Fayoum University
Faculty of Science
Mathematics Department

On Some Fractional Inequalities

By
Mohammed Ragab Zaki Kenawy

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ABSTRACT

Historically, mathematical analysis has been the major and significant branch of mathematics for the last three centuries. Indeed, inequalities became the heart of mathematical analysis. Many great mathematicians have made significant contributions to many new developments of the subject, which led to the discovery of many new inequalities with proofs and useful applications in many fields of mathematical physics, pure and applied mathematics. Indeed, mathematical inequalities became an important branch of modern mathematics in twentieth century through the pioneering work entitled "Inequalities" by G. H. Hardy, J. E. Littlewood and G. Pólya [26], which was first published treatise in 1934. This unique publication represents a paradigm of precise logic, full of elegant inequalities with rigorous proofs and useful applications in mathematics.

Fractional calculus, the study of integration and differentiation of fractional order, is nowadays one of the most intensively developing areas of mathematical analysis ([32], [46] and [72]), including several definitions of fractional operators like Riemann-Liouville, Caputo, and Grünwald-Letnikov. Operators for fractional differentiation and integration have been used in various fields, such as signal processing, hydraulics of dams, temperature field problem in oil strata, diffusion problems and waves in liquids and gases ([9],

[16] and [68]. fractional inequalities using the fractional Caputo and Riemann-Liouville derivative were studied in [13] and [73]. This Ph.D. thesis is mainly devoted to a special kind of fractional inequalities, namely on some fractional inequalities. The general idea is to prove the inequalities in fractional form by using conformable fractional calculus and from our results we obtain the classical integral inequalities as special cases. This thesis consists of five chapters and organized as follows:

Chapter 1 is an introductory chapter and contains basic concepts, definitions and preliminary results of the conformable fractional calculus which are absolutely essential for completing the results and techniques used in subsequent chapters.

Chapter 2 is divided into four parts. The first part is devoted to the start of continuous Hardy- type inequality and some of its generalizations. The second part provides the fractional version of the classical Hardy- type inequality. The third part is devoted to some fractional generalization of Hardy-type inequalities known in the literature. The last part we derive some new fractional extensions of Hardy-type inequalities and the corresponding reverse relations are also obtained by using the conformable fractional calculus.

Chapter 3 is concerned with the fractional Copson and converse Copson-type inequalities. The results as special cases can be considered as extensions of the results due to Hardy, Copson, and converse Copson inequalities.

Chapter 4 deals with deriving a new fractional Leindler-type inequality by using conformable fractional calculus and use to establish new reversed Hardy-type inequalities.

Chapter 5 is devoted to prove some new fractional dynamic inequalities on time scales. The new inequalities contain the classical Hardy, Copson, Leindler and Bennett inequalities as special cases.