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Dynamic Inequalities on Time Scales

A Thesis Submitted in Partial Fulfillment
of
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In

Pure Mathematics
(Differential Equations)

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ABSTRACT

This PhD thesis is mainly devoted to a special kind of dynamic inequalities, namely dynamic integral inequalities and their extensions to an arbitrary time scale T (which uni...es discrete and continuous calculus). The general idea is to prove a result for an inequality where the domain of the unknown function is a so-called time scale T; which is an arbitrary closed subset of the real numbers R. The study of dynamic inequalities on time scales helps avoid proving results twice - once for di¤erential inequality and once again for di¤erence inequality. In this PhD thesis we also give some new contributions to the theory of oscillation and nonoscillation of second-order half linear dynamic equations.

This thesis consists of 6 chapters and organized as follows:

Chapter 1 is an introductory chapter and contains basic concepts, de...nitions and preliminary results of the calculus on a time scale T which are absolutely essential for completing the results and techniques used in subsequent chapters.

Chapter 2 is divided into three parts. The ...rst part is devoted to some new weighted Opial type inequalities on time scales. Some of the results established in this part provide extensions and improvements of those given in earlier works. The second part provides a slight improvement to some Opial-type inequalities known in the literature. The last part is devoted to several new Opial type Diamond-alpha dynamic inequalities.

Chapter 3 is concerned with the weighted Hardy type inequalities on an arbitrary time scale T. The results as special cases can be considered as extensions of the results due to Copson, Bliss, Flett, Bennett, Leindler, Chen and Yang.

Chapter 4 deals with deriving a new dynamic Picone-type inequality on time scales and use to establish new Hardy and Wirtinger-type inequalities with two di¤erent weight functions.

Chapter 5 is devoted to Copson type inequalities on an arbitrary time scale T.

Chapter 6 is devoted to the applications of Opial-type and Hardy-type inequalities in studying the oscillatory and nonoscillatory behavior of second order dynamic equations. We also apply some of these inequalities with weighted functions to investigate the distance between zeros of a solution and/or it's derivatives. More precisely, we are concerned about the time scale versions of Opial type inequalities, Hardy type inequalities, Wirtinger type inequalities and Bennett-Copson type integral inequalities. More exactly, we discuss some applications of the new obtained inequalities to the following questions:

- study the lower bounds of the distance between zeros of a solution and/or its derivatives for a second order sublinear dynamic equation with a damping term;
- establishing some new criteria for disconjugacy and disfocality;
- new Lyapunov-type Inequalities;
- investigation of some oscillatory properties of a half-linear second order dynamic equation on time scales using Hardy-type inequality.