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**Derivation of Approximate Formula for the Coulomb  
Interaction between Two Deformed Nuclei**

**THESIS**

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## Abstract.

It is evident that the Coulomb interaction between two nuclei is an important contribution of the nucleus-nucleus interaction. It is a fundamental quantity in many problems of nuclear physics. It enters in calculating most of the physical quantities like fusion and fission cross-sections. Moreover it is needed to describe the formation and decay of new elements and is very important in the field of super-heavy nuclei. For these reasons, the present thesis concerns with testing the validity of approximate expressions of the Coulomb interaction for spherical-deformed and deformed-deformed pairs of interacting nuclei. These expressions which are simple and free from complicated integrals are used frequently in recent works.

In the first chapter, we derive in details a very simple formula; containing a small number of terms and free from complicated integrals, for the Coulomb potential between two deformed nuclei. It corresponds to the uniformly charged bodies with sharp edges in the limit where the separation distance of the center of mass of the two nuclei is larger than the sum of the two nuclear radii. This formula was first introduced by C. Y. Wong, about 37 years ago. We develop this formula to include hexadecupole deformation parameter. Also another formula for calculating the HI Coulomb potential is introduced that is derived on the basis of multipole expansion for the density of the deformed nucleus.

In chapter two we compare the Coulomb potential for deformed-spherical pair of nuclei derived from the more accurate density multipole expansion method and that obtained from analytical expression derived by taking the limit  $r > R_p + R_T$  of the Coulomb potential calculated assuming two uniformly charged bodies with sharp surfaces. We considered  $U^{238}$  as the target nucleus and  $He^4$ ,  $O^{16}$ ,  $Ca^{40}$ ,  $Zr^{91}$ ,  $Pb^{208}$  as different spherical projectiles. We found that the derived formula and Wong's formula are correctly representing the exact Coulomb potential for spherical-deformed interacting pair at the surface and tail regions with a too small error less than 0.2%. It is also found that our derived formula is better than Wong's formula in the absence of higher order deformations. When positive value of hexadecupole deformation is added to the deformed nucleus, the error in both formulas increases compared to the case of vanishing value of  $\beta_4$ . In this case our modified formula is still better than Wong's formula. The error, in this case, can be greater than 4 MeV for some projectile nuclei. Only when negative hexadecupole deformations are considered, Wong's formula becomes more accurate than our modified formula. Besides, the addition of negative sign hexadecupole deformation produces oscillations in the variation of

percentage error with the separation distance between the interacting pair of nuclei.

Due to the importance of the Coulomb potential in many problems of physics, we examine the accuracy of calculating the Coulomb potential for deformed pair of nuclei from simple formula free from integrals. In chapter three we introduce the azimuthal angle dependence in our modified formula and examine its accuracy by comparing its numerical results with more exact calculations. We examine the accuracy of the modified formula in the absence and presence of hexadecupole deformation. It was found that the strong azimuthal angle dependence occurs at orientations  $(45^\circ, 45^\circ)$  for both the cases of presence and absence of the hexadecupole deformation parameter. While the weak azimuthal angle dependence is at orientation  $(90^\circ, 90^\circ)$ . It was found also that the addition of the hexadecupole deformation parameter increases the maximum  $\varphi$ -variation in the exact formula while it is nearly the same in our modified formula. The orientations  $(90^\circ, 90^\circ)$  show the largest error among all orientations.