

# Abstract

The aim of this thesis is to relate the study of evolutions of curves and surfaces and integrability of evolutionary partial differential equations

The thesis is divided into five chapters:

## The first chapter

It is considered as an introductory one. In it, we discuss the differential geometry of curves in  $R^3$ , differential geometry of curves in  $R^4$ , characteristic properties of soliton equations, Some examples of solitons as models for integrable systems, differentiable manifolds, differential forms and their operations Jet bundles and exterior calculus.

## Chapter two

In this chapter we introduce a relationship between curve evolution and soliton equations in Minkowski 3-space in case of space-like curve with space-like principal normal.

## Chapter three

In this chapter we introduce a relationship between curve evolution and soliton equations in Euclidean 4-space.

## Chapter four

In this chapter we introduce a relationship between curve evolution and suggested integrable equations in Minkowski 4-space.

## Finally, chapter five

In this chapter we discuss the geometry of surfaces in  $R^3$ , relation between surfaces in  $R^3$  and soliton equations as well as in  $R^{2n-1}$  and higher dimension solitons.

Some new results that appeared in chapter two, three and four are formulated in research papers submitted for publication. with the titles "Moving Space Curve in Minkowski 3-Space and Soliton Equations", "Connections between moving curves and soliton equations in Euclidean 4-space" and "Moving Space Curve in Minkowski 4-Space and Soliton Equations".