L^X$ as

$I_\eta(\lambda, r) = \vee\{\mu \in L^X \mid \eta(\mu, \lambda) > r\}$.Where
(1) $I_\eta(\underline{1}, r) = 1,\ \forall r \in L_1$.
(2) $I_\eta(\lambda, r) \leq \lambda, \forall r \in L_1$.
(3) If $\lambda \leq \mu$ then $I_\eta(\lambda, r) \leq I_\eta(\mu, r), \forall r \in L_1$

(4) $I_\eta(\lambda_1 \odot \lambda_2, r) = I_\eta(\lambda_1, r) \odot I_\eta(\lambda_2, r)$.
(5) $I_\eta(\bigvee_{j\in\Gamma} \lambda_j, r) \geq \bigvee_{j\in\Gamma} I_\eta(\lambda_j, r)$.
(6) $I_\eta(I_\eta(\lambda, r), r) = I_\eta(\lambda, r)$.
Then I_η is a topological operator on X.

Proof:
(1) Since $\eta(\underline{1}, \underline{1}) = 1,\ I_\eta(\underline{1}, r) = 1,\ \forall r \in L_1$.
(2) Since $\eta(\mu, \lambda) \neq 0,\ \mu \leq \lambda$ implies $I_\eta(\lambda, r) \leq \lambda$.

(3) and (5) are easily proved.
(4) From (3), we have $I_\eta(\lambda_1 \odot \lambda_2, r) \le I_\eta(\lambda_1, r) \odot I_\eta(\lambda_2, r)$.

Conversely, suppose there exist $\lambda_1, \lambda_2 \in L^X$ and $r \in L$ such that $I_\eta(\lambda_1 \odot \lambda_2, r) \not\ge I_\eta(\lambda_1, r) \odot I_\eta(\lambda_2, r)$.
There exist $x \in X$ and $t \in L_1$ such that $I_\eta(\lambda_1 \odot \lambda_2, r)(x) < t < I_\eta(\lambda_1, r)(x) \odot I_\eta(\lambda_2, r)(x)$.
Since $I_\eta(\lambda_i, r)(x) > t$, for each $i \in \{1, 2\}$, there exist $\mu_i \in L^X$ with $\eta(\mu_i, \lambda_i) > r$ such that $I_\eta(\lambda_i, r)(x) \le \mu_i(x) > t$.
On the other hand, since by (T6) and (T3) we have
$\eta(\mu_1 \odot \mu_2, \lambda_1 \odot \lambda_2) \ge \eta(\mu_1 \odot \mu_2, \lambda_1) \odot \eta(\mu_1 \odot \mu_2, \lambda_2) \ge \eta(\mu_1, \lambda_1) \odot \eta(\mu_2, \lambda_2) \ge r$
It implies $I_\eta(\lambda_1 \odot \lambda_2, r)(x) \ge (\mu_1 \odot \mu_2)(x) > t$. It is a contradiction.

(6) Since $I_\eta(\lambda, r) \le \lambda$, $I_\eta(I_\eta(\lambda, r), r) \le I_\eta(\lambda, r)$. Suppose $I_\eta(I_\eta(\lambda, r), r) \not\ge I_\eta(\lambda, r)$. There exist $x \in X$ and $r \in]0, 1[$ such that $I_\eta(I_\eta(\lambda, r), r)(x) < t < I_\eta(\lambda, r)(x)$. Since $I_\eta(\lambda, r)(x) > t$, there exists $\mu \in L^X$ with $\eta(\mu, \lambda) > r$ such that $I_\eta(\lambda, r)(x) \ge \mu(x) > t$. Since (X, η) is a fuzzy topogenous space, by (S2) of Definition 3.7, there exist η such that $\eta \le \eta \odot \eta$. It follows $r < \eta(\mu, \lambda) \le \eta \odot \eta(\mu, \lambda)$. Since $\eta \odot \eta(\mu, \lambda) > r$, there exist $\rho \in L^X$ such that $\eta \odot \eta(\mu, \lambda) \ge \eta(\mu, \rho) \odot \eta(\rho, \lambda) > r$. Hence $\mu \le I_\eta(\rho, r), \rho \le I_\eta(\lambda, r)$. Thus $I_\eta(I_\eta(\lambda, r), r)(x) \ge \mu(x) > t$. It is a contradiction.

Theorem 3.10.
Let (X, η) be a fuzzy topogenous space. Define a map $\tau_\eta : L^X \to L$ by $\tau_\eta(\lambda) = \sup\{r \in L_1 \mid I_\eta(\lambda, r) = \lambda\}$. Then τ_η is an L-fuzzy topology on X induced by η.

Proof:
(O1) Since $I_\eta(\underline{0}, r) = \underline{0}$ and $I_\eta(\underline{1}) = \underline{1}$, for all $r \in L_1, \tau_\eta(\underline{0}) = \tau_\eta(\underline{1}) = 1$.
(O2) Suppose there exist $\lambda_1, \lambda_2 \in L^X$ and $r \in (0, 1)$ such that $\tau_\eta(\lambda_1 \odot \lambda_2) < t < \tau_\eta(\lambda_1) \odot \tau_\eta(\lambda_2)$.
Since $\tau_\eta(\lambda_1) > t$ and $\tau_\eta(\lambda_2) > t$, there exist $r_1, r_2 > t$ such that $\lambda_i = I_\eta(\lambda_i, r_i), i = 1, 2$.Put $r = r_1 \odot r_2$.By Theorem 3.9 (4), we have $I_\eta(\lambda_1 \odot \lambda_2, r) = I_\eta(\lambda_1, r) \odot I_\eta(\lambda_2, r) \ge I_\eta(\lambda_1, r_1) \odot I_\eta(\lambda_2, r_2) = \lambda_1 \odot \lambda_2$. From Theorem 3.9(2), we have $I_\eta(\lambda_1 \odot \lambda_2, r) \le \lambda_1 \odot \lambda_2$ and so $I_\eta(\lambda_1 \odot \lambda_2, r) = \lambda_1 \odot \lambda_2$. Consequently $\tau_\eta(\lambda_1 \odot \lambda_2) \ge r > t$. It is a contradiction. Hence, $\tau_\eta(\lambda_1 \odot \lambda_2) \ge \tau_\eta(\lambda_1) \odot \tau_\eta(\lambda_2)$.
(O3) Suppose there exist a family $\{\lambda_j \in L^X \mid j \in \Gamma\}$ and $r \in (0, 1)$ such that $\tau_\eta(\bigvee_{j\in\Gamma} \lambda_j) < t < \bigwedge_{j\in\Gamma} \tau_\eta(\lambda_j)$.
Since $\bigwedge_{j\in\Gamma} \tau_\eta(\lambda_j) > t$, for each $j \in \Gamma$, there exists $r_j > t$ such that $\lambda_j = I_\eta(\lambda_j, r_j)$. Put $r = \bigwedge_{j\in\Gamma} r_j$. By Theorem 2.9(5), we have $I_\eta(\bigvee_{j\in\Gamma} \lambda_j, r) \ge \bigvee_{j\in\Gamma} I_\eta(\lambda_j, r) \ge \bigvee_{j\in\Gamma} I_\eta(\lambda_j, r_j) = \bigvee_{j\in\Gamma} \lambda_j$. Consequently, $\tau_\eta(\bigvee_{j\in\Gamma} \lambda_j) \ge r > t$. It is a contradiction. Hence, $\tau_\eta(\bigvee_{j\in\Gamma} \lambda_j) \ge \bigwedge_{j\in\Gamma} \tau_\eta(\lambda_j)$.

Definition 3.11.
Let (X, η_1) and (Y, η_2) be L-fuzzy topogenous spaces. A function $f : (X, \eta_1) \to (Y, \eta_2)$ is said to be L-fuzzy topogenous continuous if $\eta_2(\lambda, \mu) \le \eta_1(f^{-1}(\lambda), f^{-1}(\mu)), \forall \lambda, \mu \in L^Y$.

Theorem 3.12.
Let $(X, \eta_1), (Y, \eta_2)$ and (Z, η_3) be L-fuzzy topogenous spaces. If $f : (X, \eta_1) \to (Y, \eta_2)$ and $g : (Y, \eta_2) \to (Z, \eta_3)$ are L-fuzzy topogenous continuous, then $g \circ f : (X, \eta_1) \to (Z, \eta_3)$ is L-fuzzy topogenous continuous.

Proof:
It follows that, for each $\lambda, \mu \in L^Z$, $\eta_1((g \circ f)^{-1}(\lambda), (g \circ f)^{-1}(\mu)) = \eta_1(f^{-1}(g^{-1}(\lambda)), f^{-1}(g^{-1}(\mu))) \ge \eta_2(g^{-1}(\lambda), g^{-1}(\mu)) \ge \eta_3(\lambda, \mu)$.

Theorem 3.13.
Let (X,η_1) and (Y,η_2) be L-fuzzy topogenous spaces. Let $f:(X,\eta_1)\to(Y,\eta_2)$ be topogenous continuous. Then it satisfies the following statements:
(1) $f^{-1}(I_{\eta_2}(\mu,r))\le I_{\eta_1}(f^{-1}(\mu),r)$, for each $\mu\in L^Y$.
(2) $f:(X,\tau_{\eta_1})\to(Y,\tau_{\eta_2})$ is a fuzzy continuous.

Proof:
(1) $f^{-1}(I_{\eta_2}(\mu,r))=f^{-1}(\vee\{\rho\in L^Y\mid\eta_2(\rho,\mu)>r\})=\vee(\{f^{-1}(\rho)\in L^X\mid\eta_2(\rho,\mu)>r\})$
$\le\vee\{f^{-1}(\rho)\in L^X\mid\eta_1(f^{-1}(\rho),f^{-1}(\mu))>r\}\le\vee\{\lambda\in L^X\mid\eta_1(\lambda,f^{-1}(\mu))>r\}=I_{\eta_1}(f^{-1}(\mu),r).$
(2) It is easily proved from Theorem 3.10.

Definition 3.14.
Let $(X_i\ ,\eta_i)_{i\in\Gamma}$ be a family of L-fuzzy topogenous spaces. Let X be a set and for each $i\in\Gamma$, f_i $:X\to X_i$ be a function. The initial structure η is the coarsest L-fuzzy topogenous on X with respect to which for each $i\in\Gamma$, f_i is an L-fuzzy topogenous map.

Theoreom 3.15.
Let $(X_i\ ,\eta_i)_{i\in\Gamma}$ be a family of L-fuzzy topogenous spaces. Let X be a set and for each $i\in\Gamma$, $f_i:X\to X_i$ a mapping. We define a map $\eta:L^X\times L^X\to L$ on X by $\eta(\lambda,\mu)=\vee\{\underset{j,k}{\wedge}\ \underset{i\in\Gamma}{\vee}\eta_i(f_i(\lambda_j),\ f_i^*(\mu_k^*))\}$, where for every finite families $\{\lambda_j\mid\lambda=\vee_{j=1}^n\lambda_j\}$ and $\{\mu_k\mid\mu=\wedge_{k=1}^m\mu_k\}$.Then
(1) A map $f:(Y,\acute{\eta})\to(X,\eta)$ is topogenous continuous iff each $f_i\circ f:(Y,\acute{\eta})\to(X,\eta)$ is topogenous continuous.
(2) $\tau_\eta=\Pi_{i\in\Gamma}\ \tau_{f_i^{-1}(\eta_i)}$.
(3) If $(X_i\ ,\eta_i)_{i\in\Gamma}$ is symmetric for each $i\in\Gamma$, then (X,η) is symmetric.

Proof:
(1) Necessity of composition condition is clear since the composition of topogenous continuous maps is topogenous continuous.
Conversely, suppose that f is not topogenous continuous map. Then there exists $\lambda,\mu\in L^X$ such that $\acute{\eta}(f^{-1}(\lambda),f^{-1}(\mu))<r<\eta(\lambda,\mu)$. Since $\eta(\lambda,\mu)>r$, therefore there are finite families $(\acute{\lambda_j}),(\acute{\mu_k})$ such that $\lambda=\vee_{j=1}^p\lambda_j$, $\mu=\wedge_{k=1}^q\mu_k$, and $\eta(\lambda,\mu)\ge\underset{j,k}{\wedge}\ \underset{i\in\Gamma}{\vee}\eta_i(f_i(\lambda_j),\ f_i^*(\mu_k^*))>r$. It follows that for any j,k, there exists $i_{jk}\in\Gamma$ such that $\eta_{i_{jk}}(f_{i_{jk}}(\lambda_j),f_{i_{jk}}^*(\mu_k^*))>r$. On the other hand, since $f_i\circ f$ is topogenous continuous and $f_i(f(f^{-1}(\lambda_j)))\le f_i(\lambda_j)$, also, $r<\underset{j,k}{\wedge}\ \eta_{i_{jk}}(f_{i_{jk}}(\lambda_j),f_{i_{jk}}^*(\mu_k^*))\le\underset{j,k}{\wedge}\ \acute{\eta}((f_{i_{jk}}\circ f)^{-1}(f_{i_{jk}}(\lambda_j)),(f_{i_{jk}}\circ f)^{-1}(f_{i_{jk}}^*(\mu_k^*)))\le\underset{j,k}{\wedge}\ \acute{\eta}(f^{-1}(\lambda_j),f^{-1}(\mu_k))$
$=\acute{\eta}(f^{-1}(\lambda),f^{-1}(\mu))$. It is a contradiction.

(2) Suppose first, $\tau_\eta\not\le\Pi_{i\in\Gamma}\tau_{f_i^{-1}(\eta_i)}$. There exist $\lambda\in L^X$ and $r\in(0,1)$ such that $\tau_\eta(\lambda)>r>\Pi_{i\in\Gamma}\tau_{f_i^{-1}(\eta_i)}(\lambda)$.There exists $r_0\in L_0$ with $r_0>r$ such that $\lambda=I_\eta(\lambda,r_0)$.It implies $\lambda=I_\eta(\lambda,r_0)=\vee\{\mu\in L^X\mid\eta(\mu,\lambda)>r_0\}$.Since $\eta(\mu,\lambda)>r_0$, there a finite families $\{\mu_j\mid\mu=\vee_{j=1}^n\mu_j\}$ and $\{\lambda_k\mid\lambda=\wedge_{k=1}^m\lambda_k\}$ such that $\eta(\mu,\lambda)\ge\underset{j,k}{\wedge}\ \underset{i\in\Gamma}{\vee}(\eta_i(f_i(\mu_j),\ f_i^*(\lambda_k^*))>r$, i.e, for all j,k, we have $\underset{i\in\Gamma}{\vee}(\eta_i(f_i(\mu_j),\ f_i^*(\lambda_k^*))>r_0$. It follows that for any j,k, there exists an $i_{jk}\in\Gamma$ such that $f_{i_{jk}}^{-1}(\eta_{i_{jk}})(\mu_j,\lambda_k)=\eta_{i_{jk}}(f_{i_{jk}}(\mu_j),f_{i_{jk}}^*(\lambda_k^*))>r_0$. It implies $I_{f_{i_{jk}}^{-1}(\eta_{i_{jk}})}(\lambda_k,r_0)\ge\mu_j$.Thus, $\lambda\ge\wedge_{k=1}^m\{\vee_{j=1}^n I_{f_{i_{jk}}^{-1}(\eta_{i_{jk}})}(\lambda_k,r_0)\}\ge\mu$. Put $\rho_{i_{jk}}=I_{f_{i_{jk}}^{-1}(\eta_{i_{jk}})}(\lambda_k,r_0)$, $\lambda=I_\eta(\lambda,r_0)=\vee\{\mu\in L^X\mid\eta(\mu,\lambda)>r_0\}=\vee\{\wedge_{i=1}^m(\vee_{j=1}^n\rho_{i_{jk}})\}$.
Since $I_{f_{i_{jk}}^{-1}(\eta_{i_{jk}})}(\lambda_k,r_0)=I_{f_{i_{jk}}^{-1}(\eta_{i_{jk}})}(I_{f_{i_{jk}}^{-1}(\eta_{i_{jk}})}(\lambda_k,r_0),r_0)$, $\tau_{f_{i_{jk}}^{-1}(\eta_{i_{jk}})}(\rho_{i_{jk}})\ge r_0>r$. It implies $\Pi_{i\in\Gamma}$ $\tau_{f_i^{-1}(\eta_i)}(\lambda)\ge r_0>r$. It is a contradiction.
Second the proof of $\tau_\eta\ge\Pi_{i\in\Gamma}\ \tau_{f_i^{-1}(\eta_i)}$ is similar to first.
(3) For every finite families $\{\lambda_j\mid\lambda=\vee_{j=1}^n\lambda_j\}$ and $\{\mu_k\mid\mu=\wedge_{k=1}^m\mu_k\}$,

$\eta(\lambda,\mu)=\vee\{\bigwedge_{j,k}\bigvee_{i\in\Gamma}(\eta_i(f_i(\lambda_j),f_i^*(\mu_k^*))\}=\vee\{\bigwedge_{j,k}\bigvee_{i\in\Gamma}(\eta_i^s(f_i(\lambda_j),f_i^*(\mu_k^*))\}=\vee\{\bigwedge_{j,k}\bigvee_{i\in\Gamma}(\eta_i(f_i(\mu_k^*),f_i^*(\lambda_j))\}=\eta(\mu^*,\lambda^*)=\eta^s(\lambda,\mu)$.

By the above Theorem, we can define the subspaces and products in the obvious way.

Definition 3.16.

Let (X,η) be an L-fuzzy topogenous structures and A be a subset of X. The pair (A,η_A) is said to be a subspace of (X,η) if it is endowed with the initial L-fuzzy topogenous structures with respect to the inclusion map.

Definition 3.17.

Let X be the product $\Pi_{i\in\Delta}X_i$ of the family $\{(X_i,\eta_i)\mid i\in\Delta\}$ of L-fuzzy topogenous structures. An initial L-fuzzy topogenous structures $\eta=\otimes\eta_i$ on X with respect to all projections $\pi_i:X\to X_i$ is called the product L-fuzzy topogenous structure $\{\eta_i\mid i\in\Delta\}$ and $(X,\otimes\eta_i)$ is called the product L-fuzzy topogenous structure.

Corollary 3.18.

Let $(X_i,\eta_i)_{i\in\Delta}$ be a family of L-fuzzy topogenous structures. Let $X=\Pi_{i\in\Delta}X_i$ be a set and for each $i\in\Delta$, $\pi_i:X\to X_i$ a mapping. The structure $\eta=\otimes\eta_i$ on X is defined by $\eta(\lambda,\mu)=\wedge\{\bigvee_{j,k}\bigwedge_{i\in\Delta}\eta_i(\pi_i(\lambda_j),\pi_i(\mu_k))\}$ where for every finite families $(\lambda_j),(\mu_k)$ such that $\lambda=\vee_{j=1}^n\lambda_j$ and $\mu=\vee_{k=1}^m\mu_k$, Then:

(1) η is the coarsest L-fuzzy topogenous on X with respect to which for each $i\in\Delta$, π_i is an L-fuzzy topogenous map.

(2) A map $f:(Y,\acute{\eta})\to(X,\eta)$ is an L-fuzzy topogenous map iff each $\pi_i\circ f:\ (Y,\acute{\eta})\to(X_i,\,\eta_i)$ is an L-fuzzy topogenous map.

Proposition 3.19.

(1) Let (X,η) be the L-fuzzy(resp. symmetric) topogenous space and let the mapping $\delta_\eta:L^X\times L^X\to L$ defined by $\delta_\eta(\mu,\lambda)=\eta^*(\mu,\lambda^*),\ \forall\lambda,\mu\in L^X$. Then δ_η is the L-fuzzy quasi-proximity (resp. L-fuzzy proximity) on X.

(2) Let δ be the L-fuzzy quasi-proximity (resp. L-fuzzy proximity) on X and let the mapping δ_η : $L^X\times L^X\to L$ defined by $\eta_\delta(\mu,\lambda)=\delta^*(\mu,\lambda^*),\ \forall\lambda,\mu\in L^X$. Then η_δ is the L-fuzzy (resp. symmetric) topogenous space.

(3) $\eta=\eta_{\delta_\eta}$ and $\delta_{\eta_\delta}=\delta$.

Proof:

It is easily proved.

4 L-fuzzy quasi-uniform spaces and L-fuzzy syntopogenous.

Definition 4.1.

Let Ψ be a fuzzy biperfect syntopogenous structure on X. A function $S:\Psi\to L$ is called L-fuzzy syntopogenous structure on X satisfying for $\eta,\eta_1,\eta_2\in\Psi$, the following conditions:

(ST1) There exists $\eta\in\Psi$ such that $S(\eta)=1$.

Body Math (ST2) $S(\eta_1)\odot S(\eta_2)\leq\vee\{S(\eta)\mid\eta_1,\eta_2\leq\eta\}$.

Body Math (ST3) $S(\eta)\leq\vee\{S(\eta_1)\mid\eta_1\circ\eta_1\leq\eta\}$.

Body Math The pair (X,S) is said to be L-fuzzy syntopogenous space.

Body Math The L-fuzzy syntopogenous space (X, S) is said to be L-fuzzy symmetric syntopogenous space if it satisfies

Body Math (ST) $S(\eta) \leq \vee\{S(\zeta) \mid \zeta \geq \eta^s\}$.

Lemma 4.2.

For every $\alpha \in \Omega_X$,we define $\eta_\alpha : L^X \times L^X \to L$ as

$$\eta_\alpha(\mu, \lambda) = \begin{cases} 1, \text{ if } \lambda \geq \alpha(\mu) \\ 0, \text{ otherwise.} \end{cases}$$

Then it satisfies the following properties:

(1) The map $\eta_\alpha \in \Psi$ is a biperfect L-fuzzy topogenous order.

(2) If $\alpha \leq \beta$, then $\eta_\beta \leq \eta_\alpha$.

(3) If $\beta \leq \alpha_1 \odot \alpha_2$, then $\eta_{\alpha_1}, \eta_{\alpha_2} \leq \eta_\beta$.

(4) For each $\alpha \in \Omega_X$, we have $\eta^*_\alpha = \eta_{\alpha^{-1}}$.

(5) If $\beta \circ \beta \leq \alpha$, then $\eta_\beta \circ \eta_\beta \geq \eta_\alpha$.

Lemma 4.3.

Let Ψ be a fuzzy biperfect syntopogenous structure on X where for each $\eta \in \Psi$, the range of η is finite. We define $\alpha_\eta(\mu) = \wedge \{\lambda \in L^X \mid \eta(\mu, \lambda) > 0\}$. Then it satisfies the following conditions:

(1) $\alpha_\eta \in \Psi$.

(2) If $\eta \leq \zeta$ and $\alpha_\eta \in \Psi$, then $\alpha_\zeta \leq \alpha_\eta$.

(3) If $\gamma, \zeta \leq \eta$ and $\alpha_\eta, \alpha_\gamma \in \Psi$, then $\alpha_\eta \leq \alpha_\zeta \odot \alpha_\gamma$.

(4) $\alpha_{\eta^s} = (\alpha_\eta)^{-1}$.where $(\alpha_\eta)^{-1}(\mu) = \wedge \{\lambda \in L^X \mid \alpha_\eta(\lambda^*) \leq \mu^*\}$
and $\alpha_{\eta^s}(\mu) = \wedge \{\lambda \in L^X \mid \eta(\lambda^*, \mu^*) > 0\}$.

(5) For each $\alpha_\eta \in \Omega_X$, there exists $\alpha_\zeta \in \Omega_X$ such that $\alpha_\zeta \odot \alpha_\zeta \leq \alpha_\eta$.

(6) $\alpha_{\eta_\alpha} = \alpha$.

Theorem 4.4.

Let $S : \Psi \to L$ be L-fuzzy syntopogenous structures (resp. L-fuzzy symmetric) on X where for each $\eta \in \Psi$,the range of η is finite. Define $B_S : \Omega_X \to L$ as $B_S(\alpha_\eta) = \vee\{S(\eta) \mid \eta \text{ induces } \alpha_\eta\}$. Then

(1) B_S is L-fuzzy quasi-uniform (resp. L-fuzzy uniform) base on X.

(2) If $B : \Omega_X \to L$ is L-fuzzy quasi-uniform base on X, then $B_{S_B} = B$.

Theorem 4.5.

Let (X, S) be L-fuzzy syntopogenous space. The mapping $C_S : L^X \times L_1 \to L^X$, is defined by $C_S(\lambda, r) = \wedge \{\mu \mid \eta(\lambda, \mu) > 0, S(\eta) > r\}$. For each $\lambda, \lambda_1, \lambda_2 \in L^X$ and $r, r_1, r_2 \in L_1$, we have the following properties:

(1) $C_S(\underline{0}, r) = \underline{0}$.

(2) $\lambda \leq C_S(\lambda, r)$.

(3) If $\lambda_1 \leq \lambda_2$ then $C_S(\lambda_1, r) \leq C_S(\lambda_2, r)$.

(4) $C_S(\lambda_1 \oplus \lambda_2, r) = C_S(\lambda_1, r) \oplus C_S(\lambda_2, r)$.

(5) If $r_1 \leq r_2$, then $C_S(\lambda, r_1) \leq C_S(\lambda, r_2)$.

(6) $C_S(C_S(\lambda, r), r) = C_S(\lambda, r)$.

Proof:

(1) Since $\eta(\underline{0}, \underline{0}) = 1$, for $S(\eta) = 1, C_S(\underline{0}, r) = \underline{0}$.

(2) Since $\lambda \leq \mu$ for $\eta(\lambda, \mu) > 0$ we have $\lambda \leq C_S(\lambda, r)$.

(3) and (5) are easily proved.

(4) First $C_S(\lambda_1 \oplus \lambda_2, r) \geq C_S(\lambda_1, r) \oplus C_S(\lambda_2, r)$, it is obvious from (3).
Second, suppose there exist $\lambda_1, \lambda_2 \in L^X$ and $r \in L_1$ such that
$C_S(\lambda_1 \oplus \lambda_2, r) \not\geq C_S(\lambda_1, r) \oplus C_S(\lambda_2, r)$.
There exist $x \in X$ and $t \in L_1$ such that
$$C_S(\lambda_1 \oplus \lambda_2, r)(x) > t > C_S(\lambda_1, r)(x) \oplus C_S(\lambda_2, r)(x). \quad (A)$$
Since $C_S(\lambda_i, r) < t$, for each $i \in \{1,2\}$, there exist $\eta_i \in L^X$ with $S(\eta_i) > r$ and $\eta(\lambda_i, \mu_i) > 0$ such that $C_S(\lambda_i, r)(x) \leq \mu_i(x) < t$. On the other hand, since $S(\eta_1) \odot S(\eta_2) > r$, By (ST2) of Definition 4.1, there exists η with $\eta \geq \eta_i$ and $S(\eta) > r$ such that $\eta(\lambda_1 \oplus \lambda_2, \mu_1 \oplus \mu_2) \geq \eta(\lambda_1, \mu_1 \oplus \mu_2) \odot \eta(\lambda_2, \mu_1 \oplus \mu_2) \geq \eta(\lambda_1, \mu_1) \odot \eta(\lambda_2, \mu_2) \geq \eta_1(\lambda_1, \mu_1) \odot \eta_2(\lambda_2, \mu_2) > 0$.
Hence $C_S(\lambda_1 \oplus \lambda_2, r)(x) \leq (\mu_1 \oplus \mu_2)(x) < t$. It is a contradiction for the equation (A).
(6) Suppose there exist $\lambda \in L^X$ and $r \in L_1$ such that $C_S(C_S(\lambda, r), r) \not\leq C_S(\lambda, r)$.There exist $x \in X$ and $t \in (0,1)$ such that $C_S(C_S(\lambda, r), r)(x) > t > C_S(\lambda, r)(x)$.Since $C_S(\lambda, r)(x) < t$, there exists $\mu \in L^X$ with $S(\eta) > r$ and $\eta(\lambda, \mu) > 0$ such that $C_S(\lambda, r)(x) \leq \mu(x) < t$. On the other hand, since $S(\eta) > r$, by (ST3) of Definition 4.1, there exist $\zeta \in \Psi$ such that $\zeta \circ \zeta(\lambda, \mu) > 0$,there exists $\rho \in L^X$ such that $\zeta(\lambda, \rho) \odot \zeta(\rho, \mu) > 0$. It implies $C_S(\lambda, r) \leq \rho, C_S(\rho, r) \leq \mu$. Hence $C_S(C_S(\lambda, r), r) \leq \mu$. Thus, $C_S(C_S(\lambda, r), r)(x) \leq \mu(x) < t$. It is a contradiction.

Theorem 4.6.
Let (X, S) be L-fuzzy syntopogenous space. Define a map $\tau_S : L^X \to L$ by $\tau_S(\lambda) = \vee \{r \in L_1 \mid C_S(\lambda^*, r) = \lambda^*\}$. Then τ_S is L-fuzzy topology on X induced by S.

Definition 4.7.
Let (X, S_1) and (Y, S_2) be two L-fuzzy syntopogenous spaces. The mapping $f : (X, S_1) \to (Y, S_2)$ is

said to be syntopogenous continuous if for each $\zeta \in \Psi_Y$, there exists $\eta \in \Psi_X$ with $\eta(f^{-1}(\mu), f^{-1}(\lambda)) \geq \zeta(\mu, \lambda)$ such that $S_2(\zeta) \leq S_1(\eta)$.

Theorem 4.8.
Let (X, S_1) and (Y, S_2) be two L-fuzzy syntopogenous spaces. Let $f : (X, S_1) \to (Y, S_2)$ be syntopogenous continuous map. Then we have the following properties:
(1) If the ranges of η and ζ are finite sets for each $\eta \in \Psi_X$ and $\zeta \in \Psi_Y$ then $f : (X, U_{S_1}) \to (Y, U_{S_2})$ is L-fuzzy quasi-uniform continuous where U_{S_i} is generated by B_{S_i} for $i \in \{1,2\}$.
(2) $f(C_{S_1}(\lambda, r)) \leq C_{S_2}(f(\lambda), r)$.
(3) $C_{S_1}(f^{-1}(\mu), r) \leq f^{-1}(C_{S_2}(\mu, r))$.
(4) $f : (X, \tau_{S_1}) \to (Y, \tau_{S_2})$ is a fuzzy continuous map.

Proof:
(1) We show that $B_{S_2}(\alpha_\zeta) \leq B_{S_1}(f^{\leftarrow}(\alpha_\zeta))$.Since $f^{\leftarrow}(\alpha_\zeta)(\lambda) = f^{-1}(\alpha_\zeta)(f(\lambda))$, and $f^{-1}(\alpha_\zeta)(f(\lambda)) = f^{-1}(\wedge \{\rho \mid \zeta(f(\lambda), \rho) > 0\}) = \wedge \{f^{-1}(\rho) \mid \zeta(f(\lambda), \rho) > 0\}$. Since f is syntopogenous continuous, for each $\zeta \in \Psi_Y$, there exists $\eta \in \Psi_X$ with $\eta(f^{-1}(f(\lambda)), f^{-1}(\rho)) \geq \zeta(f(\lambda), \rho)$ such that $S_1(\eta) \geq S_2(\zeta)$. Since $\eta(\lambda, f^{-1}(\rho)) \geq \eta(f^{-1}(f(\lambda)), f^{-1}(\rho))$, $f^{\leftarrow}(\alpha_\zeta)(\lambda) \geq \alpha_\eta(\lambda)$. It implies $B_{S_1}(f^{\leftarrow}(\alpha_\zeta)) \geq B_{S_1}(\alpha_\eta) \geq B_{S_2}(\alpha_\zeta)$.
(2) Suppose there exist $\lambda \in L^X$ and $r \in L_1$ such that $f(C_{S_1}(\lambda, r)) \not\leq C_{S_2}(f(\lambda), r)$. There exist $y \in Y$ and $t \in L_0$ such that $f(C_{S_1}(\lambda, r))(y) > t > C_{S_2}(f(\lambda), r)(y)$. Since $f^{-1}(\{y\}) = \varphi$, provides a contradiction that $f(C_{S_1}(\lambda, r))(y) = 0, f^{-1}(\{y\}) \neq \varphi$,and there exists $x \in f^{-1}(\{y\})$ such that $f(C_{S_1}(\lambda, r))(y) \geq C_{S_1}(\lambda, r))(x) > t > C_{S_2}(f(\lambda), r)(f(x))$. Since $C_{S_2}(f(\lambda), r)(f(x)) < t$, there exists $\zeta \in \Psi_Y$ with $S_2(\zeta) > r$ and $\zeta(f(\lambda), \mu) > 0$ such that $C_{S_2}(f(\lambda), r)(f(x)) \leq \mu(f(x)) < t$. On the other hand, since f is syntopogenous, for each $\zeta \in \Psi_Y$, there exists $\eta(f^{-1}(f(\lambda)), f^{-1}(\mu)) \geq \zeta(f(\lambda), \mu)$ such that $S_1(\eta) \geq S_2(\zeta) > r$. It implies $C_{S_1}(\lambda, r))(x) \leq f^{-1}(\mu)(x) < t$. It is a contradiction.

(3) and (4) are obvious.

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