Faculty of Science
Mathematics Department

New Prospective of Some Models in Quantum Information

A THESIS
Submitted to the Faculty of Science, Al-Fayoum University
In Partial Fulfillment for the Degree of Master
In Mathematics (Applied Mathematics)

BY

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1443 - 2021

In this thesis, we have studied some applications of some quantum measures of information on some different open quantum systems.

In chapter 2, we have studied the system of a moving two-level atom interacting with a single mode dissipative cavity field through multi-photon process in the presence of the Kerr medium. The atom and the field are prepared in the excited and the coherent state, respectively. We have studied the influence of the damping rates of both atom and field on the time evolution of the atomic inversion, variance squeezing and the purity function. We found that, By increasing the value of the field decay rate, the amplitudes of the revival intervals decrease which means that the interaction becomes weak. For the squeezing phenomena, we study the variances V (σˆx) and V (σˆy) against ⟨σˆz⟩, the variances V (σˆy) and V (σˆz) against ⟨σˆx⟩ and the variances V (σˆx) and V (σˆz) against ⟨σˆy⟩. We found that, in general, squeezing is negatively affected by increasing the value of damping parameter, not only the amount of squeezing in the squeezed quadratures decreases but also the maximum value of squeezing decreases in the considered different cases. We have studied the effect of the atomic decay rate on the atomic inversion and the variances squeezing, and we found that the effect is not clear. Finally, we have studied the effect of the field damping parameter Γ on the purity function to measure the degree of

entanglement between the system components, it's observed that the entanglement increases by increasing the value of Γ.

In chapter 3, we have studied the system of a moving two-level atom prepared initially in the excited state and interacting with a single mode cavity field through multi-photon process in the presence of the Kerr medium and Stark shift, taking into account the damping of the atom. The cavity field is prepared initially in the
coherent state. We have studied the effect of the Stark shift on some non-classical properties for the considered system such as the atomic inversion in the resonance and the off-resonance case, variance and entropy squeezing in the atomic components, the squeezing in the field quadratures, the degree of entanglement between
the system components by measuring the purity function and Mandel Q-parameter.
On the study of the atomic inversion, we found that, by increasing the value of the Stark parameter, the atom lost its stored energy in both cases, but the emission rate is greater in the off-resonance case. Also, we have studied the variance, entropy and normal squeezing, it's observed that the variance squeezing in the two components is negatively affected where the amount and the number of squeezing intervals decreased by increasing the value of the Stark parameter. We observed that the

effect of the shift on the entropy squeezing is different, in Sx component, the entropy squeezing occurs at certain intervals for short time through the considered interval with a great squeezing maximum value, but this value decreased as the time developed,
and had no effect on the entropy squeezing in the component Sy. By increasing the value of the Stark parameter, the amount and the maximum value decrease in the field quadratures. Using the purity function and Mandel Q-parameter to measure the degree of entanglement, we found that the entanglement increases by increasing the value of the Stark parameter.

In chapter 4, we have studied the system of two interacting two-level atoms, both interacting with one mode cavity field, in the presence of in the presence of the Kerr medium and Stark shift, taking into account the damping of the atoms.
We have studied the effect of the Stark shift on some non-classical properties for the considered system such as the atomic inversion, squeezing phenomenon and von Nuemann entropy. We observed that, at a small value of the Stark parameter, the atoms exhibit excitation and by increasing its value, the excitation decreases.
Also, we observe that when the Stark parameter takes values greater than 1, the atoms are excited again. It has been noted that, there exist more

squeezing in both components if the Stark parameter takes values greater or smaller than 1, but if the Stark parameter equal to 1, the squeezing phenomenon is negatively affected in both directions. In the end of this chapter, we closed our study by illustration the influence of the Stark parameter on the degree of entanglement using the von Nuemann entropy. It is observed that, the system displays strong entanglement by increasing the value of the Stark shift parameter.

In chapter 5, the interaction between two two-level atoms and two-mode electromagnetic field through two-photon process is studied in the presence of the Kerr medium, taking into account the coupling between the atoms and the field to be time-dependent.
We consider the atom and the field are suffering decay rates. We have studied the effect of damping rates of both atoms and field on the atomic inversion of a single atom, atomic quadrature squeezing, normal squeezing and sub-Poissonian distribution. We noted that by increasing the field damping parameter, the atomic inversion shifted down as the time developed, also the amplitude of the revival interval reduced. By taking various values for the atomic damping parameter, we noted the oscillation base line shifted downwards with the same revival amplitude. We

studied the variance and the entropy squeezing of the atomic components σx and σy for the first atom. We noted that the effect of the damping parameter of the field on the entropy squeezing is negative, and the atomic damping parameter almost has no effect on squeezing in entropy. The variance displays no squeezing neither in σx nor in σy for the considered cases. Also we considered the quadrature squeezing for the first mode and we observed that the damping parameter of the field has negative effect on squeezing in E1X component, we noted that the atomic damping parameter also has no effect on the quadrature squeezing.

Finally, we discussed the sub-Poissonian distribution, we observed that the system displays sub-Poissonian distribution if the damping parameter of the field is less than 0.00022λ the higher the value of the damping parameter of the field, the more classic characteristics the system will display. And we noted that the atomic damping parameter has random effect on the correlation function.