Fayoum University
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Spectral Galerkin Method for Solving Even-Order Differential Equations by Using Third and Fourth Kinds Chebyshev Polynomials

A Thesis

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Abstract

Many physical phenomena can be described by differential equations or their integrated forms. In this work, we are concentrating on, how to find explicit approximations for functions defined by differential equations or by their integrated forms. In particular, we consider the high even-order elliptic differential equations of one and two space variables.

One of the main objectives of this thesis is to introduce and develop new efficient spectral-Galerkin algorithms for solving high even-order elliptic differential equations in one and two dimensions by using third and fourth kinds of Chebyshev polynomials. These algorithms are based on appropriate construction of base functions for the Galerkin formulation that lead to discrete systems with specially structured matrices that can be efficiently inverted. We also aim to introduce and construct efficient spectral-Galerkin algorithms for solving the integrated forms of high even-order elliptic differential equations. We demonstrate the advantage of using the integrated forms over the differentiated ones, by noting that the algebraic systems resulting from the integrated forms are sparse and therefore cheaper to solve than those obtained from the

differentiated forms.

In Chapter 1, we give a brief introduction to the spectral methods and their advantages over the standard finite-difference and finite-element methods. Also, we clarify the differences between the three most commonly used spectral methods, namely, the Galerkin, collocation, and tau methods. A brief account of orthogonal polynomials, their properties and expansion of functions in terms of them are given. Some general properties of third and fourth kinds of Chebyshev polynomials are also considered.

In Chapter 2, two new analytical formulae expressing explicitly the derivatives of Chebyshev polynomials of third and fourth kinds of any degree and for any order in terms of Chebyshev polynomials of third and fourth kinds themselves are given. Two other explicit formulae which express the third and fourth kinds Chebyshev expansion coefficients of a general-order derivative of an infinitely differentiable function in terms of their original expansion coefficients are also given. Two new reduction formulae for some terminating hypergeometric functions of unit argument are deduced.

In Chapter 3, we introduce and develop in detail two new algorithms based on Galerkin method for solving high even-order differential equations in one and two space variables subject to homogeneous and nonhomogeneous boundary conditions using third and fourth kinds of Chebyshev polynomials. Other two algorithms based on choosing compact combinations of shifted Chebyshev polynomials of third and fourth kinds as basis functions are also considered.

In Chapter 4, We present and implement in detail two algorithms for solving the integrated forms of high even-order differential equations in one and two space variables subject to homogeneous and nonhomogeneous boundary conditions using third and fourth kinds of Chebyshev polynomials.

The numerical results obtained in this thesis are tabulated whenever possible. These results show that the proposed algorithms are efficient and accurate. Comparisons with some other techniques presented by some authors are made throughout the context whenever available.

To the best of our knowledge, this is the first work which uses the third and fourth kinds Chebyshev-Galerkin methods for solving high even-order differential equations in one and two space variables and we do believe that all the theoretical results stated and proved in this thesis are completely new. The programs used in this thesis are performed on the PC machine, with Intel(R) Pentium(R) 4 CPU 3.00 GHz, 1.00 GB of RAM, and the symbolic computation software Mathematica 7 has been also used.