SOME PROPERTIES OF r-T_2 SPACES

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Abstract. We introduce r-T_2 spaces in fuzzy topological spaces in view of Šostak[22] and investigate some properties of r-T_2 spaces. Moreover, we study properties of subspaces and products of r-T_2 spaces.

1. Introduction and preliminaries

Šostak [22] introduced the fundamental concept of a fuzzy topological structure as an extension of both crisp topology and Chang fuzzy topology [3]. It has been developed in many directions[6,8,9,11,12,13,14,18]. In [1,2,4,,5,7,10,17,21,23], the various separation axioms were introduced in fuzzy topological spaces in a sense of Chang [3] or Lowen[15]. Srivastava [24] introduced separation axioms in a view of the definition of Hazra.et.al.[8].

In this paper, we define r-T_2 space in fuzzy topological spaces in a view of the definition of Sostak. We investigate some properties of r-T_2 spaces. In particular, we study properties of subspaces and products of r-T_2 spaces.

Throughout this paper, let X be a nonempty set, $I = [0,1]$ and $I_0 = (0,1]$. For $\alpha \in I$, $\overline{\alpha}(x) = \alpha$ for all $x \in X$. A *fuzzy point* x_t for $t \in I_0$ is an element of I^X such that

$$x_t(y) = \begin{cases} t, & \text{if } y = x, \\ 0, & \text{if } y \neq x. \end{cases}$$

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The set of all fuzzy points in X is denoted by $Pt(X)$. A fuzzy point $x_t \in \lambda$ iff $t \leq \lambda(x)$. A fuzzy set λ is quasi-coincident with μ, denoted by $\lambda\, q\, \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. If λ is not quasi-coincident with μ, we denote $\lambda\, \overline{q}\, \mu$.

Definition 1.1([22]). A function $\tau : I^X \to I$ is called a *fuzzy topology* on X if it satisfies the following conditions:
(O1) $\tau(\overline{0}) = \tau(\overline{1}) = 1$.
(O2) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$ for each $\mu_1, \mu_2 \in I^X$.
(O3) $\tau(\bigvee_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \tau(\mu_i)$ for any $\{\mu_i\}_{i\in\Gamma} \subset I^X$.
The pair (X, τ) is called a *fuzzy topological space*.

Let τ_1 and τ_2 be fuzzy topologies on X. We say τ_1 is *finer* than τ_2 (τ_2 is *coarser* than τ_1) if $\tau_2(\mu) \leq \tau_1(\mu)$ for all $\mu \in I^X$.

Theorem 1.2 ([8]). Let (X, τ) be a fuzzy topological space. For each $r \in I_0, \lambda \in I^X$, we define an operator $C_\tau : I^X \times I_0 \to I^X$ as follows:

$$C_\tau(\lambda, r) = \bigwedge\{\mu \mid \mu \geq \lambda, \tau(\overline{1} - \mu) \geq r\}.$$

Then it satisfies the following properties:
(1) $C_\tau(\overline{0}, r) = \overline{0}, C_\tau(\overline{1}, r) = \overline{1}$, for all $r \in I_0$.
(2) $C_\tau(\lambda, r) \geq \lambda$.
(3) $C_\tau(\lambda_1, r) \leq C_\tau(\lambda_2, r)$, if $\lambda_1 \leq \lambda_2$.
(4) $C_\tau(\lambda \vee \mu, r) = C_\tau(\lambda, r) \vee C_\tau(\mu, r)$, for all $r \in I_0$.
(5) $C_\tau(\lambda, r) \leq C_\tau(\lambda, r')$, if $r \leq r'$, where $r, r' \in I_0$.
(6) $C_\tau(C_\tau(\lambda, r), r) = C_\tau(\lambda, r)$.

Definition 1.3 ([11]). Let $\overline{0} \notin \Theta_X$ be a subset of I^X. A function $\beta : \Theta_X \to I$ is called a *fuzzy topological base* on X if it satisfies the following conditions:
(B1) $\beta(\overline{1}) = 1$.
(B2) $\beta(\mu_1 \wedge \mu_2) \geq \beta(\mu_1) \wedge \beta(\mu_2)$, for all $\mu_1, \mu_2 \in \Theta_X$.

A fuzzy topological base β always *generates* a fuzzy topology τ_β on X in the following sense:

Theorem 1.4 ([11]). Let β be a fuzzy topological base on X. Define the function $\tau_\beta : I^X \to I$ as follows: for each $\mu \in I^X$,

$$\tau_\beta(\mu) = \begin{cases} \bigvee\{\bigwedge_{i\in J} \beta(\mu_i)\} & \text{if } \mu = \bigvee_{i\in J} \mu_i, \quad \mu_j \in \Theta_X, \\ 1 & \text{if } \mu = \overline{0}, \\ 0 & \text{otherwise} \end{cases}$$

where the first $\bigvee$ is taken over all families $\{\mu_i \in \Theta_X \mid \mu = \bigvee_{i\in J} \mu_i\}$.
Then (X, τ_β) is a fuzzy topological space.

Let (X, τ_1) and (Y, τ_2) be fuzzy topological spaces. A function $f : (X, \tau_1) \to (Y, \tau_2)$ is called *fuzzy continuous* if $\tau_2(\mu) \leq \tau_1(f^{-1}(\mu))$ for all $\mu \in I^Y$.

Theorem 1.5 ([11]). Let $(X_i, \tau_i)_{i\in\Gamma}$ be fuzzy topological spaces and X a set and $f_i : X \to X_i$ a function , for each $i \in \Gamma$. Let $\Theta_X = \{\overline{0} \neq \mu = \bigwedge_{i\in F} f_i^{-1}(\nu_i) \mid \tau_i(\nu_i) > 0,\ i \in F\}$ be given, for every finite index set $F \subset \Gamma$. Define a function $\beta : \Theta_X \to I$ on X by

$$\beta(\mu) = \bigvee\{\bigwedge_{i\in F} \tau_i(\nu_i) \mid \mu = \bigwedge_{i\in F} f_i^{-1}(\nu_i)\}$$

where the first $\bigvee$ is taken over all finite index subset F of Γ. Then:

(1) β is a fuzzy topological base on X.

(2) The fuzzy topology τ_β generated by β is the coarsest fuzzy topology on X for which each $i \in \Gamma$, f_i is fuzzy continuous.

(3) A map $f : (Z, \tau_Z) \to (X, \tau_\beta)$ is fuzzy continuous iff for each $i \in \Gamma$, $f_i \circ f$ is fuzzy continuous.

From Theorem 1.5, we can define the following definitions.

Definition 1.6 ([11]). Let (X, τ) be a fuzzy topological space and A be a subset of X. The pair $(A, \tau|_A)$ is said to be a *subspace* of (X, τ) if $\tau|_A$ is the coarsest fuzzy topology on A for which the inclusion map i is fuzzy continuous.

Definition 1.7 ([11]). Let X be the product $\prod_{i\in\Gamma} X_i$ of the family $\{(X_i, \tau_i) \mid i \in \Gamma\}$ of fuzzy topological spaces. The the coarsest fuzzy topology $\tau = \bigotimes \tau_i$ on X for which each the projections $\pi_i : X \to X_i$ is fuzzy continuous is called the *product fuzzy topology* of $\{\tau_i \mid i \in \Gamma\}$, and (X, τ) is called the *product fuzzy topology space.*

2. The properties of r-T_2 spaces

Definition 2.1. Let (X, τ) be a fuzzy topological space. A fuzzy set $\mu \in I^X$ is called a *r-$\mathcal{Q}_\tau$ open neighborhood* of x_t if $x_t\ q\ \mu$ and $\tau(\mu) \geq r$. We denote

$$\mathcal{Q}_\tau(x_t, r) = \{\mu \in I^X \mid x_t\ q\ \mu,\ \tau(\mu) \geq r\}.$$

Definition 2.2. A fuzzy topological space (X, τ) is said to be a *r-T_2-space* if for each $x_t, y_s \in Pt(X)$ such that $x \neq y$, there exist $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_s, r)$ such that $\lambda \wedge \mu = \overline{0}$.

Theorem 2.3. A fuzzy topological space (X, τ) is r-T_2 iff for each $x_t, y_s \in Pt(X)$ such that $x \neq y$, and $t, s < 1$, there exist $\lambda, \mu \in I^X$ such that $x_t \in \lambda$, $\tau(\lambda) \geq r$, $y_s \in \mu$, $\tau(\mu) \geq r$ and $\lambda \wedge \mu = \overline{0}$.

Proof. ($\Rightarrow$) For each $x_t, y_s \in Pt(X)$ such that $x \neq y$, and $t, s < 1$, $x_{1-t}, y_{1-s} \in Pt(X)$. Since (X, τ) is r-T_2, there exist $\lambda \in Q_\tau(x_{1-t}, r)$ and $\mu \in Q_\tau(y_{1-s}, r)$ such that $\lambda \wedge \mu = \overline{0}$. Thus, $\lambda \in Q_\tau(x_{1-t}, r)$ implies $x_t \in \lambda$ and $\tau(\lambda) \geq r$. Thus, $\mu \in Q_\tau(y_{1-s}, r)$ implies $y_s \in \mu$ and $\tau(\mu) \geq r$.

($\Leftarrow$) Let $x_t, y_s \in Pt(X)$ such that $x \neq y$. Let $t, s < 1$. For $x_{1-t}, y_{1-s} \in Pt(X)$, there exist $\lambda, \mu \in I^X$ such that $x_{1-t} \in \lambda, \tau(\lambda) \geq r$, $y_{1-s} \in \mu, \tau(\mu) \geq r$ and $\lambda \wedge \mu = \overline{0}$. It implies $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_s, r)$.

If $t = 1$ or $s = 1$, let $t = 1$ and $s < 1$. There exists $0 < p < 1$ such that $x_p, y_{1-s} \in Pt(X)$. Then there exist $\lambda, \mu \in I^X$ such that $x_p \in \lambda \quad \tau(\lambda) \geq r$, $y_{1-s} \in \mu \quad \tau(\mu) \geq r$ and $\lambda \wedge \mu = \overline{0}$. Thus, $x_t \; q \; \lambda$ and $y_s \; q \; \mu$. Hence $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_s, r)$ such that $\lambda \wedge \mu = \overline{0}$. Hence (X, τ) is r-T_2. □

Theorem 2.4. A fuzzy topological space (X, τ) is r-T_2 iff for each $x_t, y_t \in Pt(X)$ such that $x \neq y$, there exist $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_t, r)$ such that $\lambda \wedge \mu = \overline{0}$.

Proof. ($\Rightarrow$) It is trivial.

($\Leftarrow$) Let $x_t, y_s \in Pt(X)$ such that $x \neq y$ and $t < s$. Since for each $x_t, y_t \in Pt(X)$, there exist $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_t, r)$ such that $\lambda \wedge \mu = \overline{0}$, then $y_t \; q \; \mu$ implies $y_s \; q \; \mu$. Hence (X, τ) is r-T_2. □

Definition 2.5 ([16]). Let D be a directed set. A function $S : D \to Pt(X)$ is called a *fuzzy net.*

Definition 2.6. Let (X, τ) be a fuzzy topological space, $\mu \in I^X$, $x_t \in Pt(X)$ and $r \in I_0$. A fuzzy point x_t is called a *fuzzy r-limit point* of S, denoted by $S \xrightarrow{r} x_t$, if for every $\mu \in Q_\tau(x_t, r)$, there exists $n_0 \in D$ such that for each $n \in D$ with $n \geq n_0$, we have $S(n) \; q \; \mu$.

We denote

$$lim_\tau(S, r) = \bigvee \{x_t \in Pt(X) \mid x_t \text{ is a fuzzy r-limit point of } S\}.$$

For $\lambda \in I^X$, we denote $supp(\lambda) = \{x \in X \mid \lambda(x) > 0\}$ and $|supp(\lambda)|$ is the cardinal number of $supp(\lambda)$.

Theorem 2.7. Let (X, τ) be a fuzzy topological space. Then the following statements are equivalent.

(1) (X, τ) is r-T_2.

(2) For each fuzzy net S, $|supp(lim_\tau(S, r))| \leq 1$.

Proof. (1)$\Rightarrow$ (2) Suppose there exists a fuzzy net $S : D \to Pt(X)$ such that $|supp(lim_\tau(S, r))| \geq 2$. There exist $x \neq y \in supp(lim_\tau(S, r))$ such that $S \xrightarrow{r} x_t, y_s$. Since (X, τ) is r-T_2, there exist $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_s, r)$ such that $\lambda \wedge \mu = \overline{0}$. Since $S \xrightarrow{r} x_t, y_s$, there exist n_1, n_2 such that

$$\forall n \geq n_1, \; S(n) \; q \; \lambda,$$

$$\forall n \geq n_2, \; S(n) \; q \; \mu.$$

Since D is a directed set, there exists $n_3 \geq n_1, n_2$ such that

$$\forall n \geq n_3,\ S(n)\ q\ \lambda,\ S(n)\ q\ \mu.$$

It implies $S(n)\ q\ \lambda \wedge \mu$, for all $n \geq n_3$. Since $\lambda \wedge \mu = \overline{0}$, it is a contradiction.

(2)$\Rightarrow$ (1) Let (X, τ) be not r-T_2. Then there exist $x_t, y_s \in Pt(X)$ with $x \neq y$ such that for all $\lambda \in Q_\tau(x_t, r)$ and for all $\mu \in Q_\tau(y_s, r)$, we have $\lambda \wedge \mu \neq \overline{0}$. Define a relation on $D = \{\lambda \wedge \mu \mid \lambda \in Q_\tau(x_t, r), \mu \in Q_\tau(y_s, r)\}$ by

$$\lambda_1 \wedge \mu_1 \prec \lambda_2 \wedge \mu_2 \text{ iff } \lambda_1 \geq \lambda_2, \mu_1 \geq \mu_2.$$

Then $(D, \prec)$ is a directed set. For each $\lambda \wedge \mu \in D$, since $\lambda \wedge \mu \neq \overline{0}$, there exist $z \in X$ and $p \in I_0$ such that $(\lambda \wedge \mu)(z) > 1 - p > 0$. Then $z_p\ q\ \lambda \wedge \mu$. Thus, we can define a fuzzy net $S : D \to Pt(X)$ by

$$S(\lambda \wedge \mu) = z_p, \text{ that is}, S(\lambda \wedge \mu) = \lambda \wedge \mu.$$

For every $\lambda \in Q_\tau(x_t, r)$, there exists $\lambda = \lambda \wedge \overline{1} \in D$ such that for all $\rho \in D$ with $\lambda \prec \rho$, we have $S(\rho)\ q\ \rho$. Since $\rho \leq \lambda$, we have $S(\rho)\ q\ \lambda$. Hence $x_t \in lim_\tau(S, r)$. Similarly, $y_s \in lim_\tau(S, r)$. Thus, $|supp(lim_\tau(S, r))| \geq 2$. □

Example 2.8. Let $X = \{x, y\}$ be a set. We define a fuzzy topology $\tau : I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \lambda = x_1, \\ \frac{1}{2}, & \text{if } \lambda = y_1, \\ 0, & \text{otherwise.} \end{cases}$$

For each $x_t, y_s \in Pt(X)$ such that $x \neq y$, for $0 < r \leq \frac{1}{2}$, there exist $x_1 \in Q_\tau(x_t, r)$ and $y_1 \in Q_\tau(y_s, r)$ such that $x_1 \wedge y_1 = \overline{0}$. Hence (X, τ) is r-T_2, for $0 < r \leq \frac{1}{2}$. Moreover, we easily show that (X, τ) is not r-T_2, for $r > \frac{1}{2}$.

Let N be a natural number set. Define a fuzzy net $S : N \to Pt(X)$ by

$$S(n) = \begin{cases} x_{0.4}, & \text{if } n = 2m, \\ y_{0.3}, & \text{if } n = 2m + 1, \end{cases}$$

We can show $lim_\tau(S, \frac{1}{2}) = \overline{0}$ from (1) to (2).

(1) x_t for $t \in I_0$ is not a fuzzy $\frac{1}{2}$-limit point of S, for $x_1 \in Q_\tau(x_t, \frac{1}{2})$ and for each $n \in N$, there exists $2n + 1 \in N$ such that $2n + 1 \geq n$ and $S(2n + 1)\ \bar{q}\ x_1$.

(2) y_s for $s \in I_0$ is not a fuzzy $\frac{1}{2}$-limit point of S, for $y_1 \in Q_\tau(y_s, \frac{1}{2})$ and for each $n \in N$, there exists $2n + 1 \in N$ such that $2n + 1 \geq n$ and $S(2n)\ \bar{q}\ y_1$.

Thus, $|supp(lim_\tau(S, \frac{1}{2}))| = 0$. □

Example 2.9. Let $X = \{x, y\}$ be a set. We define a fuzzy topology $\tau : I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{1}{2}, & \text{if } \lambda = x_{0.4}, \\ 0, & \text{otherwise.} \end{cases}$$

For $x_{0.3}, y_{0.5} \in Pt(X)$, since $Q_\tau(x_{0.3}, r) = Q_\tau(y_{0.5}, r) = \{\overline{1}\}$, for each $r \in I_0$, (X, τ) is not r-T_2.

Let N be a natural number set. Define a fuzzy net $S : N \to Pt(X)$ by

$$S(n) = x_{a_n},\ a_n = 0.5 + (-1)^n 0.2.$$

(1) x_t for $t \leq 0.6$ is a fuzzy r-limit point of S, for $\overline{1} \in Q_\tau(x_t, r)$ and for all $n \in N$,we have $S(n)\ q\ \overline{1}$.

(2) x_t for $0.6 < t$ and $0 < r \leq \frac{1}{2}$ is not a fuzzy r-limit point of S, for $x_{0.4} \in Q_\tau(x_t, r)$ and for each $n \in N$, there exists $2n+1 \in N$ such that $2n+1 \geq n$ and $S(2n+1) = x_{0.3}\ \overline{q}\ x_{0.4}$.

(3) y_s for $s \in I_0$ is a fuzzy r-limit point of S, for $\overline{1} \in Q_\tau(y_s, r)$ and for all $n \in N$,we have $S(n)\ q\ \overline{1}$.

From (1) to (3), put $\mu(x) = 0.6$ and $\mu(y) = 1$, we obtain

$$lim_\tau(S, r) = \begin{cases} \mu, & \text{if } 0 < r \leq \frac{1}{2}, \\ \overline{1}, & \text{if } r > \frac{1}{2}. \end{cases}$$

Thus, $|supp(lim_\tau(S, r))| = 2$. □

Define $\triangle_X \in I^{X \times X}$ as follows:

$$\triangle_X(x, y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{if } x \neq y. \end{cases}$$

Theorem 2.10. Let (X, τ) be r-T_2. Then $C_{\tau\otimes\tau}(\triangle_X, r) = \triangle_X$ where $\tau \otimes \tau$ is a product fuzzy topology on $X \times X$.

Proof. We only show that $C_{\tau\otimes\tau}(\triangle_X, r) \leq \triangle_X$ from Theorem 1.2 (2).

Suppose $C_{\tau\otimes\tau}(\triangle_X, r) \not\leq \triangle_X$. Then there exist $(x, y) \in X \times X$ and $t \in I_0$ such that

$$C_{\tau\otimes\tau}(\triangle_X, r)(x, y) > t > \triangle_X(x, y). \qquad (1)$$

Since $\triangle_X(x, y) < t$. Then $x \neq y$. Since (X, τ) is r-T_2, for $x_t, y_t \in Pt(X)$, there exist $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_t, r)$ such that $\lambda \wedge \mu = \overline{0}$. Put $\rho = \pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu)$. Then $\tau \otimes \tau(\rho) \geq \tau(\lambda) \wedge \tau(\mu) \geq r$. Moreover, since $x_t\ q\ \lambda$ and $y_t\ q\ \mu$, we have

$$(\pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu))(x, y) + t = \lambda(x) \wedge \mu(y) + t > 1.$$

Thus, $\rho \in Q_{\tau\otimes\tau}((x,y)_t, r)$. Since, for all $x \in X$,

$$\rho(x,x) = (\pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu))(x,x) = \lambda(x) \wedge \mu(x) = 0,$$

we have $\rho \leq \overline{1} - \triangle_X$. So, $\triangle_X \leq \overline{1} - \rho$ and $\tau \otimes \tau(\rho) \geq r$ implies

$$\triangle_X \leq C_{\tau\otimes\tau}(\triangle_X, r) \leq \overline{1} - \rho.$$

Since $(x,y)_t \; q \; \rho$,

$$C_{\tau\otimes\tau}(\triangle_X, r)(x,y) \leq (\overline{1} - \rho)(x,y) < t.$$

It is a contradiction for the equation (1). □

Theorem 2.11. Let $\tau \otimes \tau$ be a product fuzzy topology on $X \times X$ of a fuzzy topological space (X,τ). If $C_{\tau\otimes\tau}(\triangle_X, r) = \triangle_X$, then (X,τ) is $(r-\epsilon)$-T_2, for arbitrary $\epsilon > 0$.

Proof. Let $x_t, y_t \in Pt(X)$ with $x \neq y$. Since $C_{\tau\otimes\tau}(\triangle_X, r) = \triangle_X$, by the definition $C_{\tau\otimes\tau}$ of Theorem 1.2 and Definition 1.1 (O3), we have $\tau \otimes \tau(\overline{1} - \triangle_X) \geq r$. Put $\rho = \overline{1} - \triangle_X$. Then $\rho(x,y) = 1$ implies $(x,y)_t \; q \; \rho$. Let β be a base for $\tau \otimes \tau$. Since $\tau \otimes \tau(\rho) \geq r$, by Theorem 1.4, for $\epsilon > 0$, there exists a family $\{\rho_i \mid \rho = \bigvee_{i\in\Gamma} \rho_i\}$ such that

$$\tau \otimes \tau(\rho) \geq \bigwedge_{i\in\Gamma} \beta(\rho_i) > r - \epsilon.$$

Since $(x,y)_t \; q \; (\rho = \bigvee_{i\in\Gamma} \rho_i)$, there exists $i \in \Gamma$ such that $(x,y)_t \; q \; \rho_i$ and $\beta(\rho_i) > r - \epsilon$. From Theorem 1.5, there exist $\lambda, \mu \in I^X$ such that

$$\rho_i = \pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu), \; \beta(\rho_i) \geq \tau(\lambda) \wedge \tau(\mu) \geq r - \epsilon.$$

Therefore $\tau(\lambda) \geq r - \epsilon, \; \tau(\mu) \geq r - \epsilon$. Furthermore, since $(x,y)_t \; q \; \rho_i$, we have

$$(x,y)_t \; q \; \pi_1^{-1}(\lambda) \Rightarrow (\pi_1^{-1}(\lambda)(x,y) = \lambda(x)) + t > 1,$$

$$(x,y)_t \; q \; \pi_2^{-1}(\mu) \Rightarrow (\pi_2^{-1}(\mu)(x,y) = \mu(y)) + t > 1.$$

Hence $\lambda \in Q_\tau(x_t, r - \epsilon)$ and $\mu \in Q_\tau(y_t, r - \epsilon)$. Moreover, for each $x \in X$,

$$\begin{aligned}(\lambda \wedge \mu)(x) &= \pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu)(x,x) \\ &= \rho_i(x,x) \leq \rho(x,x) = 0,\end{aligned}$$

because $\rho(x,x) = (\overline{1} - \triangle_X)(x,x) = 0$. Thus, by Theorem 2.4, (X,τ) is $(r-\epsilon)$-T_2, for arbitrary $\epsilon > 0$. □

Example 2.12. Let $X = \{a, b\}$ and $X \times X$ be sets. We define a fuzzy topology $\tau : I^X \to I$ as follows:

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{3}{4}, & \text{if } \lambda = b_1, \\ \frac{1}{2}, & \text{if } \lambda = a_1, \\ 0, & \text{otherwise.} \end{cases}$$

For each $a_t, b_s \in Pt(X)$ and $0 < r \leq \frac{1}{2}$, there exist $a_1 \in Q_\tau(a_t, r)$ and $b_1 \in Q_\tau(b_s, r)$ such that $a_1 \wedge b_1 = \overline{0}$. Hence (X, τ) is r-T_2, for $0 < r \leq \frac{1}{2}$.

Let β be a base of product fuzzy topological space $(X \times X, \tau \otimes \tau)$. From Theorem 1.5, since

$$(a, b)_1 = \pi_1^{-1}(a_1) \wedge \pi_2^{-1}(b_1),$$

$$\beta((a, b)_1) = \tau(a_1) \wedge \tau(b_1) = \frac{1}{2}.$$

Since

$$(b, a)_1 = \pi_2^{-1}(a_1) \wedge \pi_1^{-1}(b_1),$$

$$\beta((b, a)_1) = \tau(a_1) \wedge \tau(b_1) = \frac{1}{2}.$$

Thus,

$$\tau \otimes \tau((a, b)_1 \vee (b, a)_1) = \frac{1}{2}.$$

Since $\triangle_X = \overline{1} - (a, b)_1 \vee (b, a)_1$, $C_{\tau \otimes \tau}(\triangle_X, \frac{1}{2}) = \triangle_X$. $\square$

Theorem 2.13. Let $f : (X, \tau) \to (Y, \tau_1)$ and $g : (X, \tau) \to (Z, \tau_2)$ be fuzzy continuous. Define a function $h : X \to Y \times Z$ by

$$h(x) = (f(x), g(x)).$$

Then $h : (X, \tau) \to (Y \times Z, \tau_1 \otimes \tau_2)$ is fuzzy continuous where $\tau_1 \otimes \tau_2$ is a product fuzzy topology of (Y, τ_1) and (Z, τ_2).

Proof. Suppose there exists $\rho \in I^{Y \times Z}$ such that

$$\tau(h^{-1}(\rho)) < \tau_1 \otimes \tau_2(\rho).$$

Let β be a base for $\tau_1 \otimes \tau_2$. By the definition of $\tau_1 \otimes \tau_2$, there exists a family $\{\rho_i \mid \rho = \bigvee_{i \in \Gamma} \rho_i\}$ such that

$$\tau(h^{-1}(\rho)) < \bigwedge_{i \in \Gamma} \beta(\rho_i) \leq \tau_1 \otimes \tau_2(\rho).$$

By the definition of β of Theorem 1.5, for each $i \in \Gamma$, there exist $\lambda_i \in I^Y$ and $\mu_i \in I^Z$ with $\rho_i = \pi_1^{-1}(\lambda_i) \wedge \pi_2^{-1}(\mu_i)$ such that

$$\tau(h^{-1}(\rho)) < \bigwedge_{i\in\Gamma}(\tau_1(\lambda_i) \wedge \tau_2(\mu_i)) \leq \bigwedge_{i\in\Gamma}\beta(\rho_i). \tag{2}$$

On the other hand, $(\pi_1 \circ h)^{-1}(\lambda_i)(x) = \lambda_i(\pi_1(h(x))) = \lambda_i(f(x)) = f^{-1}(\lambda_i)(x)$ for all $x \in X$, similarly, $(\pi_2 \circ h)^{-1}(\mu_i) = g^{-1}(\mu_i)$. Thus, we have

$$\begin{aligned} h^{-1}(\rho_i) &= h^{-1}(\pi_1^{-1}(\lambda_i) \wedge \pi_2^{-1}(\mu_i)) \\ &= h^{-1}(\pi_1^{-1}(\lambda_i)) \wedge h^{-1}(\pi_2^{-1}(\mu_i)) \\ &= (\pi_1 \circ h)^{-1}(\lambda_i) \wedge (\pi_2 \circ h)^{-1}(\mu_i) \\ &= f^{-1}(\lambda_i) \wedge g^{-1}(\mu_i). \end{aligned}$$

It follows

$$\begin{aligned} \tau(h^{-1}(\rho)) &= \tau(h^{-1}(\bigvee_{i\in\Gamma}\rho_i)) \\ &\geq \bigwedge_{i\in\Gamma}\tau(h^{-1}(\rho_i)) \\ &= \bigwedge_{i\in\Gamma}(\tau(f^{-1}(\lambda_i) \wedge g^{-1}(\mu_i))) \\ &\geq \bigwedge_{i\in\Gamma}(\tau(f^{-1}(\lambda_i)) \wedge \tau(g^{-1}(\mu_i))) \end{aligned}$$

(Since f and g are fuzzy continuous,)

$$\geq \bigwedge_{i\in\Gamma}(\tau_1(\lambda_i) \wedge \tau_2(\mu_i)).$$

It is a contradiction for the equation (2). □

Theorem 2.14. Let $f, g : (X, \tau_1) \to (Y, \tau_2)$ be fuzzy continuous. Let (Y, τ) be r-T_2 and $C_{\tau_1}(\chi_A, r) = \chi_X$ where A is a subset of X ,χ_A and χ_X are characteristic functions. If $f(a) = g(a)$ for all $a \in A$, then $f(x) = g(x)$ for all $x \in X$.

Proof. Suppose that there exists $x \in X - A$ with $f(x) \neq g(x)$. Since (Y, τ_2) is r-T_2, for $f(x)_1, g(x)_1 \in Pt(Y)$, there exist $\lambda \in Q_{\tau_2}(f(x)_1, r)$ and $\mu \in Q_{\tau_2}(g(x)_1, r)$ such that $\lambda \wedge \mu = \overline{0}$. Since $f(x)_1 \; q \; \lambda$ and $g(x)_1 \; q \; \mu$, we have

$$(f^{-1}(\lambda) \wedge g^{-1}(\mu))(x) > 0 \qquad (3).$$

On the other hand, let $\pi_i : Y \times Y \to Y$ be projection maps for each $i \in \{1, 2\}$ and $h : X \to Y \times Y$ defined by $h(x) = (f(x), g(x))$. We have

$$\begin{aligned}
&\lambda \wedge \mu = \overline{0} \\
&\Rightarrow \pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu) \wedge \triangle_Y = \overline{0} \\
&\Rightarrow h^{-1}(\pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu)) \wedge h^{-1}(\triangle_Y) = \overline{0} \\
&\Rightarrow f^{-1}(\lambda) \wedge g^{-1}(\mu) \wedge h^{-1}(\triangle_Y) = \overline{0} \\
&\Rightarrow h^{-1}(\triangle_Y) \leq \overline{1} - (f^{-1}(\lambda) \wedge g^{-1}(\mu)).
\end{aligned}$$

Since (Y, τ_2) is r-T_2, by Theorem 2.10, $C_{\tau_2 \otimes \tau_2}(\triangle_Y, r) = \triangle_Y$. Since $h : (X, \tau_1) \to (Y \times Y, \tau_2 \otimes \tau_2)$ is fuzzy continuous, by Theorem 1.5(3),

$$C_{\tau_1}(h^{-1}(\triangle_Y), r) \leq h^{-1}(C_{\tau_2 \otimes \tau_2}(\triangle_Y, r)) = h^{-1}(\triangle_Y).$$

Hence, by Theorem 1.2(2), $C_{\tau_1}(h^{-1}(\triangle_Y), r) = h^{-1}(\triangle_Y)$. Furthermore, Since $C_{\tau_1}(\chi_A, r) = \chi_X$ and $\chi_A \leq h^{-1}(\triangle_Y)$, we have

$$\begin{aligned}
\chi_X = C_{\tau_1}(\chi_A, r) \leq C_{\tau_1}(h^{-1}(\triangle_Y), r) &= h^{-1}(\triangle_Y) \\
&\leq \overline{1} - (f^{-1}(\lambda) \wedge g^{-1}(\mu)).
\end{aligned}$$

Thus, $\chi_X(x) = 1$ but $\overline{1}(x) - (f^{-1}(\lambda) \wedge g^{-1}(\mu))(x) < 1$. It is a contradiction for the equation (3). □

Theorem 2.15. If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy continuous and (Y, τ_2) is r-T_2, then $C_{\tau_1 \otimes \tau_2}(\chi_{G(f)}, r) = \chi_{G(f)}$ where $G(f) = \{(x, y) \in X \times Y |\ y = f(x)\}$.

Proof. Let $id_Y : (Y, \tau_2) \to (Y, \tau_2)$ be an identity map. Since $f \circ \pi_1 : (X \times Y, \tau_1 \otimes \tau_2) \to (Y, \tau_2)$ and $id_Y \circ \pi_2 : (X \times Y, \tau_1 \otimes \tau_2) \to (Y, \tau_2)$ are fuzzy continuous, by Theorem 2.13, $f \times id_Y : (X \times Y, \tau_1 \otimes \tau_2) \to (Y \times Y, \tau_2 \otimes \tau_2)$ is fuzzy continuous. From Theorem 1.5(3), it implies

$$C_{\tau_1 \otimes \tau_2}((f \times id_Y)^{-1}(\triangle_Y), r) \leq (f \times id_Y)^{-1}(C_{\tau_2 \otimes \tau_2}(\triangle_Y, r)).$$

Since (Y, τ_2) is r-T_2, we have $C_{\tau_2 \otimes \tau_2}(\triangle_Y, r) = \triangle_Y$. By Theorem 1.3(2),

$$C_{\tau_1 \otimes \tau_2}((f \times id_Y)^{-1}(\triangle_Y), r) = (f \times id_Y)^{-1}(\triangle_Y).$$

Since $(f \times id_Y)^{-1}(\triangle_Y) = \chi_{G(f)}$, we have $C_{\tau_1 \otimes \tau_2}(\chi_{G(f)}, r) = \chi_{G(f)}$ □

Theorem 2.16. Every subspace of r-T_2 spaces is a r-T_2 space.

Proof. Let $(A, \tau|_A)$ be a subspace of a r-T_2 space (X, τ). Let $a_t, b_s \in Pt(A)$ such that $a \neq b$. Then $a_t, b_s \in Pt(X)$ such that $a \neq b$. Since (X, τ) is r-T_2, there exists $\lambda \in Q_\tau(a_t, r)$ $\mu \in Q_\tau(b_s, r)$ such that $\lambda \wedge \mu = \overline{0}$. Since $\tau_A(i^{-1}(\lambda)) \geq \tau(\lambda) \geq r$ and a_t q $i^{-1}(\lambda)$, we have $i^{-1}(\lambda) \in Q_{\tau|_A}(a_t, r)$. Similarly, $i^{-1}(\mu) \in Q_{\tau|_A}(b_s, r)$. Moreover, $i^{-1}(\lambda) \wedge i^{-1}(\mu) = \overline{0}$. Hence $(A, \tau|_A)$ is r-T_2. □

Theorem 2.17. Let $\{(X_i,\tau_i) \mid i \in \Gamma\}$ be a family of r-T_2 spaces. Let (X,τ) be the product fuzzy topological space of $\{(X_i,\tau_i) \mid i \in \Gamma\}$. Then (X,τ) is r-T_2.

Proof. Let $x_t, y_s \in Pt(X)$ such that $x \neq y$. Then there exists $i \in \Gamma$ such that $(\pi_i(x))_t, (\pi_i(y))_s \in Pt(X_i)$ with $\pi_i(x) \neq \pi_i(y)$. Since (X_i,τ_i) is r-T_2, there exist $\lambda \in Q_{\tau_i}((\pi_i(x))_t, r)$ and $\mu \in Q_{\tau_i}((\pi_i(y))_s, r)$ with $\lambda \wedge \mu = \overline{0}$.

Since $\pi_i(x_t) = (\pi_i(x))_t \; q \; \lambda$ iff $x_t \; q \; \pi_i^{-1}(\lambda)$, we have

$$\pi_i^{-1}(\lambda) \in Q_\tau(x_t, r).$$

Similarly, $\pi_i^{-1}(\mu) \in Q_\tau(y_s, r)$. Moreover, $\pi_i^{-1}(\lambda) \wedge \pi_i^{-1}(\mu) = \overline{0}$. Therefore, (X,τ) is a r- T_2 space. $\square$

Theorem 2.18. Let $\{(X_i,\tau_i) \mid i \in \Gamma\}$ be a family of fuzzy topological spaces. Let (X,τ) be their product fuzzy topological space. If (X,τ) is a r-T_2 space, then (X_j,τ_j) is a $(r-\epsilon)$-T_2 space for each $\epsilon > 0$ and for each $j \in \Gamma$.

Proof. Let $(x^j)_t, (y^j)_s \in Pt(X_j)$ such that $x^j \neq y^j$. Then there exist $x^i \in X_i$ for all $i \in \Gamma - \{j\}$ such that $x \neq y \in X$ with

$$\pi_i(x) = \begin{cases} x^i, & \text{if } i \in \Gamma - \{j\}, \\ x^j, & \text{if } i = j, \end{cases} \qquad \pi_i(y) = \begin{cases} x^i, & \text{if } i \in \Gamma - \{j\}, \\ y^j, & \text{if } i = j. \end{cases}$$

Since (X,τ) is r-T_2 space , there exist

$$\rho \in Q_\tau(x_t, r),\ \omega \in Q_\tau(y_s, r),\ \rho \wedge \omega = \overline{0}$$

Let β be a base for τ. Since $\tau(\rho) \geq r$ and $\tau(\omega) \geq r$, for $\epsilon > 0$, there exists two families $\{\rho_k \mid \rho = \bigvee_{k\in K} \rho_k\}$ and $\{\omega_m \mid \omega = \bigvee_{m \in M} \omega_m\}$ such that

$$\tau(\rho) \geq \bigwedge_{k \in K} \beta(\rho_k) > r - \epsilon,$$

$$\tau(\omega) \geq \bigwedge_{m \in M} \beta(\omega_m) > r - \epsilon.$$

Since $x_t \; q \; (\rho = \bigvee_{k\in K} \rho_k)$ and $y_s \; q \; (\omega = \bigvee_{m\in M} \omega_m)$, there exist $k \in K$ and $m \in M$ such that

$$x_t \; q \; \rho_k,\ \beta(\rho_k) > r - \epsilon,$$
$$y_s \; q \; \omega_m,\ \beta(\omega_m) > r - \epsilon,$$
$$\rho_k \wedge \omega_m = \overline{0}. \tag{4}$$

Then there exist two family:

$\{\lambda_i \mid \rho_k = \bigwedge_{i\in F_1} \pi_i^{-1}(\lambda_i)\}$ and $\{\mu_l \mid \omega_m = \bigwedge_{l\in F_2} \pi_l^{-1}(\mu_l)\}$, where F_1 and F_2 are finite subsets of Γ such that

$$\beta(\rho_k) \geq \bigwedge_{i\in F_1} \tau_i(\lambda_i) > r - \epsilon,$$

$$\beta(\omega_m) \geq \bigwedge_{l\in F_2} \tau_l(\mu_l) > r - \epsilon.$$

Without loss of generality, we may assume $j \in F = F_1 = F_2$ because ,if necessary, we can take $F = F_1 \cup F_2 \cup \{j\}$ such that

$$\lambda_i = \overline{1},\ \forall\, i \in F_2 \cup \{j\} \text{ and } \mu_l = \overline{1},\ \forall\, l \in F_1 \cup \{j\}.$$

Hence we have

$$\beta(\rho_k) \geq \bigwedge_{i\in F} \tau_i(\lambda_i) > r - \epsilon,$$

$$\beta(\omega_m) \geq \bigwedge_{i\in F} \tau_i(\mu_i) > r - \epsilon.$$

Since $x_t\ q\ \rho_k$ and $y_s\ q\ \omega_m$, for each $j \in F$,

$$x_t\ q\ \pi_j^{-1}(\lambda_j),\ y_s\ q\ \pi_j^{-1}(\mu_j).$$

Hence

$$\lambda_j \in Q_{\tau_j}(x_t^j, r-\epsilon),\ \mu_j \in Q_{\tau_j}(y_s^j, r-\epsilon).$$

We only show that $\lambda_j \wedge \mu_j = \overline{0}$.

Suppose there exists $z^j \in X^j$ such that

$$(\lambda_j \wedge \mu_j)(z^j) > 0. \tag{5}$$

Then there exist $x^i \in X_i$ for all $i \in \Gamma - \{j\}$ and $z \in X$ with

$$\pi_i(z) = \begin{cases} x^i, & \text{if } i \in \Gamma - \{j\}, \\ z^j, & \text{if } i = j. \end{cases}$$

Since $x_t\ q\ \rho_k$ and $y_s\ q\ \omega_m$, we have

$$t > (\bigvee_{i\in F-\{j\}} (\overline{1} - \lambda_i)(\pi_i(x))) \vee (\overline{1} - \lambda_j)(x^j)$$

$$s > (\bigvee_{i\in F-\{j\}} (\overline{1} - \mu_i)(\pi_i(x)) \vee (\overline{1} - \mu_j)(y^j).$$

It implies

$$t > \bigvee_{i\in F-\{j\}} (\overline{1} - \lambda_i)(\pi_i(x)), \tag{6}$$

$$s > \bigvee_{i\in F-\{j\}} (\overline{1} - \mu_i)(\pi_i(x)). \tag{7}$$

Moreover, from (5),

$$((\overline{1} - \lambda_j) \vee (\overline{1} - \mu_j))(z^j) < 1. \tag{8}$$

Hence, by (6),(7) and (8), we have

$$(\overline{1} - \rho_k)(z) = (\bigvee_{i\in F-\{j\}} (\overline{1} - \lambda_i)(x^i)) \vee (\overline{1} - \lambda_j)(z^j) < 1$$

$$(\overline{1} - \omega_m)(z) = (\bigvee_{i\in F-\{j\}} (\overline{1} - \mu_i)(x^i) \vee (\overline{1} - \mu_j)(z^j) < 1.$$

Therefore

$$(\rho_k \wedge \omega_m)(z) > 0.$$

It is a contradiction for the equation (4). Hence (X_j, τ_j) is $(r - \epsilon)$-T_2. □

Example 2.19. Let $X = \{a\}$, $Y = \{b, c\}$ and $X \times Y = \{(a, b), (a, c)\}$ be sets. We define fuzzy topologies τ_1, τ_2 as follows:

$$\tau_1(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ 0, & \text{otherwise,} \end{cases}$$

$$\tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{3}{4}, & \text{if } \lambda = c_1, \\ \frac{1}{2}, & \text{if } \lambda = b_1, \\ 0, & \text{otherwise.} \end{cases}$$

Then (X, τ_1) is r-T_2-space for all $r \in I_0$ and (Y, τ_2) is r-T_2-space for all $0 < r \leq \frac{1}{2}$.

We obtain the product fuzzy topology $\tau_1 \times \tau_2 : I^{X\times Y} \to I$ as follows:

$$\tau_1 \otimes \tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda = \overline{0} \text{ or } \overline{1}, \\ \frac{3}{4}, & \text{if } \lambda = (a, c)_1, \\ \frac{1}{2}, & \text{if } \lambda = (a, b)_1, \\ 0, & \text{otherwise.} \end{cases}$$

Hence $(X \times Y, \tau_1 \otimes \tau_2)$ is r-T_2 for $0 < r \leq \frac{1}{2}$. □

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