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ON SOME BOUNDARY VALUE PROBLEMS FOR ELLIPTIC EQUATIONS IN NONSMOOTH REGIONS, IN SOBOLEV AND HÖLDER SPACES

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Abstract

In this dissertation, we study the regularity of the solution of the first boundary value problem for the equation

$$\Delta u + a(x,y)u_x + b(x,y)u_y + c(x,y)u = f(x,y) \qquad (١)$$

with mixed boundary conditions in some domain G with boundary Γ which is assumed to be decomposed as $\Gamma = N \cup D$, where $N \cap D = \emptyset$.

Given functions φ on D and Ψ on N. We wish to find a function u which is harmonic in G, which satisfies $u = \varphi$ on D, and $\frac{\partial u}{\partial n} = \Psi$ on N, where $\frac{\partial u}{\partial n}$ is the interior normal derivative on Γ.

In particular, we consider a domain which satisfy an additional condition which means, roughly that the sets D and N meets at an angle $w = \frac{\pi}{٢}$.

For such domain, we will show that if: the coefficients and the right hand side of (١) is in the Hölder space $C_\alpha(\bar{G})$, and that the boundary function φ is continuous on Γ and belong to $C_{٢+\alpha}(\Gamma \backslash \{٠\})$. Then the mixed boundary problem has a solution u for which $u(x,y) \in C_{٢+\alpha}(\bar{G})$.

Our process adapts the techniques of **(Azzam, ١٩٨٠)** and **(Volkov, ١٩٦٥)** in their study of Dirichlet and Neumann problems, respectively.

We propose – **Chapter ٢** – to give necessary and sufficient conditions for the solution of the first boundary value problem for the equation (١) with dirichlet and mixed boundary conditions to be as smooth as they would be in the absence of corners.

We investigate – **Chapter ٣** – to give numerical examples for the results approved in **(Azzam, ١٩٨٠)** and that is approved in previous chapter using Finite Element Method and Matlab Program.