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APPLICATIONS OF DIFFERENTIAL SUBORDINATION ON CERTAIN SUBCLASSES OF $p-$ VALENT MEROMORPHIC FUNCTIONS

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Abstract. This paper gives some subordination and convolution properties of certain subclasses of $p-$valent meromorphic functions which are defined by using the linear operator $Q^{p,\mu}_{\alpha,\beta,\gamma}$.

1. Introduction

For any integer $m > -p$, let $\Sigma_{p,m}$ denote the class of all meromorphic functions f of the form:

$$f(z) = z^{-p} + \sum_{k=m}^{\infty} a_k z^k \qquad (p \in \mathbb{N} = \{1, 2, ...\}), \tag{1.1}$$

which are analytic and $p-$valent in the punctured disc $U^* = \{z \in \mathbb{C} : 0 < |z| < 1\} = U\backslash\{0\}$. For convenience, we write $\Sigma_{p,-p+1} = \Sigma_p$. If f and g are analytic in U, we say that f is subordinate to g, written symbolically as, $f \prec g$ or $f(z) \prec g(z)$, if there exists a Schwarz function w, which (by definition) is analytic in U with $w(0) = 0$ and $|w(z)| < 1\,(z \in U)$ such that $f(z) = g(w(z))\,(z \in U)$. In particular, if the function g is univalent in U, we have the equivalence (see for example [5]):

$$f(z) \prec g(z) \Leftrightarrow f(0) = g(0) \text{ and } f(U) \subset g(U).$$

For functions $f \in \Sigma_{p,m}$ given by (1.1), and $g \in \Sigma_{p,m}$ defined by

$$g(z) = z^{-p} + \sum_{k=m}^{\infty} b_k z^k \qquad (m > -p, p \in \mathbb{N}), \tag{1.2}$$

then the Hadamard product (or convolution) of f and g is given by

$$(f * g) = z^{-p} + \sum_{k=m}^{\infty} a_k b_k z^k = (g * f)(z) \quad (m > -p, p \in \mathbb{N}). \tag{1.3}$$

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