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Pseudospherical Planes and Evolution Equations in Higher Dimensions II

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Abstract : *In this paper, the study of evolution equations with two independent variables which are related to pseudospherical surfaces in* R^3*, is extended to evolution equations with more than two independent variables. Equations of the type*

$$u_t = \psi(u, u_x, \dots\dots, \frac{\partial^k u}{\partial x^k}, u_y, \dots\dots\dots, \frac{\partial^{k'} u}{\partial y^{k'}})$$

are studied and characterized. Some features and results on properties of these equations are given via this study.

Keywords: *Evolution equations, Pseudospherical surfaces, Riemannian manifold, Solitons and* differential equations.

I. Introduction

As well known now, he study of non-linear evolution equations has been closely related to the study of soliton phenomena. The traced properties of 2-dimensional (one spatial variable and the time variable) soliton equations: such as having Bäcklund transformation $[2-10, 14, 21, 26]$being solvable by the inverse scattering method $[1, 7]$, having infinite number of conservation laws $[7, 22]$, satisfying the painleve $[9]$ and describing Pseudospherical surfaces$[5,12,15-21]$; have been under extensivem studies till now. The interrelations between these properties also are well established, $[3, 11, 14, 19]$. However, in higher dimensions, the studies of solitons $[13, 23-26]$, accordingly of the non- linear evolution equations with two or more spatial variables are less developed and remain one of the interesting present and future field of studies .In fact the studies in higher dimensions take the form of studying each single traced property in itself, then searching for relations to other properties, $[7, 9, 19, 22]$. From geometric point of view, the properties of describing Pseudospherical plane as well as Bäcklund transformations interest us more. In a previous study, $[12]$, EI- Sabbagh et al showed what necessary and sufficient conditions for the evolution equations

$$u_{xt} = \psi(u, u_x, u_{xx}, \dots\dots, \frac{\partial^k u}{\partial x^k}, u_y, u_{yy}, \dots\dots\dots, \frac{\partial^{k'} u}{\partial y^{k'}})$$

To describe a two-parameter3-dimensional Pseudospherical plane (P.S.P) in R^5, (i.e 3-dim plane with constant sectional curvature -1 isometrically imbedded in R^5).

In the present paper, we carry a similar study for other types of evolution equations with two or more spatial variables which have the form

$$u_t = \psi(u, u_x, u_{xx}, \dots\dots, \frac{\partial^k u}{\partial x^k}, u_y, u_{yy}, \dots\dots\dots, \frac{\partial^{k'} u}{\partial y^{k'}}) \qquad (1)$$

In section II, we give necessary definitions and notations to lay the ground for the main study in section III, where conditions on equations (1) to describe(η, ξ) 3-dim. P.S.P in R^5 are given. Some features and comments are also given.

II. Basic Notations And Definitons

For the present paper to be self contained, we give a simple review of needed geometry. Let M be an n-dimensional Riemannian manifold with constant negative sectional curvature K isometrically imbedded in the (2n-1) Eculidian spaceR^{2n-1}.The dimension (2n-1) is the least possible dimension so that an isometric imbedding can exist[7.19]. Let $e_1, e_2, \dots, e_{2n-1}$ be an orthonormal frame on an open set of R^{2n-1}so that at points of M,$e_1, e_2, \dots, e_n$are tangents toM.

Let $\omega_A$be the dual orthonormal coframe and consider ω_{AB}defined by

$$de_A = \sum_B \omega_{AB}\, e_B$$

Thus for R^{2n-1} we have