Fayoum University
Faculty of Sciences

ON DIFFERENTIAL EQUATIONS DESCRIBING SURFACESOF CONSTANT CURVATURE, INTEGRABILITYAND SOLITONS

By

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Abstract

In the last twenty years, many mathematical methods, for solving as well as obtaining special types (travelling-solitary-non travelling) of solutions of non-linear ordinary as well as partial differential equations, have been introduced. Some relationships between local differential geometry of curves (as well as surfaces) and integrability of evolutionary partial differential equations have been traced and studied

In our thesis, we will concentrate on the relationships between the geometry of surfaces of constant curvature (pseudospherical surfaces) and the concepts of integrability of evolutionary partial differential equations (solitons) as well as their other well known characteristic properties .

The thesis is consisted of five Chapters:

Firstly in chapter 1, we present an introduction to the Local Theory of Surfaces, Differentiable Manifolds, Differential forms and their operations, Jet bundles and Partial differential equations on manifolds, Bäcklund maps, Exterior differential systems and differential ideals, Soliton equations and Geometrization of 2-dimension soliton equations, Relation between surfaces in R^3 and integrable systems, finally Geometry of n-pseudospherical surfaces in R^{2n-1} and its relation with integrable systems in higher dimensions.

In chapter 2, we extended the evolution equations with two independent variables which are related to pseudospherical surfaces in R^3, to evolution equations with more than two independent variables and we concentrated on studying equations of the type

$$u_{xt} = \psi\left(u, u_x, \dots\dots.., \frac{\partial^k u}{\partial\, x^k}, u_y, \dots\dots\dots, \frac{\partial^{k'} u}{\partial\, y^{k'}}\right).$$

And we successfully get some new features and results on properties of these equations.

In the work of chapter 3, the study of evolution equations with two independent variables which are related to pseudospherical surfaces in R^3, is extended to evolution equations with more than two independent variables. westudied Equations of the type

$$u_t = \psi\left(u, u_x, \dots\dots.., \frac{\partial^k u}{\partial\, x^k}, u_y, \dots\dots\dots, \frac{\partial^{k'} u}{\partial\, y^{k'}}\right).$$

And we conclude some features and good results on properties of these equations.

In chapter 4, we concentrated on the evolution equations with two or more spatial variables, which describe pseudospherical planes in higher dimensions. Consequently we obtain the necessary and sufficient conditions for equations of type

$$u_{tt} = \psi\left(u, u_x, \dots\dots, \frac{\partial^k u}{\partial\, x^k}, u_y, \dots\dots, \frac{\partial^{k'} u}{\partial\, y^{k'}}, u_t\right)$$

to describe a 3-dimensionalpseudospherical plane of R^5.

Finally, in chapter 5,we will generalize Bäcklund transformations and conservation laws based on geometrical properties of evolution equations with more than two independent variables that describe pseudospherical surfaces, specially equations of types

$$u_{xt} = \psi\left(u, u_x, \dots\dots.., \frac{\partial^k u}{\partial\, x^k}, u_y, \dots\dots\dots, \frac{\partial^{k'} u}{\partial\, y^{k'}}\right)$$

$$u_t = \psi\left(u, u_x, \dots\dots.., \frac{\partial^k u}{\partial\, x^k}, u_y, \dots\dots\dots, \frac{\partial^{k'} u}{\partial\, y^{k'}}\right)$$

$$u_{tt} = \psi\left(u, u_x, \dots\dots, \frac{\partial^k u}{\partial\, x^k}, u_y, \dots\dots, \frac{\partial^{k'} u}{\partial\, y^{k'}}, u_t\right)$$

This thesis including new and satisfied results appeared in chapters 2, 3,4 and 5 which are published in international journals.