



Name of Candidat : Heba Mohammed Mohammed Rezk

Degree M.Sc

Title of Thesis

STUDY OF SMOOTH PROXIMITY AND SMOOTH UNIFORMITY  
STRUCTURES IN  $P^*(L)$ -FUZZY SUBSETS

Supervisors:

1- Prof. Dr. Kamal Ahmed Dib

2- Dr. Salama Mohamed El-deeb

Approval /9/2015

Department Mathematics

### ABSTRACT

In this thesis, the extension of the proximity on the fuzzy space  $L^Y$  to a proximity on the fuzzy space  $L^X$ ;  $Y \subset X$  and the restriction of the fuzzy proximity on  $L^X$  to a fuzzy proximity on  $L^Y$  are defined and studied, together with the relations between their closure operators. The induced basic proximity on  $P(\Lambda)^X$  for each given basic proximity on  $L^X$  is also defined where  $L \in \mathcal{L}(\Lambda)$  and  $\mathcal{L}(\Lambda)$  is the family of all complete lattices defined on the nonempty set  $\Lambda$  and fundamental relations between their closure operators are obtained. Furthermore, the  $(L, M)$ -fuzzy proximity is approached. Also, the restriction and the extension of  $(L, M)$ -fuzzy proximities and the induced  $(L, M)$ -fuzzy proximity on  $P^*(L)^X$  and the induced  $(L, M)$ -fuzzy proximity on  $P(\Lambda)^X$  corresponding to each  $(L, M)$ -fuzzy proximity on  $L^X$ ;  $L \in \mathcal{L}(\Lambda)$  are clearly undertaken. Moreover, we have showed that the family of the categories of proximities on  $L^X$ ;  $L \in \mathcal{L}(\Lambda)$  is embedded in the category of proximities on  $P(\Lambda)^X$ . Here, the extension of the L-fuzzy uniformity on the fuzzy space  $L^Y$  to a L-fuzzy uniformity on the fuzzy space  $L^X$ ;  $Y \subset X$ , and the restriction of the L-fuzzy uniformity on  $L^X$  to a L-fuzzy uniformity on  $L^Y$  have been taken into account. In each case the relation between their interior operators is obtained. Also, we have defined the induced  $P^*(L)$ -quasi uniformity on  $P^*(L)^X$  for each given L-quasi uniformity on  $L^X$  and a fundamental relation between their interior operators is acquired. Consequently, we have carried out more studies on the induced  $(P^*(L), M)$ -fuzzy quasi-uniformity on  $P^*(L)^X$  for each given  $(L, M)$ -fuzzy quasi-uniformity on  $L^X$ . Moreover, the relation between there interior operators were approached and procured.