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Spectral Methods for Solving Differential Equations Using Orthogonal Polynomials

By

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Abstract

The main objectives of this work is to introduce and develop new efficient spectral algorithms based on orthogonal polynomials for solving special types of linear or nonlinear ordinary differential equations subject to initial and homogeneous or nonhomogeneous boundary conditions. Also, we extend this work to include some types of partial differential equations.

In Chapter 1, we give a brief introduction to the spectral methods and their advantages. Also, we clarify the differences between the three most commonly used spectral methods, namely, the Galerkin, collocation, and tau methods. A brief account of orthogonal polynomials, their properties and expansion of functions in terms of them are given. General and useful needed properties of Legendre, Chebyshev; in particular first, third and fourth kinds; and Laguerre polynomials are also considered.

In Chapter 2, we developed the shifted third and fourth kinds Chebyshev operational matrices of derivatives for choices of Galerkin/tau basis functions. Therefore, effective numerical algorithms for solving Lane-Emdan type equations are introduced. We have ascertained that the introduced truncated shifted third and fourth Chebyshev expansions of a function u(x) converges uniformly to u(x).

In Chapter 3, we developed two efficient spectral-Galerkin methods for handling Linear hyperbolic telegraph equation with initial and boundary conditions. The first one is based on applying Legendre-Galerkin method (LGM); while for the second method, we constructed appropriate choice of basis functions and deduced their operational matrices. This enables one to apply the Galerkin method. We extended the uses of these operational matrices along with collocation method to solve the nonlinear telegraph equation. The Convergence and error analysis of the suggested approximate double expansions is carefully investigated.

In Chapter 4, We constructed appropriate basis functions for the Legendre-Laguerre Galerkin method (LLPGM) to solve the uniform Euler-Bernoulli beam model. These choices led to systems with specially structured matrices that can be efficiently inverted.

We end this thesis by Chapter 5. In this chapter, we presented two spectral algorithms for the solution of high odd-order differential equations based on Legendre-dual-Petrov-Galerkin method. The two presented algorithms are very reliable and efficient. The main advantage of our algorithms is that all of the resulting systems are band and this of course simplify the numerical computational efforts required to solve them.

The numerical results obtained in this thesis are tabulated whenever possible. These results show that the proposed algorithms are efficient and accurate. Comparisons with some other techniques presented by some other authors are made throughout the context whenever available. The programs used in this thesis are performed on the PC machine, with Intel(R) core(TM) i5 CPU 2.40 GHz, 4.00 GB of RAM, and the symbolic computation software Mathematica 10 has been also used.